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GENERATION OF ANNUAL RAINFALL DATA FOR AUSTRALIAN STATIONS

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1. INTRODUCTION

Even though generated annual rainfall data have little direct application, the modelling of annual rainfall data serves two purposes. Firstly, it enables the understanding of the stochastic nature of the annual rainfall data and its implications for long periods of low and high rainfall. This understanding is necessary to manage water supply systems during low rainfall periods. Secondly, any stochastic model should be able to maintain its statistical characteristics at different time scales and a good annual rainfall model allows one to disaggregate the generated annual rainfall data into monthly data. In this case, the annual data becomes the input to various disaggregation schemes.

The work reported here forms part of Project 5.2 - National Data Bank of Stochastic Climate and Streamflow Models - of the Climate Variability Program. The review (Srikanthan and McMahon, 2000) carried out as part of the project recommended an autoregressive time series model or the Hidden State Markov (HSM) model to generate annual rainfall data. In this report, these two models are applied to 44 stations located in various parts of Australia. The performance of the models is assessed using a number of basic and other statistics. Based on this, recommendations are made as to the appropriate model for the generation of annual data.

The effect of parameter uncertainty is not considered in this report, as the aim of the report is to compare the performance of different models. It is assumed that the parameters of the models estimated from the historic data are the true values and the sampling variability is ignored. Once an appropriate model is chosen, the parameter uncertainty will be incorporated later.

2. RAINFALL DATA

Forty four rainfall stations with long records were selected. The locations of the selected rainfall stations are shown in Figure 1 and the details are given in Table 1. Because the strength of persistence in the annual data changed with the starting month of forming the annual totals, the starting months (Table 1) were obtained by calibrating the HSM model for annual data formed by different starting months. For more details on this, the reader is referred to Srikanthan et al (2002). The parameters of the annual data are given in Table 2.

The data length varies from 69 to 143 years long. In order to have at least one rainfall station in each of the CRCCH focus catchments, the station Tongala (80056) was included although it has only 69 years of data. The mean annual rainfall varies from 164 to 1550 mm. The coefficient of variation (C_v) is in the range 0.19 to 0.62. The large values of C_v are associated with low rainfall. All the stations have skewness significantly different from zero at 5% level except for two stations (shown in bold). Only four stations (shown in bold) have lag one autocorrelation coefficient significantly different from zero.

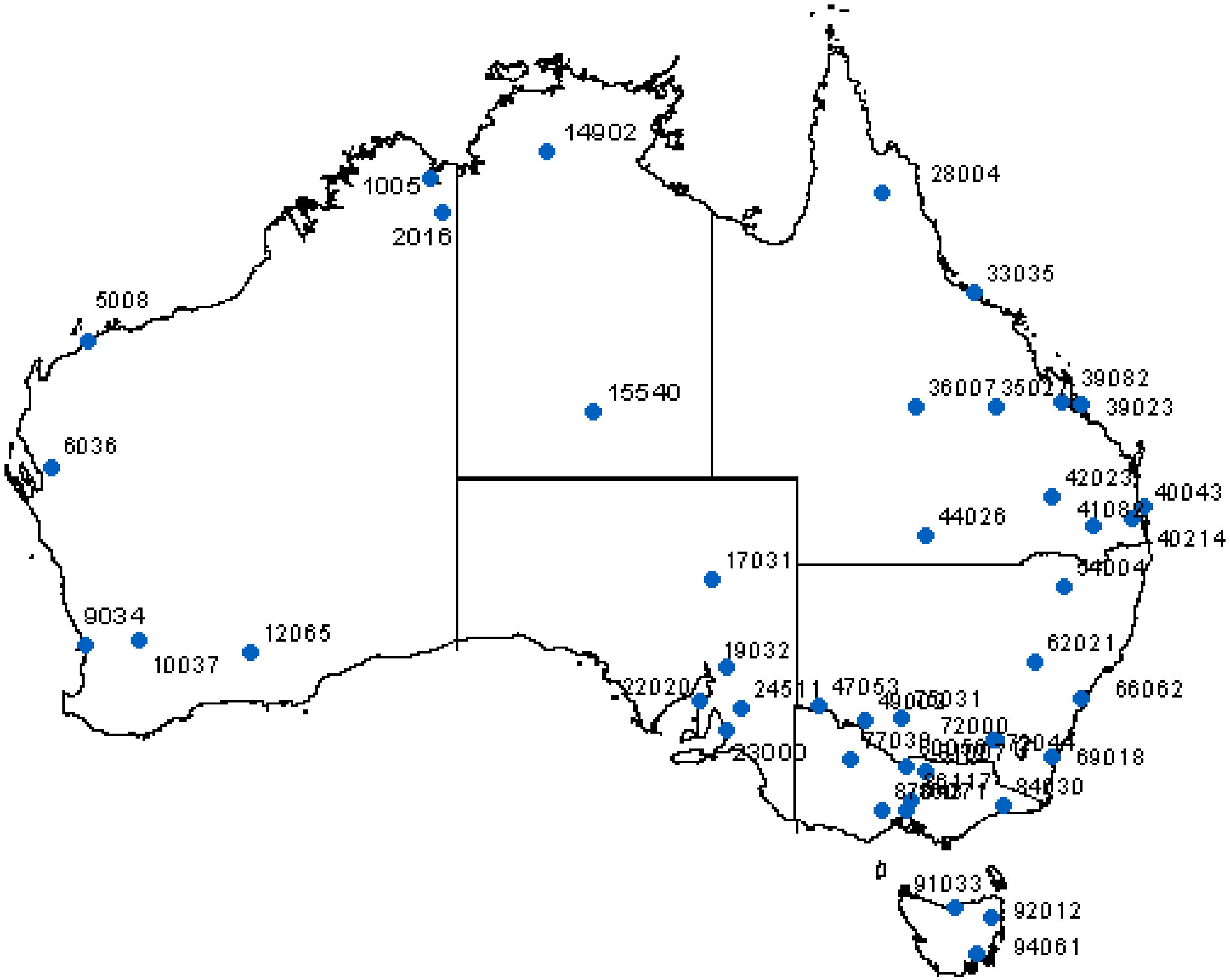

Figure 1. The locations of the selected rainfall stations.

Table 1. The details of the rainfall stations used in the study. The length of record is in years.

No	Number	Name	Latitude	Longitude	Length	Start month
1	5008	Mardie	-21.19	115.98	108	4
2	6036	Meedo	-25.66	114.62	94	4
3	9034	Perth	-34.93	138.58	115	6
4	10037	Cuttening	-31.73	117.76	96	6
5	12065	Norseman Post Office	-32.20	121.78	102	9
6	17031	Marree	-29.65	138.06	113	6
7	19032	Orroroo	-32.74	138.61	118	6
8	22020	Wallaroo	-33.93	137.63	135	12
9	23000	Adelaide	-31.95	115.84	139	6
10	24511	Eudunda	-34.18	139.09	118	8
11	28004	Palmerville	-16.00	144.08	109	9
12	33035	Kalamia Estate	-19.54	147.41	112	4
13	35027	Emerald Post Office	-23.53	148.16	108	8
14	36007	Barcaldine Post Office	-23.55	145.29	112	11
15	39023	Cape Capricorn Lighthouse	-23.48	151.23	87	7
16	39082	Rockhampton Post Office	-23.40	150.50	96	7
17	40043	Cape Moreton Lighthouse	-27.03	153.47	129	7
18	40214	Brisbane	-27.48	153.03	133	7
19	41082	Pittsworth Post Office	-27.71	151.63	112	8
20	42023	Miles Post Office	-26.66	150.18	114	2
21	44026	Cunnamulla Post Office	-28.07	145.68	120	8
22	47053	Wentworth Post Office	-34.11	141.91	132	1
23	49002	Balranald RSL	-34.64	143.56	121	11
24	54004	Bingara Post Office	-29.87	150.57	113	8
25	62021	Mudgee (George Street)	-32.59	149.58	122	7
26	66062	Sydney	-33.86	151.20	140	8
27	69018	Moruya Heads Pilot Station	-35.91	150.15	123	5
28	72000	Adelong	-35.31	148.06	115	8
29	72044	Tumut	-35.30	148.22	113	8
30	75031	Hay Miller Street	-34.52	144.85	119	1
31	77030	Narraport	-36.01	143.03	112	8
32	80056	Tongala	-36.25	144.95	69	8
33	81007	Caniambo	-36.46	145.66	95	8
34	84030	Orbost	-37.63	148.46	115	8
35	86071	Melbourne	-37.81	144.97	143	6
36	86117	Toorourrong Reservoir	-37.48	145.15	106	9
37	87043	Meredith (Darra)	-37.82	144.15	124	6
38	91033	Frankford (Rossville)	-41.32	146.73	106	12
39	92012	Fingal (Forestry Legge Street)	-41.64	147.97	110	8
40	94061	Sandford (Maydena)	-42.93	147.52	111	12
41	1005	Wyndham Port	-15.46	128.10	79	3
42	2016	Lissadell	-16.67	128.57	105	1
43	14902	Katherine Council	-14.46	132.26	111	4
44	15540	Alice Springs Post Office	-23.71	133.87	112	2

Table 2. The parameters of the annual data.

Number	Name	Mean (mm)	C_v	Skew	Lag one autocorrel
5008	Mardie	276	0.62	0.79	-0.11
6036	Meedo	216	0.42	0.88	**0.24**
9034	Perth	868	0.18	0.72	-0.02
10037	Cuttening	312	0.24	0.51	0.06
12065	Norseman Post Office	287	0.32	1.24	0.01
17031	Marree	164	0.51	1.53	0.06
19032	Orroroo	341	0.32	1.44	0.02
22020	Wallaroo	360	0.23	**0.29**	-0.05
23000	Adelaide	530	0.20	0.63	0.09
24511	Eudunda	446	0.25	1.21	0.07
28004	Palmerville	1034	0.31	0.62	0.07
33035	Kalamia Estate	1085	0.46	0.85	0.06
35027	Emerald Post Office	642	0.33	0.75	**0.22**
36007	Barcaldine Post Office	496	0.43	0.88	0.17
39023	Cape Capricorn Lighthouse	801	0.33	0.69	0.09
39082	Rockhampton Post Office	946	0.38	0.97	0.09
40043	Cape Moreton Lighthouse	1550	0.26	0.95	0.00
40214	Brisbane	1154	0.32	1.01	0.05
41082	Pittsworth Post Office	703	0.23	0.86	-0.12
42023	Miles Post Office	661	0.32	0.51	-0.12
44026	Cunnamulla Post Office	374	0.42	1.25	0.10
47053	Wentworth Post Office	288	0.35	1.17	0.16
49002	Balranald RSL	322	0.34	1.00	0.18
54004	Bingara Post Office	745	0.27	0.54	0.04
62021	Mudgee (George Street)	670	0.26	0.79	0.09
66062	Sydney	1226	0.27	0.73	0.16
69018	Moruya Heads Pilot Station	972	0.32	0.71	**0.27**
72000	Adelong	795	0.24	0.72	0.08
72044	Tumut	822	0.23	0.67	0.05
75031	Hay Miller Street	369	0.34	1.22	0.16
77030	Narraport	354	0.29	0.48	0.17
80056	Tongala	443	0.28	1.56	0.23
81007	Caniambo	524	0.27	0.99	0.06
84030	Orbost	855	0.24	0.74	-0.16
86071	Melbourne	657	0.19	0.45	0.10
86117	Toorourrong Reservoir	804	0.20	**0.43**	0.03
87043	Meredith (Darra)	685	0.19	0.36	0.09
91033	Frankford (Rossville)	1069	0.22	0.52	0.01
92012	Fingal (Forestry Legge Street)	611	0.27	1.29	0.11
94061	Sandford (Maydena)	578	0.22	0.63	0.10
1005	Wyndham Port	695	0.32	0.78	0.06
2016	Lissadell	616	0.30	0.89	0.14
14902	Katherine Council	974	0.28	0.61	0.04
15540	Alice Springs Post Office	280	0.50	1.48	**0.28**

3. MODEL EVALUATION

The following parameters are used to evaluate the data generation model for annual rainfall:

- Mean ($\bar{x}$)
- Standard deviation (s)
- Coefficient of skewness (g)
- Lag one autocorrelation coefficient (r)
- Maximum rainfall
- Minimum rainfall
- Adjusted range (R)
- 2, 3, 5, 7 and 10-year low rainfall totals

For convenience, the last four items in the above list are standardised by dividing by the historical mean annual rainfall. The first four items are usually estimated from the following expressions.

$$\bar{x} = \frac{1}{n}\sum_{t=1}^{n} x_t \tag{1}$$

$$s = \sqrt{\frac{1}{(n-1)}\sum_{t=1}^{n}(x_t - \bar{x})^2} \tag{2}$$

$$g = \frac{n}{(n-1)(n-2)s^3}\sum_{t=1}^{n}(x_t - \bar{x})^3 \tag{3}$$

$$r = \frac{1}{(n-1)s^2}\sum_{t=1}^{n-1}(x_{t+1} - \bar{x})(x_t - \bar{x}) \tag{4}$$

In the above equations, x_t represents the annual rainfall for year t and n the number of years of data. Stedinger and Taylor (1982) suggest the use of population mean (i. e. the mean estimated from the historic data) rather than the generated mean to overcome the bias in the standard deviation, skewness and correlation. The adjusted range (R) is obtained from

$$R = \max\{D_k\} - \min\{D_k\} \qquad k = 1, 2, \dots, n \tag{5}$$

where $D_k = \sum_{t=1}^{k}(x_t - \bar{x})$

The maximum and minimum rainfalls, adjusted range and the low rainfall sums are standardised by dividing by the observed annual mean.

4. TIME SERIES MODEL

Unlike streamflow data, rainfall data are less variable with little correlation between the values in successive years. Hence an AR(1) or a random model is adequate for most cases. The AR(1) model is of the form

$$X_t = rX_{t-1} + (1 - r^2)^{½} \eta_t \quad (6)$$

where X_t standardised rainfall in year t
η_t normally distributed random component with zero mean and unit variance
r lag one autocorrelation coefficient

The annual rainfall amount is obtained from

$$x_t = \bar{x} + s X_t \quad (7)$$

If the annual data are skewed, the skewness in the data can be modelled through the Wilson-Hilferty transformation.

$$\varepsilon_t = \frac{2}{g_\varepsilon}\left\{\left(1 + \frac{g_\varepsilon \eta_t}{6} - \frac{g_\varepsilon^2}{36}\right)^3 - 1\right\} \quad (8)$$

where g_ε is the skewness of ε_t which is related to the skewness (g) of annual data through

$$g_\varepsilon = \frac{(1 - r^3)}{(1 - r^2)^{3/2}} g \quad (9)$$

Since only two stations have skewness which is not significantly different from zero, the Wilson-Hilferty transformation is used for all the stations. Most of the stations have small lag one autocorrelation coefficients. An AR(1) model is used for the stations with lag one autocorrelation coefficient greater than 0.05 and a random model for the other stations.

4.1 Autoregressive model

An autoregressive model of order p, denoted by AR(p), is defined as

$$z_t = \phi_1 z_{t-1} + \ldots + \phi_p z_{t-p} + a_t \quad (10)$$

where z_t is a zero mean AR process, $\phi_1, \phi_2, \ldots, \phi_p$ the AR parameters and a_t the noise term with zero mean and variance σ_a^2. The parameters of the model are the mean of z_t, the variance of the random variable and the coefficients $\phi_1, \ldots, \phi_p$. A total of (p+2) parameters must be evaluated from the data.

The variance of the AR(p) process is

$$\sigma_z^2 = \frac{\sigma_a^2}{1 - \rho_1\phi_1 - \rho_2\phi_2 - \ldots - \rho_p\phi_p} \quad (11)$$

where ρ_k is the autocorrelation coefficient at lag k.

The autocorrelation function (acf) of the AR(p) process is given by

$$\rho_k = \phi_1\rho_{k-1} + \phi_2\rho_{k-2} + \dots + \phi_p \rho_{k-p} \quad \text{or} \quad \phi(B)\, \rho_k = 0 \tag{12}$$

where $\phi(B) = 1 - \phi_1 B - \dots - \phi_p B^p$ and B is the backward shift operator operating on k.

If a pair of roots of the characteristic equation $\phi(B) = 0$ is complex, the acf follows a damped sine wave and the AR process displays pseudo periodic behaviour (Box and Jenkins, 1976).

The properties of the AR(1) model are described above and is not repeated here. The properties of an AR(2) model are described below.

An AR(2) model takes the form

$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + a_t \tag{13}$$

The variance of the process is given by

$$\sigma_z^2 = \frac{\sigma_a^2}{1-\rho_1\phi_1-\rho_2\phi_2} = \left(\frac{1-\phi_2}{1+\phi_2}\right)\frac{\sigma_a^2}{\{(1-\phi_2)^2-\phi_1^2\}} \tag{14}$$

The autocorrelation function satisfies the second order difference equation

$$\rho_k = \phi_1\rho_{k-1} + \phi_2\rho_{k-2} \tag{15}$$

with starting values $\rho_0 = 1$ and $\rho_1 = \phi_1/(1 - \phi_2)$.

If the roots of the characteristic equation $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 = 0$ are complex, a second order autoregressive process displays pseudo cyclic behaviour. In this case ($\phi_1^2 + 4\phi_2 < 0$) and the acf behaves like a damped sine wave with damping factor d, frequency f and phase F.

$$\begin{aligned} d &= \sqrt{-\phi_2} \\ f &= \frac{1}{2\pi}\cos^{-1}\left(\frac{|\phi_1|}{2\sqrt{-\phi_2}}\right) \\ F &= \tan^{-1}\left(\frac{1+d^2}{1-d^2}\tan 2\pi f\right) \end{aligned} \tag{16}$$

4.2 Moving average model

A moving average model of order q, denoted by MA(q), takes the form

$$z_t = a_t - \theta_1 a_{t-1} - \dots - \theta_q a_{t-q} \quad \text{or } z_t = \theta(B)\, a_t \tag{17}$$

where θ_1, θ_2, … , θ_q are the MA parameters and $\theta(B) = 1 - \theta_1 B - \dots - \theta_q B^q$. The parameters of the model are the mean of z_t, the variance of the random variable and the coefficients θ_1, … , θ_q. A total of (q+2) parameters must be evaluated from the data.

The variance of a MA(q) process is

$$\sigma_z^2 = (1 + \theta_1^2 + \theta_2^2 + \dots + \theta_q^2)\sigma_a^2 \tag{18}$$

The autocorrelation function is

$$\rho_k = \begin{cases} \dfrac{-\theta_k + \theta_1\theta_{k+1} + \dots + \theta_{q-k}\theta_q}{1 + \theta_1^2 + \dots + \theta_q^2} & k = 1, 2, \dots, q \\ 0 & k > q \end{cases} \tag{19}$$

A first order moving average process is of the form

$$z_t = a_t - \theta_1 a_{t-1} \quad \text{or} \quad z_t = (1 - \theta_1 B)\, a_t \tag{20}$$

The variance of the process is

$$\sigma_z^2 = (1 + \theta_1^2)\sigma_a^2 \tag{21}$$

and the autocorrelation function is

$$\rho_k = \begin{cases} \dfrac{-\theta_1}{1 + \theta_1^2} & \text{k} = 1 \\ 0 & \text{k} > \text{q} \end{cases} \tag{22}$$

A second order moving average process is of the form

$$z_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} \quad \text{or} \quad z_t = (1 - \theta_1 B - \theta_2 B^2)\, a_t \tag{23}$$

The variance of the process is

$$\sigma_z^2 = (1 + \theta_1^2 + \theta_2^2)\sigma_a^2 \tag{24}$$

and the autocorrelation function is

$$\begin{aligned} \rho_1 &= \frac{\theta_1(1 - \theta_2)}{1 + \theta_1^2 + \theta_2^2} \\ \rho_2 &= \frac{-\theta_2}{1 + \theta_1^2 + \theta_2^2} \\ \rho_k &= 0 \qquad \text{k} \geq 3 \end{aligned} \tag{25}$$

4.3 Autoregressive moving average model

The autoregressive moving average model of order (p,q), denoted by ARMA(p,q), is of the form

$$z_t = \phi_1 z_{t-1} + \ldots + \phi_p z_{t-p} + a_t - \theta_1 a_{t-1} - \ldots - \theta_q a_{t-q} \qquad (26)$$

where z_t is a zero mean process and a_t white noise. The parameters of the model are the mean of z_t, the variance of the random variable and the coefficients $\phi_1, \ldots, \phi_p, \theta_1, \ldots, \theta_q$. A total of $p+q+2$ parameters must be evaluated from the data.

The variance of z_t is given by

$$\sigma_z^2 = \sigma_a^2 + \sum_{i=1}^{p} \phi_i \gamma_i - \sum_{i=1}^{q} \theta_i \gamma_{za}(-i) \qquad (27)$$

where $\gamma_z(i)$ is the covariance of z_t and $\gamma_{za}(k)$ is the cross covariance between z_t and a_t, which is defined by $\gamma_{za}(k) = E[z_{t-k}a]$.

$\gamma_{za}(k) = 0 \qquad k > 0$
$\gamma_{za}(k) \neq 0 \qquad k \leq 0$

A first order autoregressive moving average or ARMA(1,1) model is of the form

$$z_t = \phi_1 z_{t-1} + a_t - \theta_1 a_{t-1} \qquad (28)$$

The variance of the process is

$$\sigma_z^2 = \frac{1+\theta_1^2 - 2\phi_1\theta_1}{1-\phi_1^2}\sigma_a^2 \qquad (29)$$

and the autocorrelation function is

$$\rho_k = \begin{cases} \dfrac{(\phi_1 - \theta_1)(1-\phi_1\theta_1)}{1+\theta_1^2 - 2\phi_1\theta} & \mathrm{k} = 1 \\ \phi_1 \rho_{\mathrm{k-1}} & \mathrm{k} \geq 2 \end{cases} \qquad (30)$$

4.4 Model fitting

Box and Jenkins (1976) suggested a three step iterative approach to model building: Identification, Estimation and Diagnostic checking.

The characteristic behaviours of the theoretical autocorrelation function (acf) and partial autocorrelation function (pacf) for AR, MA and ARMA processes are used in model identification. The lag k partial autocorrelation coefficient of a series is defined as the kth autoregressive parameter of a kth order AR model fitted to the series. The acf of an AR process of order p tails off while its pacf has a cut off after lag p. On the other hand, for a MA process of order q, the acf has a cut off after lag q, while its pacf tails off. If both the acf and pacf tail off, an ARMA process is suggested. Furthermore, for an ARMA(p,q) process, the acf is a mixture of exponentials and damped sine waves after the first (p-q) lags, while the pacf is a mixture of exponentials and damped sine waves after the first (p-q) lags (Table 3.2 in Box and Jenkins, 1976).

Once a model is identified for a particular time series, the next step is to estimate the parameters of the model. The parameters of the model can be estimated by using the method of moments or the method of maximum likelihood (ML). The moments estimates are easier to obtain but they do not possess the same asymptotic efficiency as maximum likelihood estimates. Box and Jenkins (1976) have shown that moment estimates of AR parameters closely approximate the ML estimates, while the moment estimates of MA parameters do not.

The moment estimates of the ARMA model may be obtained from the estimates of the (p+q) autocorrelation coefficients or equivalently the autocovariances. Since the acf at lags greater than q depend only on AR parameters, the AR parameters can be obtained by solving a system of linear equations. The MA parameters are then obtained by an iterative method.

The unconditional log-likelihood for an ARMA model is given by

$$L(\phi,\theta,\sigma_a = f(\phi,\theta) - n\ln\sigma_a - \frac{S(\phi,\theta)}{2\sigma_a^2} \quad (31)$$

where f(ϕ,θ) is a function of ϕ and θ. The unconditional sum of squares function is given by

$$S(\phi,\theta) = \sum_{t=1}^{n}[a_t \mid \phi,\theta,z]^2 \quad (32)$$

where $[a_t|\phi,\theta,z] = E[a_t|\phi,\theta,z]$ denotes the expectation of conditional on ϕ, θ and z.

For moderate to large n, Eq (31) is dominated by S(ϕ,θ). Hence the set of parameters ϕ and θ which minimises the sum of squares function is the maximum likelihood estimates. The variance of the residuals is then estimated from

$$\sigma_a^2 = \frac{1}{n}S(\phi,\theta) \quad (33)$$

One criteria for selecting among competing ARMA(p,q) models is the Akaike Information Criterion (AIC) which is defined (Akaike, 1974) as

$$AIC = n\ln\sigma_a^2 + 2(p+q) \quad (34)$$

Since AIC has a tendency to overestimate the order of the model, the correction given by Hurvich and Tsai (1989) can be used. The corrected AIC is given by

$$AICc = n\ln\sigma_a^2 + n(n+p+q)/(n-p-q-2) \quad (35)$$

An approach for interpreting the differences in the AIC values between two competing models is to consider plausibility (Hipel and McLeod, 1994). The plausibility of model i versus model j can be calculated using

$$\text{Plausibility} = \exp[0.5(AIC_j - AIC_i)] \tag{36}$$

where AIC_i is the value of AIC for model *i*. If the two models are very similar, the plausibility will tend to 1. The model j (which has the lower AIC value than model i) is considered to be significantly different from model i if the value of plausibility is less than 0.05.

The autocorrelation and partial autocorrelation functions for the annual rainfall data are presented in Appendix A. Following the Box and Jenkins approach (Table 3.2 in Box and Jenkins, 1976), suitable models were identified by the using the autocorrelation and partial autocorrelation functions and the identified models are given in Table 3. No dependence is found in the data for all the stations except for 6 stations. Of the 6 stations, one needed an AR(2) model while for the remaining 5, an AR(1) model was identified. Since the AR(1) model has already been used to generate data for all the stations, we need to generate data for only one station (35027) which has a different model.

In addition, ARMA model is fitted to annual rainfall data using the corrected Akaike Information Criterion. The corrected AIC was calculated for ARMA models with p and q varying from 0 to 2. This resulted in 9 different models including the white noise case. The models selected with the minimum AICc are also given in Table 3.

Table 3. The models identified by the acf-pacf and AICc.

No	Number	Name	acf - pacf	AICc
1	005008	Mardie	WN	WN
2	006036	Meedo	AR(1)	MA(1)
3	009034	Perth	WN	ARMA(2,2)
4	010037	Cuttening	WN	WN
5	012065	Norseman Post Office	WN	WN
6	017031	Marree	WN	WN
7	019032	Orroroo	WN	WN
8	022020	Wallaroo	WN	WN
9	023000	Adelaide	WN	WN
10	024511	Eudunda	WN	MA(2)
11	028004	Palmerville	WN	WN
12	033035	Kalamia Estate	WN	WN
13	035027	Emerald Post Office	AR(2)	AR(2)
14	036007	Barcaldine Post Office	AR(1)	AR(1)
15	039023	Cape Capricorn Lighthouse	WN	WN
16	039082	Rockhampton Post Office	WN	WN
17	040043	Cape Moreton Lighthouse	WN	WN
18	040214	Brisbane	WN	WN
19	041082	Pittsworth Post Office	WN	WN
20	042023	Miles Post Office	WN	MA(1)
21	044026	Cunnamulla Post Office	WN	WN
22	047053	Wentworth Post Office	WN	WN
23	049002	Balranald RSL	WN	WN
24	054004	Bingara Post Office	WN	ARMA(2,1)
25	062021	Mudgee (George Street)	WN	ARMA(1,2)
26	066062	Sydney	AR(1)	MA(1)
27	069018	Moruya Heads Pilot Station	AR(1)	AR(1)
28	072000	Adelong	WN	MA(2)
29	072044	Tumut	WN	WN
30	075031	Hay Miller Street	WN	ARMA(1,1)
31	077030	Narraport	WN	MA(1)
32	080056	Tongala	WN	AR(1)
33	081007	Caniambo	WN	ARMA(1,1)
34	084030	Orbost	WN	AR(1)
35	086071	Melbourne	WN	WN
36	086117	Toorourrong Reservoir	WN	WN
37	087043	Meredith (Darra)	WN	WN
38	091033	Frankford (Rossville)	WN	WN
39	092012	Fingal (Forestry Legge Street)	WN	WN
40	094061	Sandford (Maydena)	WN	ARMA(1,2)
41	001005	Wyndham Port	WN	WN
42	002016	Lissadell	WN	WN
43	014902	Katherine Council	WN	WN
44	015540	Alice Springs Post Office	AR(1)	AR(1)

WN - White noise
AR(p) - Autoregressive model of order p
MA(q) - Moving average model of order q
ARMA(p,q) - Autoregressive moving average model of order (p.q)

It can be seen from Table 3 that the random noise model was selected for 26 stations. Of the remaining 18 stations, five indicated AR(1) model. Since this model has already been run, it was decided to generate data for the 13 stations using the models identified by the AICc. The AICc also identified the same model [AR(2)] for the station Emerald (35027) as the Box and Jenkins approach. Hence running the models for the 13 stations covers all the newly identified models. The parameters for the 14 stations are given in Table 4. For only two cases (Emerald and Bingara), the roots of the characteristic equation are complex ($\phi_1^2 + 4\phi_2 < 0$). The expected frequency of the pseudo periodicity is 1/4.8 for Emerald and 1/10.5 for Bingara.

The plausibility of AR(1) model over the above selected models was assessed. The selected model was significantly better in three cases only, which are shown in bold in Table 4. However, all the selected 14 models were used to generate annual rainfall data.

Table 4. The parameters of the identified models.

Number	Name	Plausibility	ϕ_1	ϕ_2	θ_1	θ_2
6036	Meedo	0.466			-0.3039	
9034	Perth	0.332	0.1103	0.8581	-0.0076	0.9799
24511	Eudunda	0.253			-0.1631	0.1897
35027	Emerald Post Office	**0.058**	0.2776	-0.2704		
42023	Miles Post Office	0.766			0.1703	
54004	Bingara Post Office	0.142	-0.6997	-0.1799	-0.7655	
62021	Mudgee (George Street)	**0.019**	-0.7754		-0.9486	0.0399
66062	Sydney	0.739			-0.1642	
72000	Adelong	0.408			-0.1817	0.1645
75031	Hay Miller Street	**0.005**	-0.7710		-0.9876	
77030	Narraport	0.944			-0.1996	
81007	Caniambo	0.284	0.8798		0.9942	
94061	Sandford (Maydena)	0.327	-0.8749		-1.0980	-0.3136

The standard deviation and the coefficient of skewness of the noise term were calculated wherever possible using the expressions given in Appendix B and also empirically from the residual series (Table 5). The skewness was calculated only for the moving average and ARMA(1,1) models.

Table 5. The standard deviation and coefficient of skewness of a_t.

		Standard deviation (mm)		Skewness	
Number	Name	Theoretical	Empirical	Theoretical	Empirical
6036	Meedo	93.5	86.0	1.029	0.942
9034	Perth	101.3	164.2	-	0.651
24511	Eudunda	117.0	111.4	1.323	1.126
35027	Emerald Post Office	199.2	200.1	-	0.683
42023	Miles Post Office	216.8	212.0	0.526	0.557
54004	Bingara Post Office	222.2	185.5	-	0.441
62021	Mudgee (George Street)	165.3	170.8	-	0.586
66062	Sydney	339.0	331.5	0.762	0.695
72000	Adelong	196.8	186.9	0.788	0.565
75031	Hay Miller Street	119.8	154.4	1.424	0.357
77030	Narraport	104.5	101.0	0.515	0.530
81007	Caniambo	135.5	136.0	1.088	0.854
94061	Sandford (Maydena)	119.4	121.9	-	0.656

Since theoretical values were available for all the moving average models, these were used to generate the data. For the remaining models, data were generated using both the theoretical and empirical values of the skewness for the random numbers. The basic parameters for these two runs are compared in Table 6 to select the best estimates. The theoretical values were selected for all the stations in Table 6 except for Perth (009034).

Table 6. Comparison of basic parameters

Number		Mean	Std Dev	CV	Skew
9034	Historical	868	158	0.18	0.72
	Theoretical	868	104	0.12	0.55
	Empirical	869	169	0.19	0.55
35027	Historical	642	212	0.33	0.75
	Theoretical	642	211	0.33	0.53
	Empirical	642	212	0.33	0.53
54004	Historical	745	201	0.27	0.54
	Theoretical	745	231	0.31	0.37
	Empirical	745	193	0.26	0.37
62021	Historical	670	174	0.26	0.79
	Theoretical	671	174	0.26	0.48
	Empirical	671	179	0.27	0.48
75031	Historical	369	127	0.34	1.22
	Theoretical	369	126	0.34	1.08
	Empirical	370	163	0.44	0.32
81007	Historical	524	140	0.27	0.99
	Theoretical	524	138	0.26	0.92
	Empirical	524	139	0.27	0.73
94061	Historical	578	126	0.22	0.63
	Theoretical	579	126	0.22	0.51
	Empirical	579	128	0.22	0.51

For MA(q) models, the data were generated directly as the lagged q values of random numbers can be generated. However, for the AR and ARMA models, the previous values of the AR and ARMA processes are unknown and as a result, their unconditional expectations were used. To minimise the effect of the starting values, the first generated replicate was discarded. One hundred replicates were generated for all the 14 stations and the parameters estimated and averaged for comparison.

5. HIDDEN STATE MARKOV MODEL

The HSM model (Figure 4) assumes the climate is in one of two states: wet (W) or dry (D). Each state has an independent rainfall distribution, assumed to be Gaussian. The time spent in each state is governed by the state transition probabilities. This provides an explicit mechanism to replicate the variable length of wet and dry cycles.

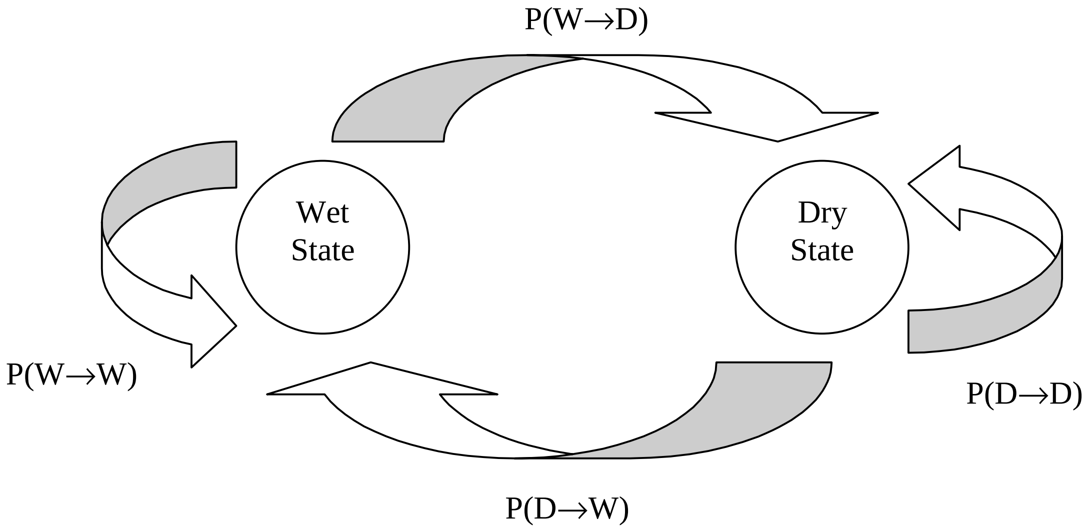

Figure 4. Schematic representation of the HSM model

The simulation of annual rainfall time series is a two-step process. In the first step the climate state at year t, s_t, is simulated by a Markovian process:

$$s_t \mid s_{t-1} \sim \text{Markov}(\boldsymbol{P}) \tag{37}$$

where $\boldsymbol{P}$ is a (2x2) state transition probability matrix whose elements are:

$$p_{ij} = Pr(s_t = j \mid s_{t-1} = i) \qquad i, j = W, D \tag{38}$$

Once the state for year t is known, the rainfall is simulated using:

$$y_t \sim \begin{cases} \text{N}(\mu_W, \sigma_W^2) & \text{if } s_t = W \\ \text{N}(\mu_D, \sigma_D^2) & \text{if } s_t = D \end{cases} \tag{39}$$

where $\text{N}(\mu, \sigma^2)$ denotes a Gaussian distribution with mean μ and variance σ^2. Therefore the vector of unknown parameters for the HSM model, $\boldsymbol{\theta}$, is composed of the rainfall distribution parameters for each state, the transition probabilities, and the hidden state time series, $S_N = \{s_1, s_2, ... s_N\}$, where:

$$\boldsymbol{\theta}' = (\mu_W, \sigma_W, \mu_D, \sigma_D, \boldsymbol{P}, S_N) \tag{40}$$

Prior to model calibration the hidden state time series is unknown. Thus it is included as a model parameter to be estimated during the calibration process.

5.1 Model Calibration

For model calibration a Bayesian framework is used to infer the distribution of the model parameters, $\boldsymbol{\theta}$, for the given time series data, Y_N. This distribution is referred to as the posterior distribution of the model parameters, $p(\boldsymbol{\theta} \mid Y_N)$. For the HSM model it is not possible to derive an analytical expression for the posterior distribution. Thus Markov Chain Monte Carlo (MCMC) simulation methods are employed to draw samples from the posterior distribution. The basic idea of MCMC methods is to simulate a Markov chain iterative sequence, where at each iteration a sample of the model parameters, $\boldsymbol{\theta}$, is generated. Given certain conditions the distribution of these samples converges to a stationary distribution which is the posterior distribution, $p(\boldsymbol{\theta} \mid Y_N)$. To calibrate the HSM model, the MCMC method known as the Gibbs sampler is applied. The details of the calibration process are given in Thyer and Kuczera (2000).

5.2 Application of HSM model

The HSM model was calibrated for the 40 Australian rainfall stations using twelve different months for forming the annual rainfall totals. Several indices are used to interpret the results and these are briefly defined below.

The Wet And Dry Separation Index (*WADSI*) is defined as

$$WADSI = \frac{\mu_W - \mu_D}{\sqrt{\left(\sigma_W^2 + \sigma_D^2\right)}} \tag{41}$$

This index is a convenient measure of the separation between the wet and dry states. If the difference between the wet and dry means is large then the value of *WADSI* will be relatively high. As the value of *WADSI* increases, the wet and dry distributions become more separated and easier to identify. At a value of *WADSI* = 1, the probability of wet mean less than dry mean is only 16% and the wet and dry distributions would be easier to identify (Thyer, 2001).

The state stability index (*SSI*) is defined as follows:

$$SSI = \frac{\sum \left| P(W) - 0.5 \right|}{N} \tag{42}$$

where *P(W)* is the probability of wet state and *N* the number of years of data.

Values of *SSI* close to zero indicate no persistence in the rainfall to stay in either wet or dry state. Values of *SSI* around 0.3 generally indicates persistence, but this needs to be confirmed with a visual inspection of a plot of {*P(W)-0.5*} versus year.

The strength of persistence structure is assessed using the expected state resident times (SRT) and is obtained as the reciprocal of the transition probabilities.

$$E(SRT_D) = \frac{1}{p_{DW}}$$
$$E(SRT_W) = \frac{1}{p_{WD}} \quad (43)$$

The strength of persistence structure (*SPS*) was classified from the state residence times using the following table (Thyer, 2001).

Table 7. The classification of the strength of persistence structure.

Expected residence time (years)	Classification
1 - 4	Weak (W)
4 - 10	Medium (M)
10 - 25	Strong (S)
> 25	Very Strong (VS)

In Table 8, the two state persistence structure is denoted by [SPS_W,SPS_D,WADSI] where the subscripts denote the wet and dry states. The full set of the calibration results are given in Srikanthan et al (2001). Of the 44 stations used, 12 stations indicated that it is highly unlikely to have two-state persistence structure (Figure 5). Of the remaining 32 stations, 15 stations indicated that it is highly likely to have two-state persistence and 17 stations possibly have two-state persistence structure. These 32 stations with the chosen starting months and the two-state persistence structure are given in Table 8.

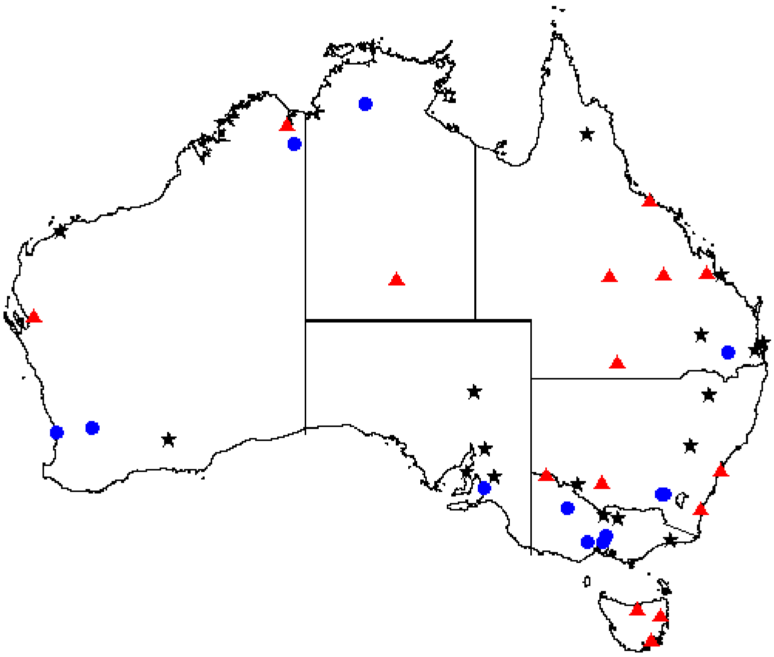

Figure 5. The locations of the stations that are highly unlikely to have two-state persistence (circles), highly likely to have two-state persistence (triangles) and possibly have two-state persistence (stars).

The estimated parameters of the HSM model are given in Table 9. One hundred replicates each of length equal to the historic record were generated for the 23 stations and the parameters listed in section 3 were calculated for model evaluation.

Table 8. The rainfall stations indicating (i) highly likely to have and (ii) possibly have two-state persistence structure.

Number	Name	Starting month	Two state persistence structure
Stations highly likely to have two-state persistence			
006036	Meedo	4	[W, M, 0.991]
033035	Kalamia Estate	4	[W, M, 1.173]
035027	Emerald Post Office	8	[W, M, 0.924]
036007	Barcaldine Post Office	11	[W, M, 1.176]
039082	Rockhampton Post Office	7	[W, M, 1.183]
044026	Cunnamulla Post Office	8	[W, M, 1.183]
047053	Wentworth Post office	1	[W, S, 0.922]
066062	Sydney	8	[W, M, 0.993]
069018	Moruya Heads Pilot Station	5	[W, M, 1.323]
075031	Hay Miller Street	1	[W, M, 0.926]
091033	Frankford (Rossville)	12	[W, M, 0.910]
092012	Fingal (Forestry Legge Street)	8	[W, M, 0.986]
094061	Sandford (Maydena)	12	[W, M, 1.429]
001005	Wyndham	3	[W, W, 1.203]
015540	Alice Springs	1	[W, S, 1.236]
Stations possibly to have two-state persistence			
005008	Mardie	4	[W, M, 0.970]
012065	Norseman Post Office	9	[W, M, 0.834]
017031	Marree	6	[W, M, 1.116]
019032	Orroroo	6	[W, M, 0.794]
022020	Wallaroo	12	[W, M, 1.014]
024511	Eudunda	8	[W, M, 0.822]
028004	Palmerville	9	[W, M, 0.724]
039023	Cape Capricorn	7	[W, M, 0.839]
040043	Cape Moreton	7	[W, W, 0.842]
040214	Brisbane	7	[W, M, 0.890]
054004	Bingara	8	[W, M, 0.826]
062021	Mudgee	7	[W, W, 0.764]
042023	Miles Post Office	2	[W, M, 1.013]
049002	Balranald RSL	11	[W, M, 0.938]
080056	Tongala	8	[W, M, 0.940]
081007	Caniambo	8	[W, M, 0.842]
084030	Orbost	8	[W, W, 0.808]

Table 9. The estimated parameters for the HSM model.

Number	p_{WD}	p_{DW}	μ_W (mm)	μ_D (mm)	σ_W (mm)	σ_D (mm)
001005	0.55	0.38	874	579	211	125
005008	0.76	0.62	389	191	178	101
006036	0.48	0.21	297	185	98	63
012065	0.62	0.22	372	261	119	59
015540	0.57	0.15	473	235	175	82
017031	0.75	0.17	267	144	122	56
019032	0.81	0.24	453	315	157	76
022020	0.46	0.58	405	309	75	57
024511	0.75	0.21	566	420	157	82
028004	0.38	0.25	1228	943	369	244
033035	0.53	0.30	1524	844	503	291
035027	0.46	0.23	822	568	237	140
036007	0.57	0.32	693	398	219	123
039023	0.42	0.36	957	680	280	175
039082	0.52	0.32	1255	765	365	196
040043	0.72	0.39	1819	1430	478	278
040214	0.66	0.20	1515	1059	469	267
042023	0.57	0.58	791	542	204	136
044026	0.66	0.14	593	330	198	102
047053	0.55	0.10	418	268	145	72
049002	0.53	0.10	475	305	160	85
054002	0.64	0.26	896	697	225	151
062021	0.49	0.30	787	609	195	124
066062	0.43	0.16	1539	1112	363	232
069018	0.31	0.21	1236	792	285	177
075031	0.69	0.14	537	344	186	95
080056	0.65	0.14	606	416	186	80
081007	0.61	0.25	653	486	174	96
084030	0.77	0.50	985	781	219	147
091033	0.50	0.48	1205	954	227	158
092012	0.60	0.25	772	549	200	104
094061	0.48	0.32	691	507	111	65

6. DISCUSSION OF RESULTS

The average of the parameters from the 100 replicates along with the historical values are presented in Table C1 – C12 in Appendix C. The range of the parameters from the 100 replicates are shown in Figures C1 to C12. For easier comparison, the generated means and standard deviations in Figures C1 and C2 are divided by the corresponding historical values.

The mean was well reproduced by all the models (Table C1). However, a slight over estimation of the mean was evident in some cases for the HSM model. None of the means was outside the 95 % confidence limits (Figure C1).

The standard deviation was well reproduced by all the models (Table C2). As in the case of the mean, HSM model overestimated the standard deviation for a few stations. The variation in the values for standard deviation estimated from the replicates is shown in Figure C2. The historical standard deviation is within the 95 % confidence limits for all the stations. The skewness was preserved satisfactorily by all the models (Table C3). Except for a few cases for the ARMA and HSM models, the lag one autocorrelation coefficient was satisfactorily reproduced (Table C4).

The maximum annual rainfall was generated satisfactorily by all the models (Table C5). Figure 5 shows that all the historical maximum values were within the 95 % confidence limits (Figure C5). Except for a few stations, the minimum annual rainfall was satisfactorily generated (Table C6). The AR model gave larger average values for Adelong, Tumut, Hay and Tongala while for Mardie a small average value. The HSM model performed better for these sites. Except for Mardie, the historical minimum vales were within the 95 % confidence limits. The observed minimum for Mardie was greater than all of the generated minimum values.

For all the sites except Mardie, Perth and Orbost, the average value of the adjusted range was larger than the corresponding values (Table C7). However, the observed values were within the 95 % confidence limits for these sites (Figure C7).

The minimum 2-year rainfall was preserved for all the sites except for a few cases (Table C8). The historical values of the 2-year sums were outside the 95 % confidence limits for 7 and 6 stations respectively for both the AR(1) and ARMA models while all the historical values were within the were within the 95 % confidence limits for the HSM model. The number of historical minimum 2-, 3-, 5-, 7- and 10-year rainfall sums that were outside the were within the 95 % confidence limits are given in Table 10.

Table 10. The number of historical minimum 2-, 3-, 5-, 7- and 10-year rainfall sums that were outside the 95 % confidence limits.

Model	Number of	Duration (years)				
	stations	2	3	5	7	10
AR(1)	44	9	12	0	0	0
ARMA	14	6	3	1	1	0
HSM	32	0	0	0	0	0

Using binomial distribution, the number of stations expected to lie outside the 95% confidence limits for 44 stations is 4 and for 14 the expected number is 1 (may be 2). The AR(1) and ARMA results for the 2- and 3-year sums suggest the models are not consistent with the data. The probability of no stations in 32 falling outside the 95% confidence limits if the model were true is about 19%. Therefore the HSM model performance can be considered as consistent with the data.

7. CONCLUSIONS

The lag one Markov or AR(1) model, appropriate ARMA model as identified by the corrected Akaike information criterion and the HSM model were applied to annual rainfall data from 44 stations located in various parts of Australia. Only 3 stations indicated that a model other than

AR(1) was significantly better than the AR(1) model. For the HSM model, it is either highly likely or possible to have two-state persistence structure for 32 stations. One hundred replicates, each of length equal to the historical record were generated and several model evaluation statistics computed. The AR(1) model was found to be generally adequate for all the stations. It was found that there was no advantage in using a higher order ARMA model as the persistence structure in annual rainfall data is rather weak and does not warrant a higher order model. However, the HSM model performed better than the AR(1) model for a few stations with respect to 2- and 3-year low rainfall sums.

The observed statistics cannot discriminate between AR(1) and HSM (Thyer, 2001). However, the autocorrelation function of the HSM model has a fatter tail and this may be significant for extremely rare drought sequences. In this study, the parameter uncertainty was not taken into account and further work is in progress to take into account the parameter uncertainty.

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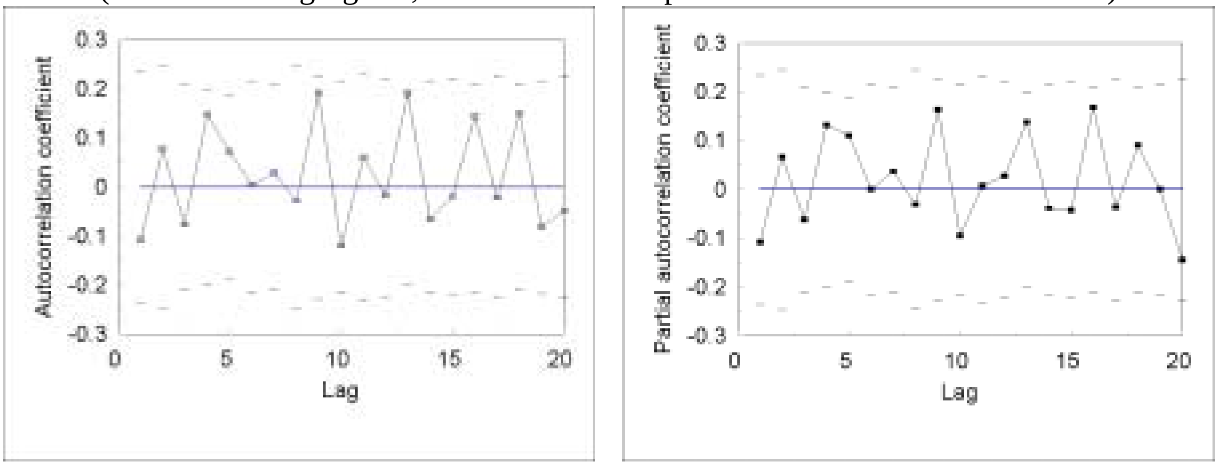
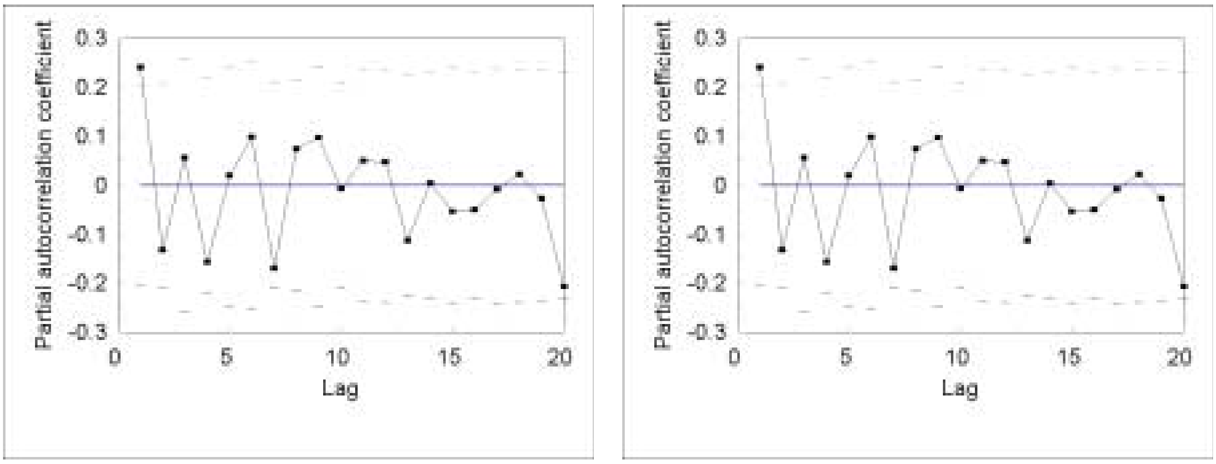
APPENDIX A
AUTOCORRELATION AND PARTIAL AUTOCORRELATION FUNCTIONS

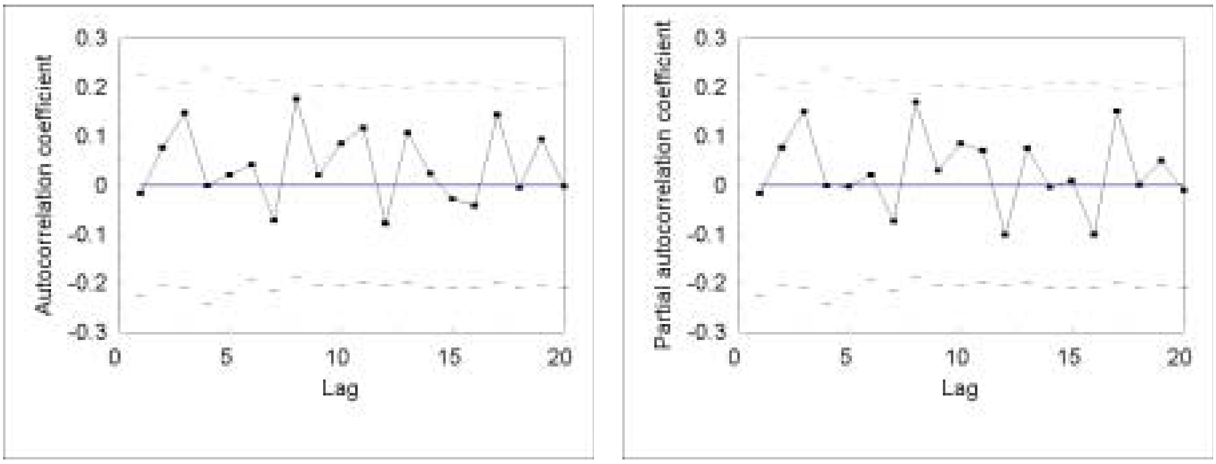
(In the following figures, the dashed lines represent the 95 % confidence limits)



1. Mardie

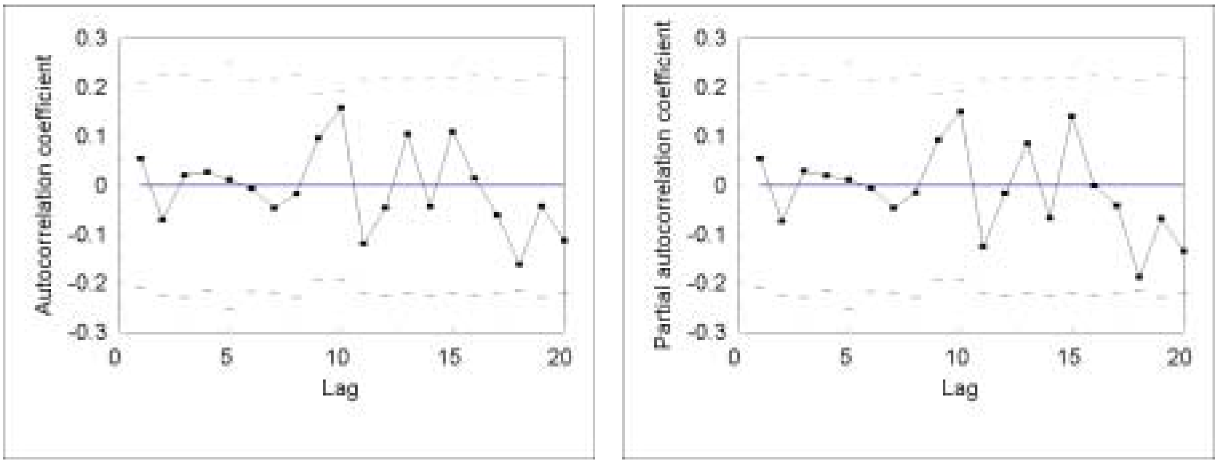


2. Meedo

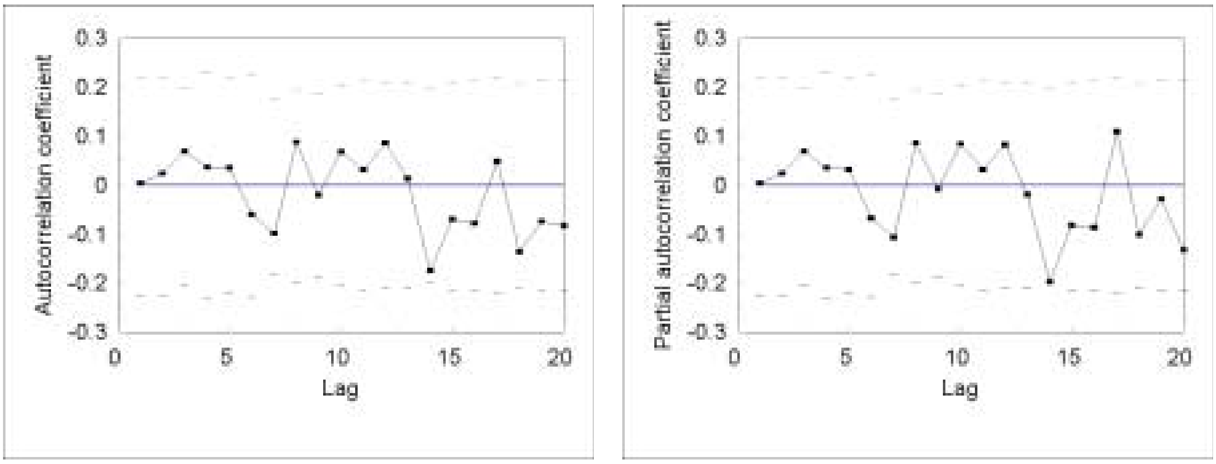


3. Perth

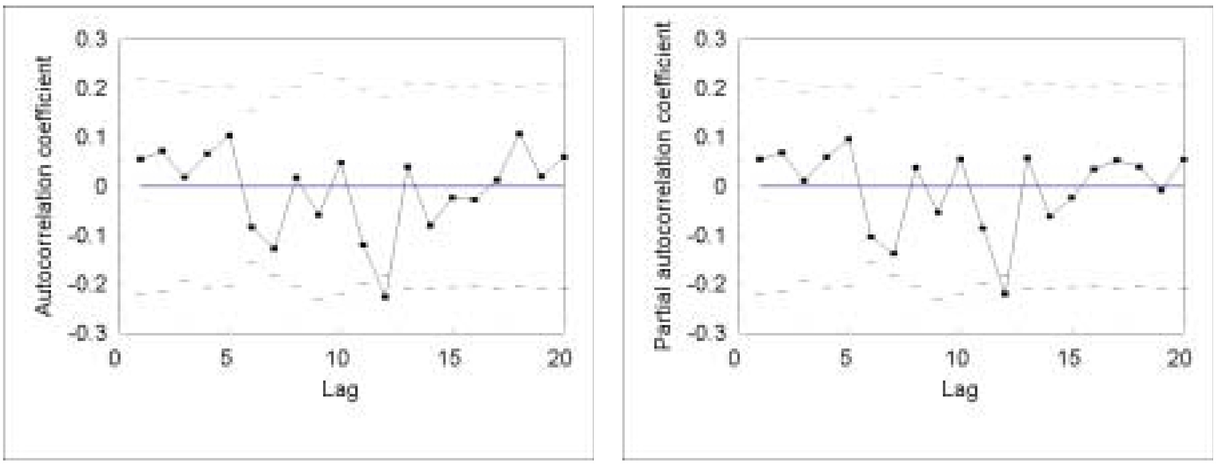
Figure A1. Plots of autocorrelation and partial autocorrelation functions.



4. Cuttening

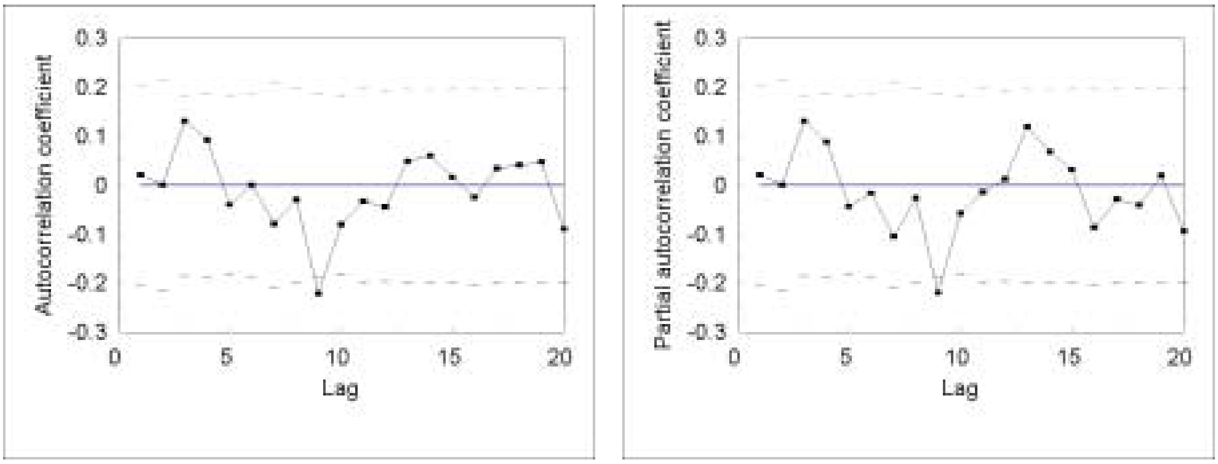


5. Norseman Post Office

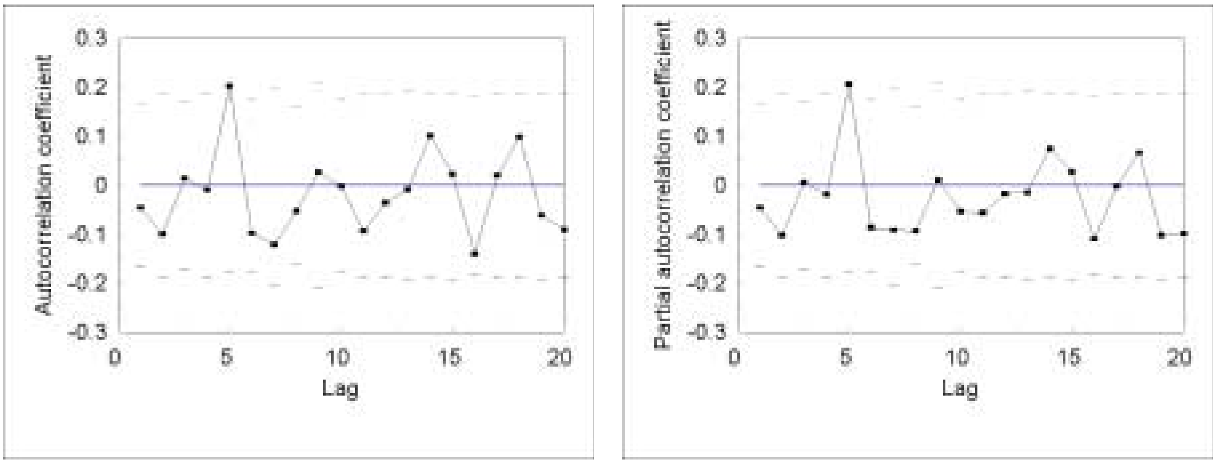


6. Marree

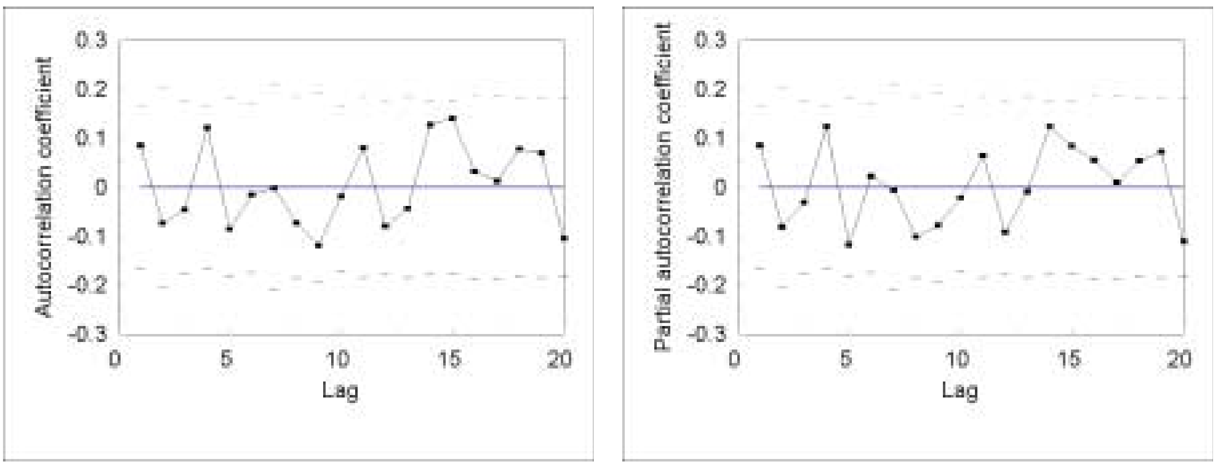
Figure A1. (Cont)



7. Orroroo

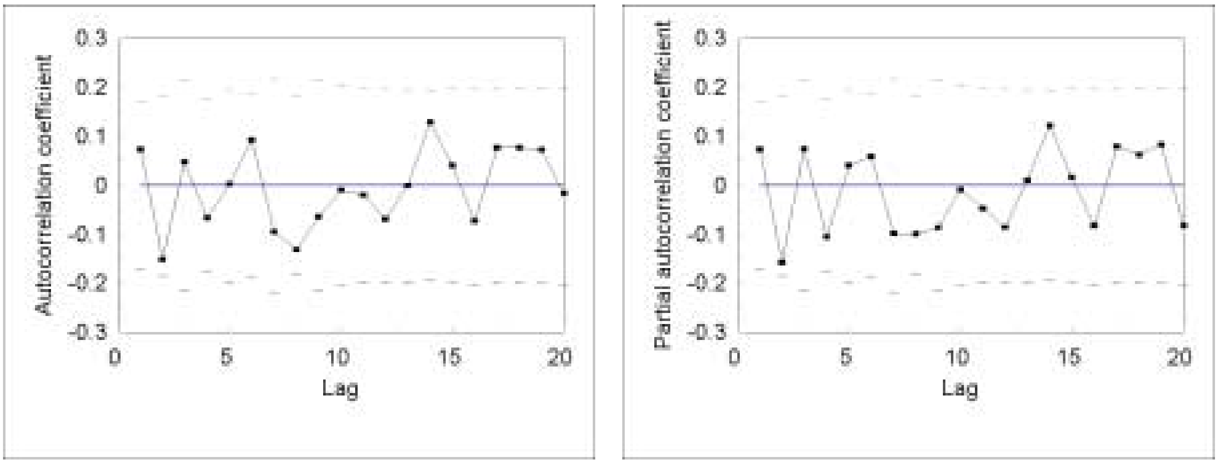


8. Wallaroo

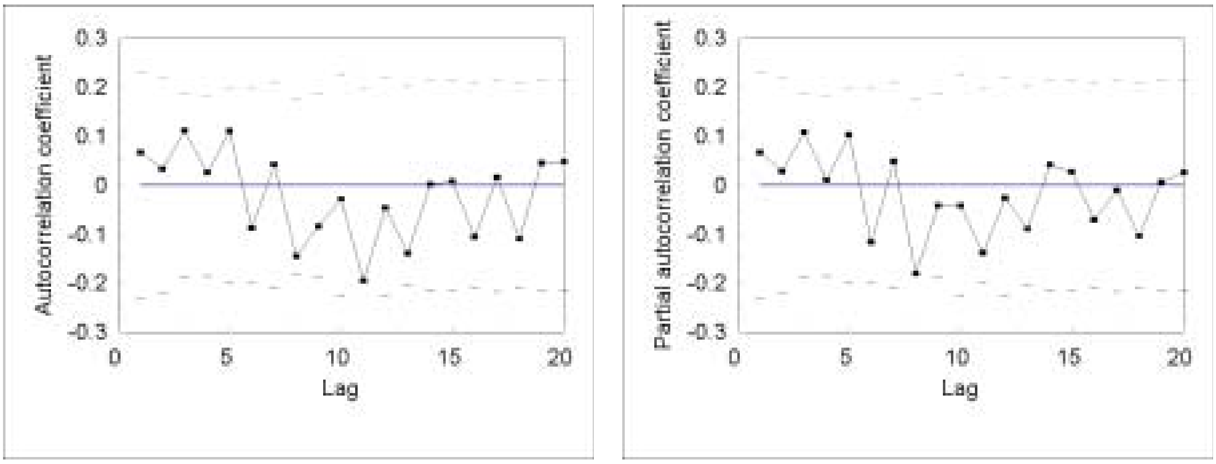


9. Adelaide

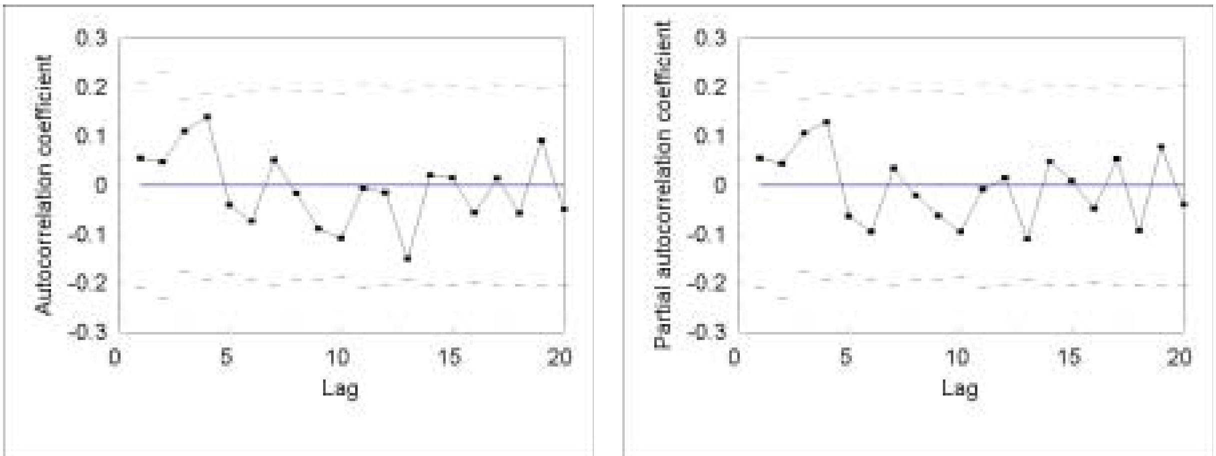
Figure A1. (Cont)



10. Eudunda

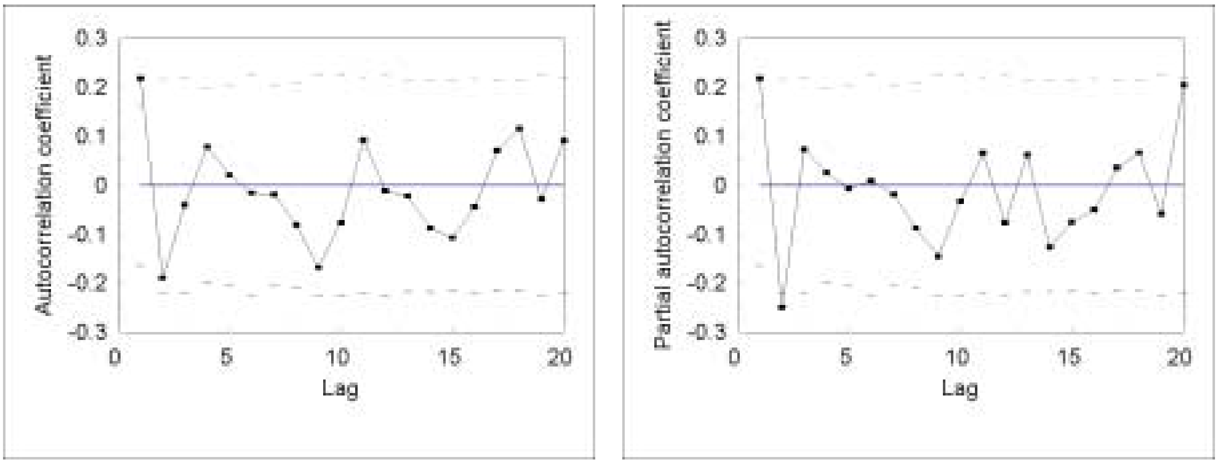


11. Palmerville

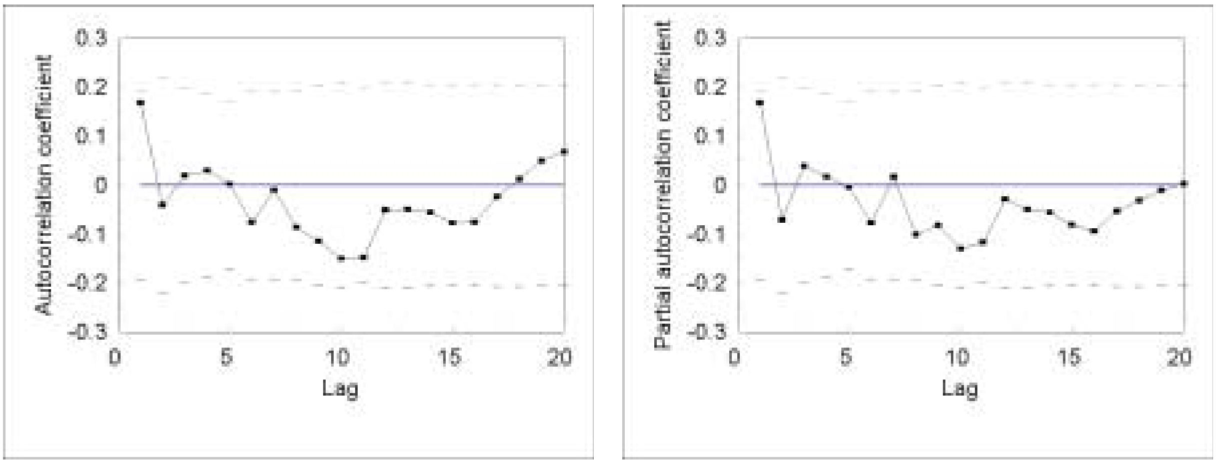


12. Kalamia Estate

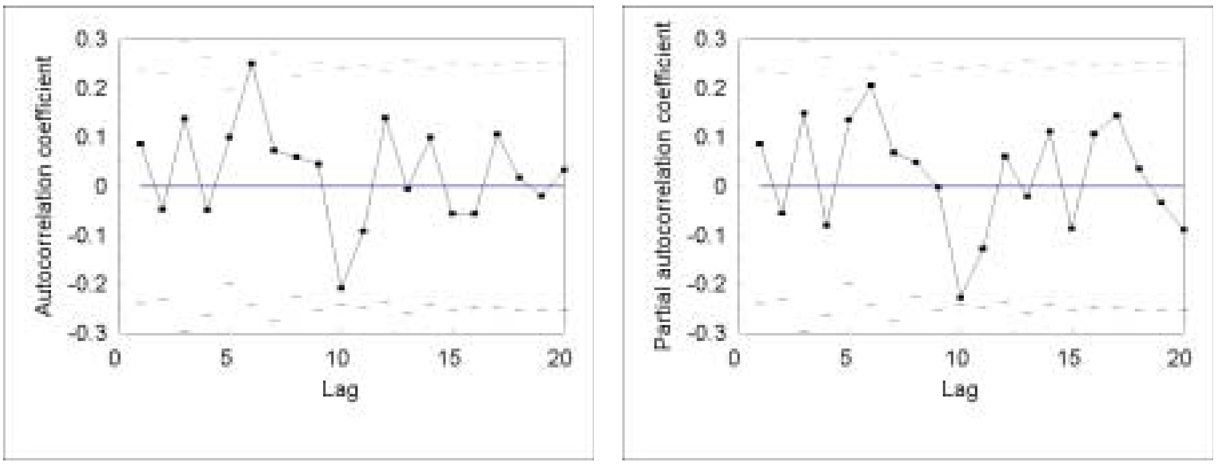
Figure A1. (Cont)



13. Emerald Post Office

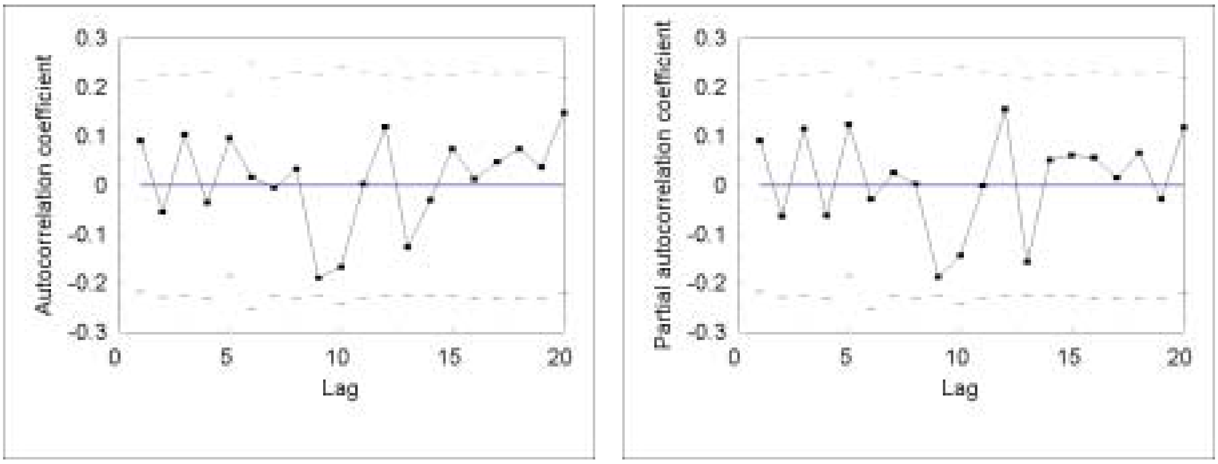


14. Barcaldine Post Office

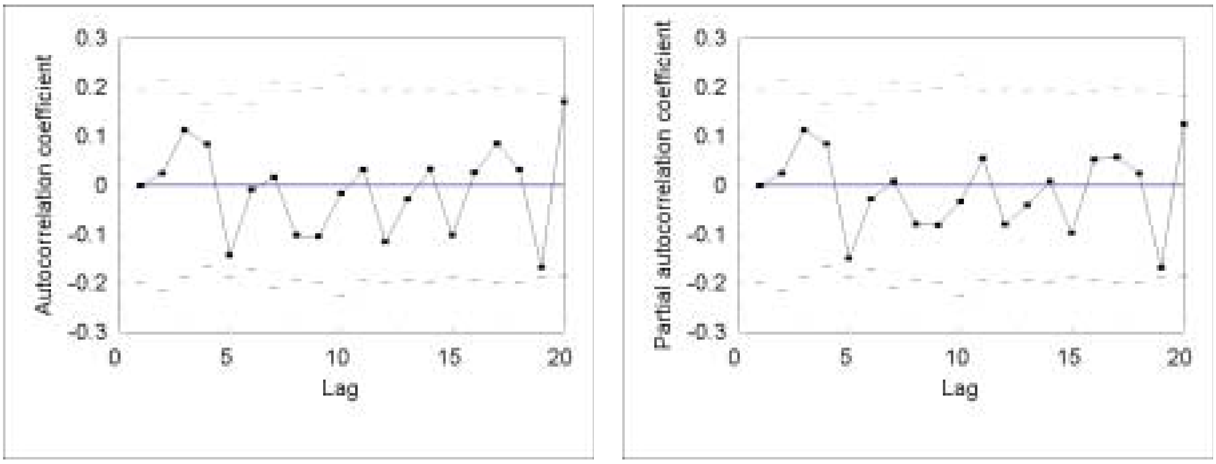


15. Cape Capricorn Lighthouse

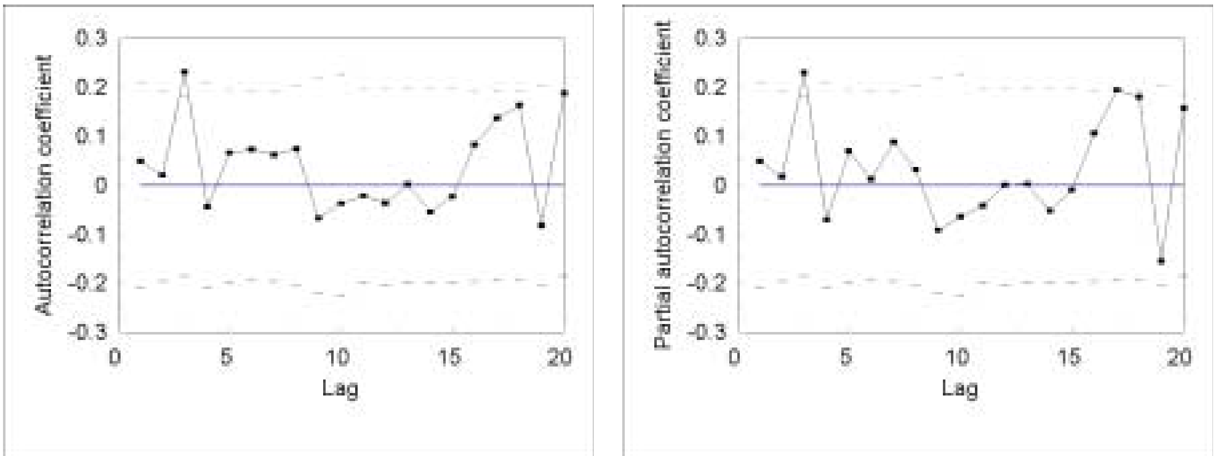
Figure A1. (Cont)



16. Rockhampton Post Office

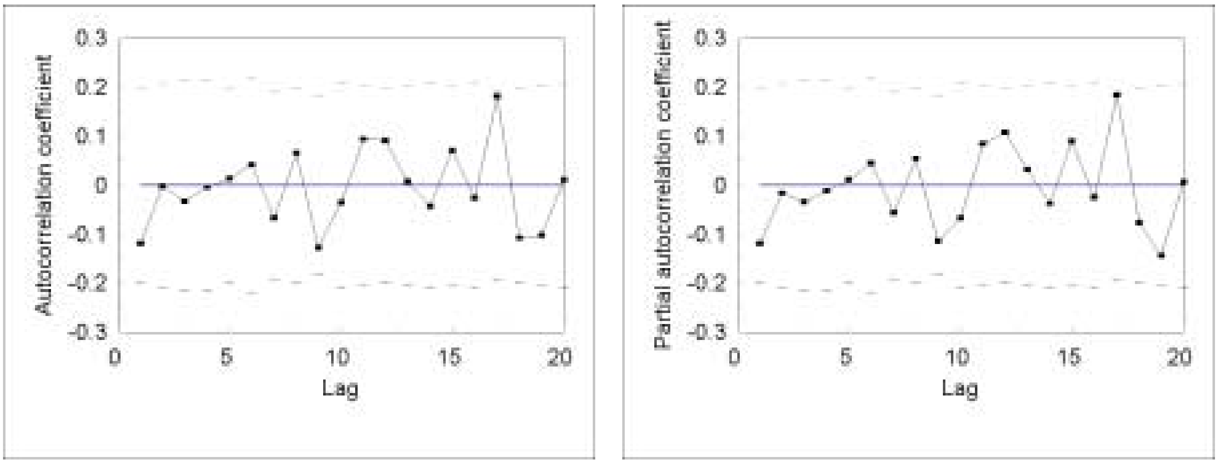


17. Cape Moreton Lighthouse

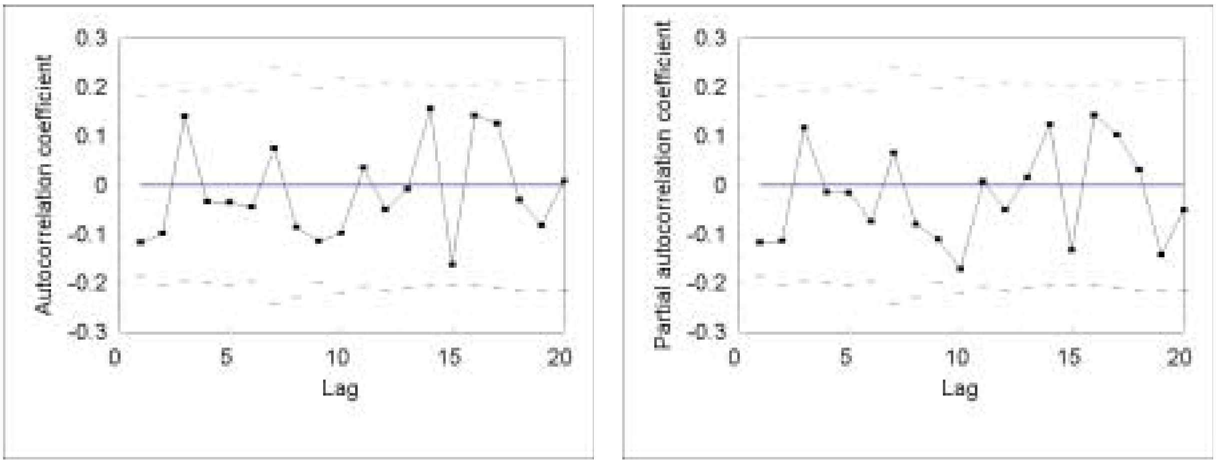


18. Brisbane

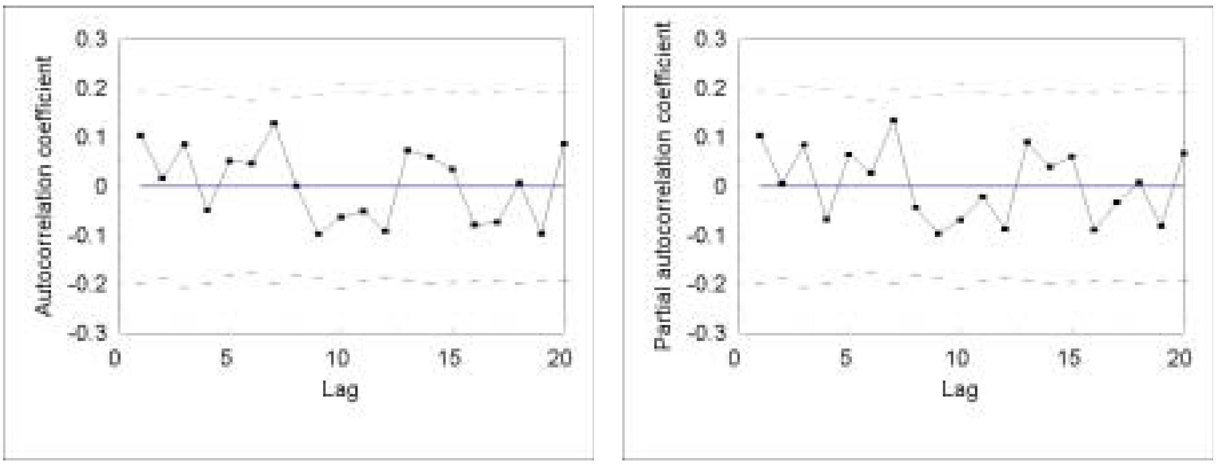
Figure A1. (Cont)



19. Pittsworth Post Office

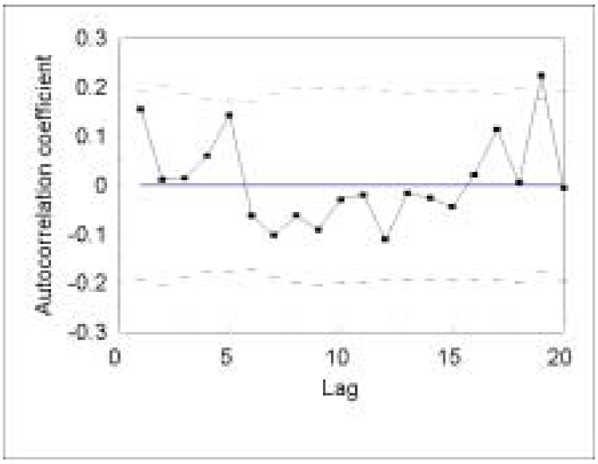


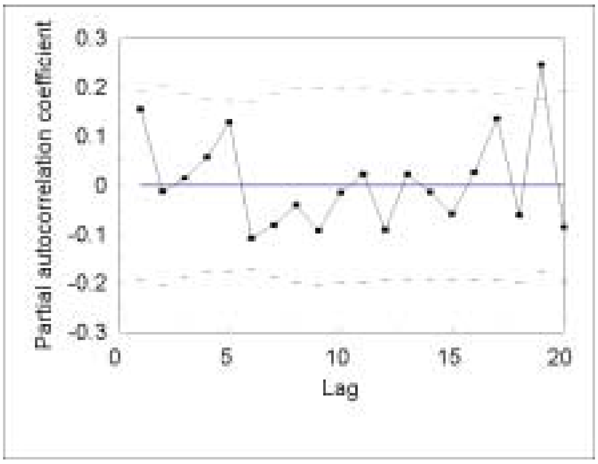
20. Miles Post Office



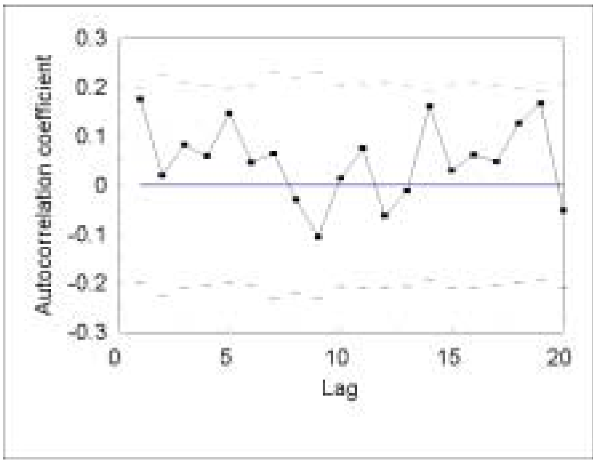
21. Cunnamulla Post Office

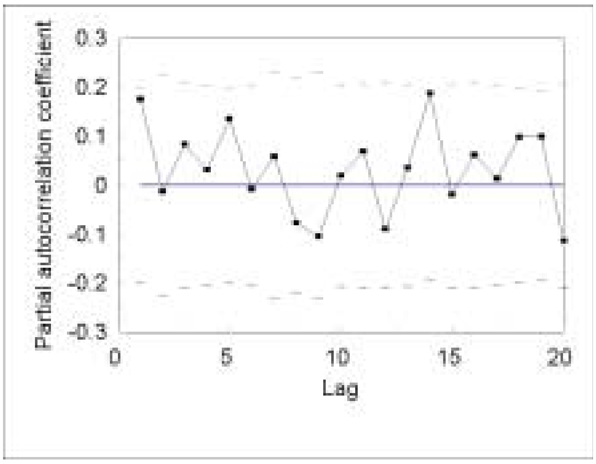
Figure A1. (Cont)



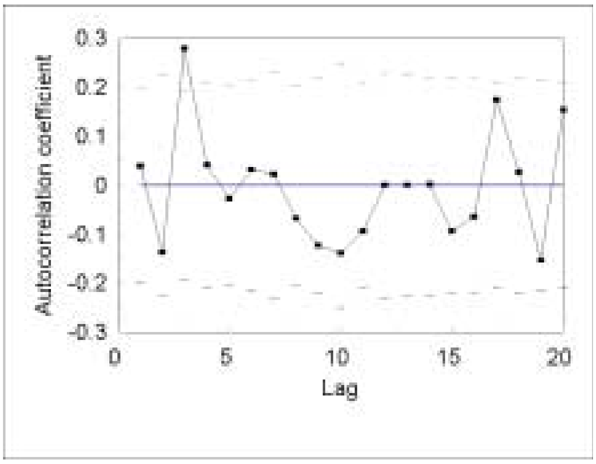


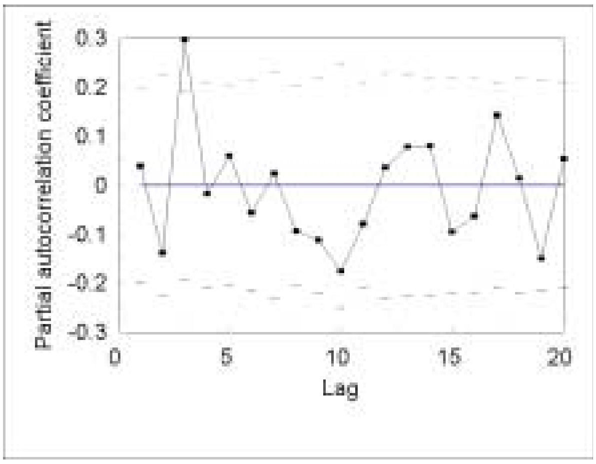
22. Wentworth Post Office





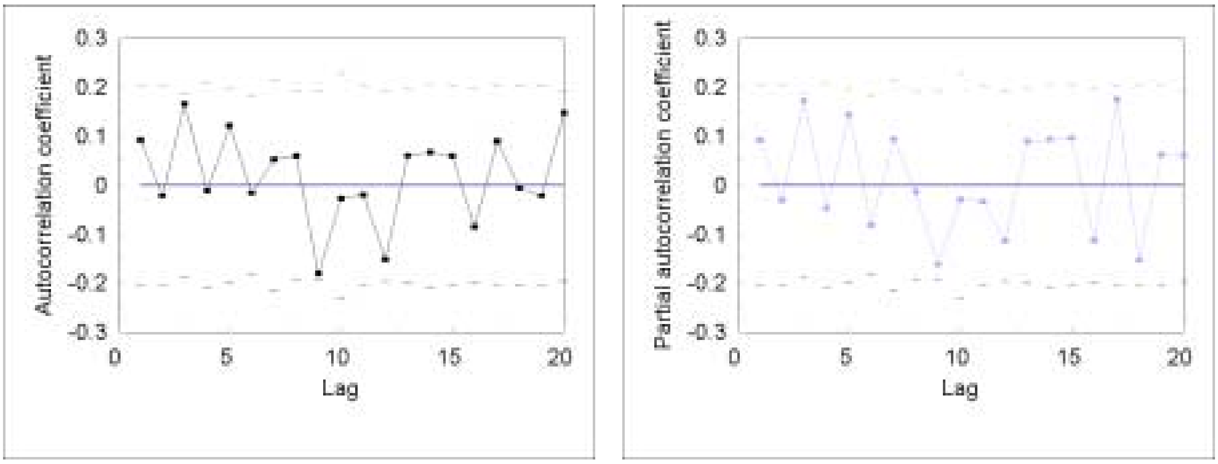
23. Balranald RSL



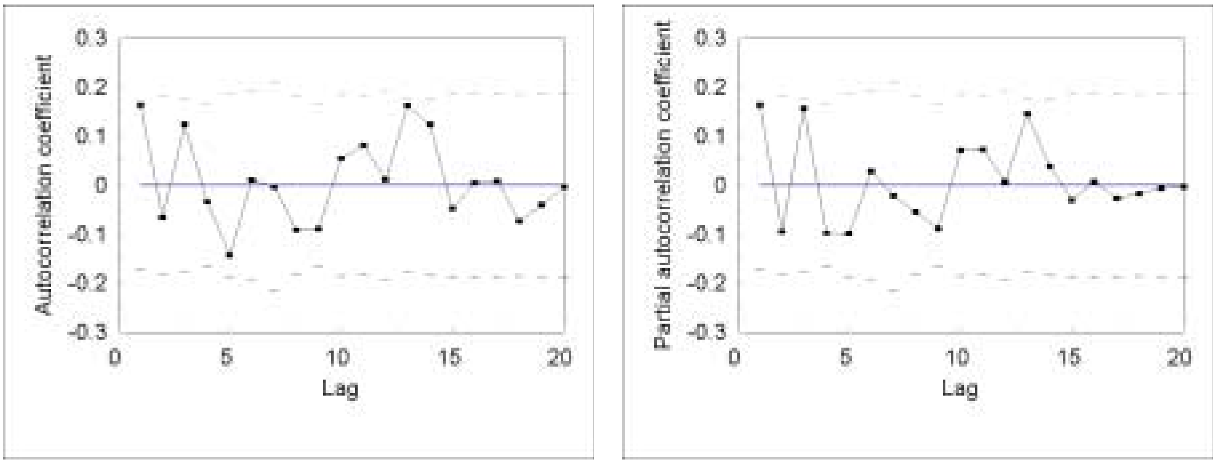


24. Bingara Post Office

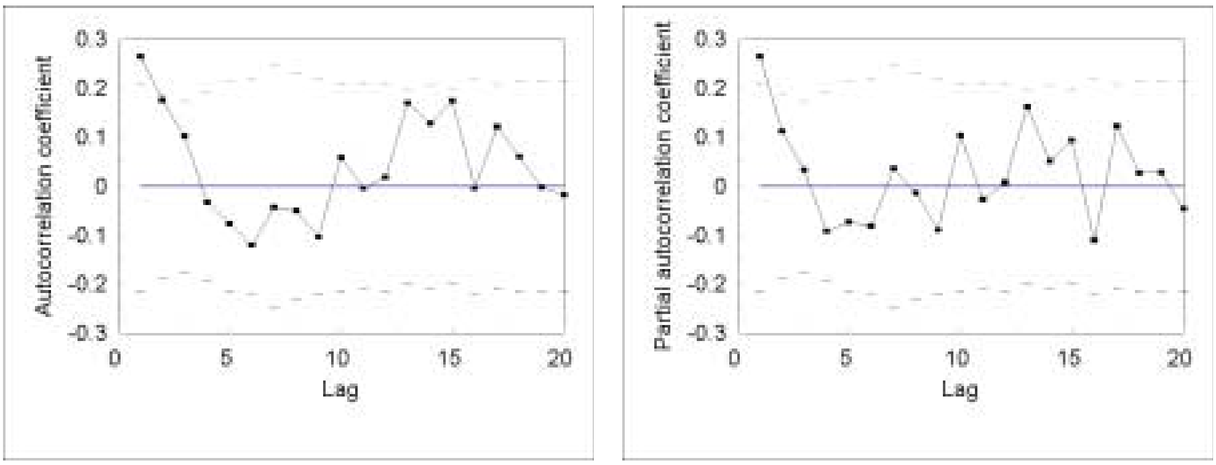
Figure A1. (Cont)



25. Mudgee

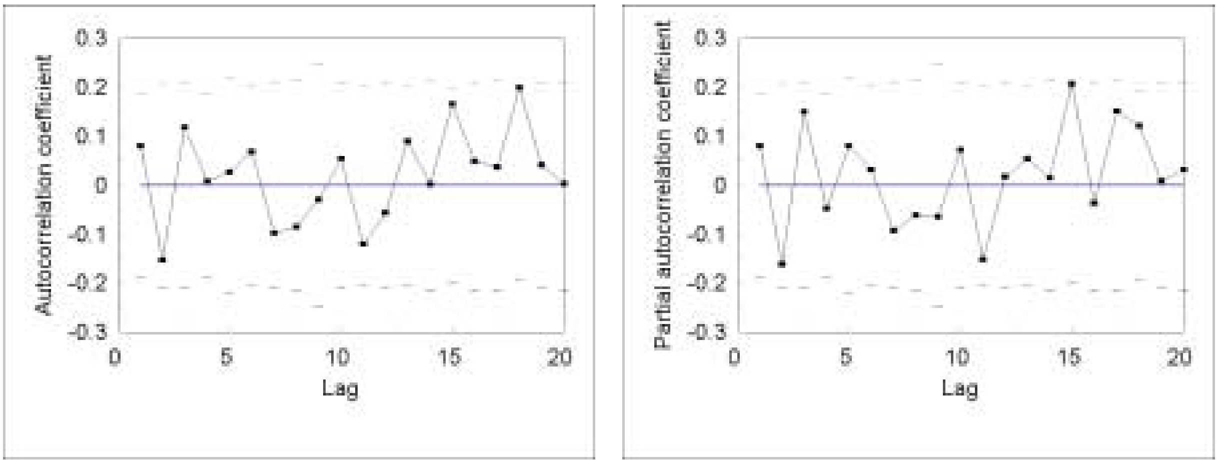


26. Sydney

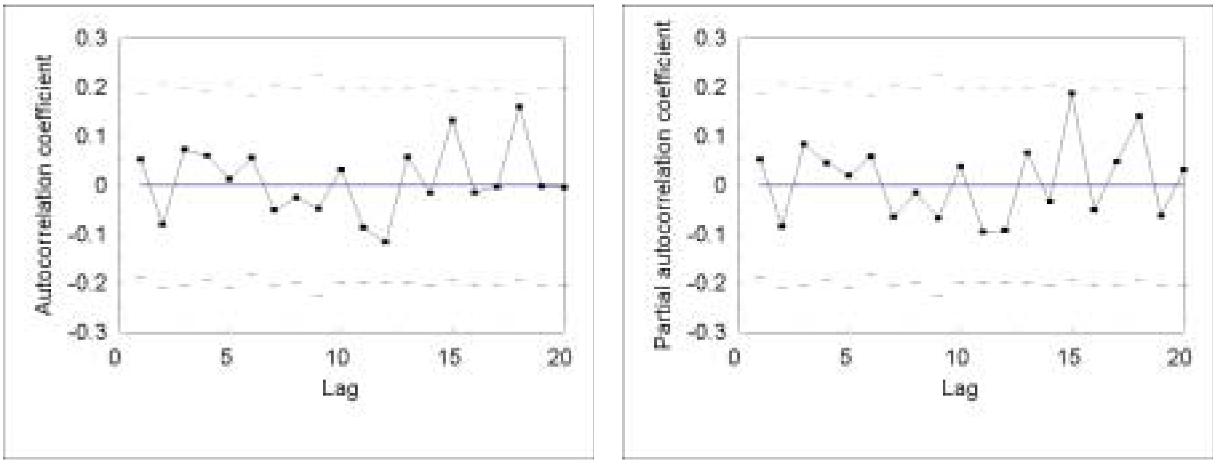


27. Moruya Heads Pilot Station

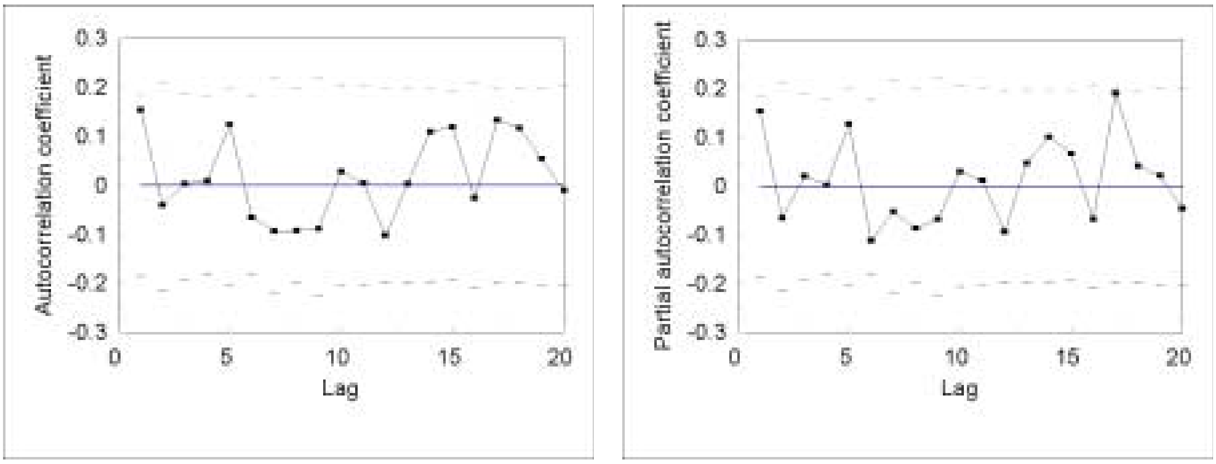
Figure A1. (Cont)



28. Adelong

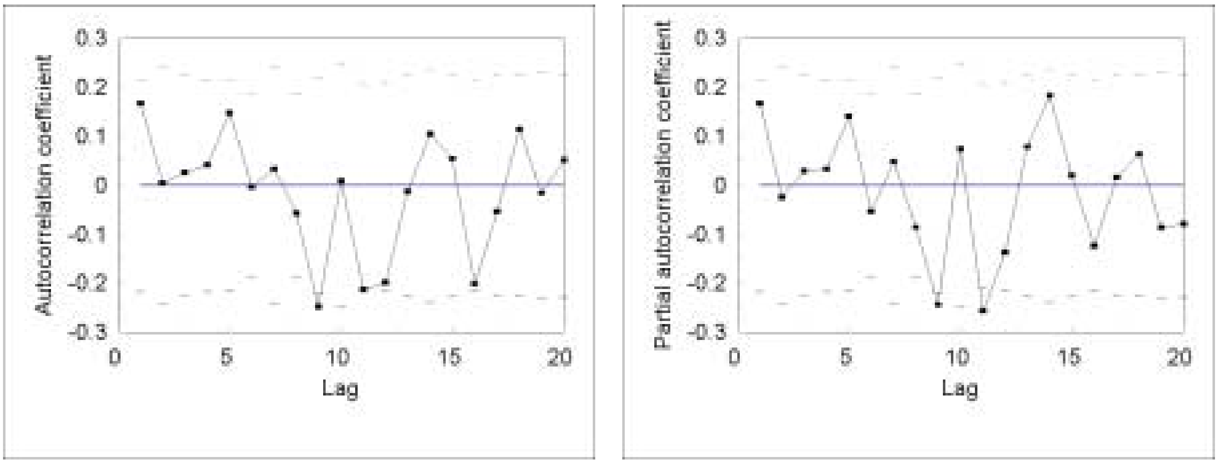


29. Tumut

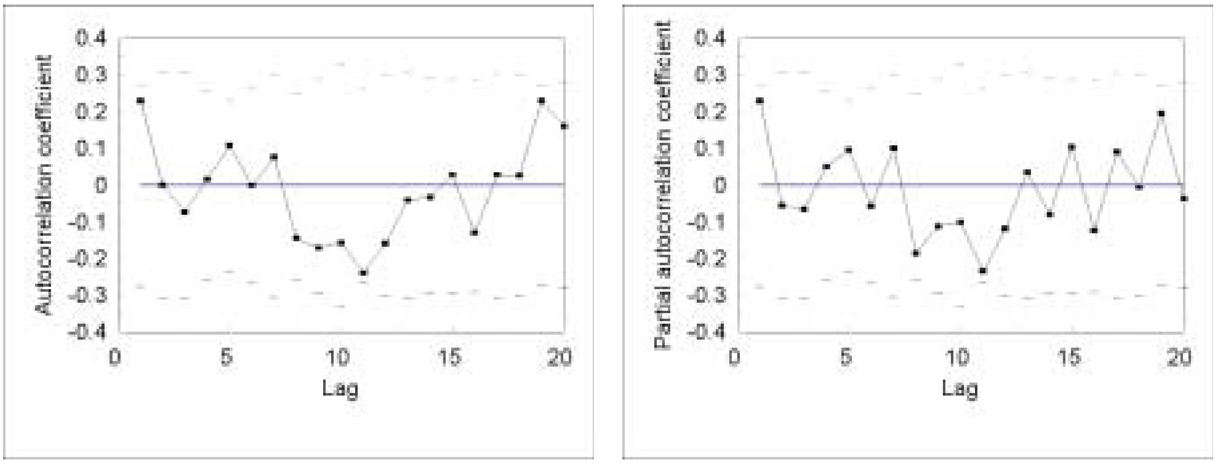


30. Hay Miller Street

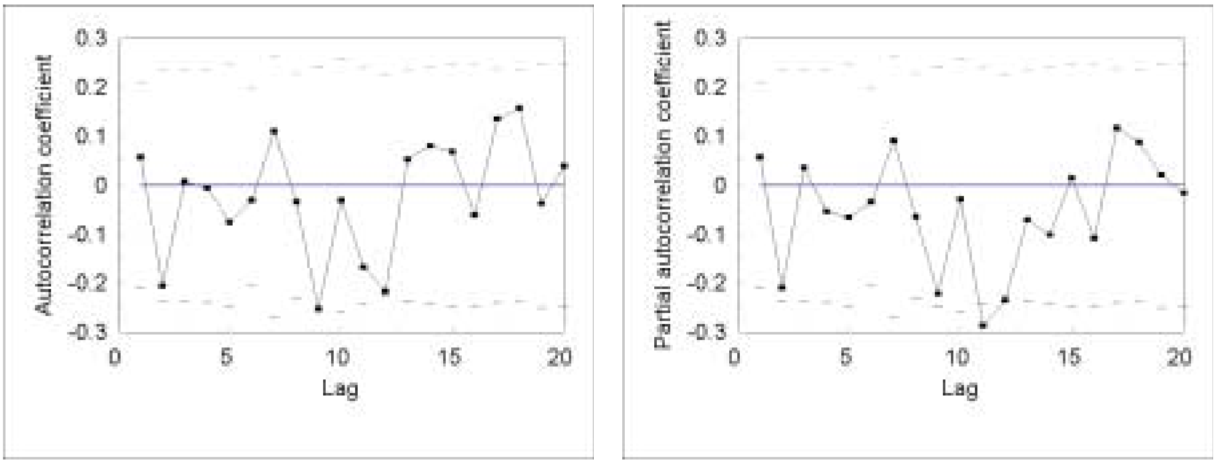
Figure A1. (Cont)



31. Narraport

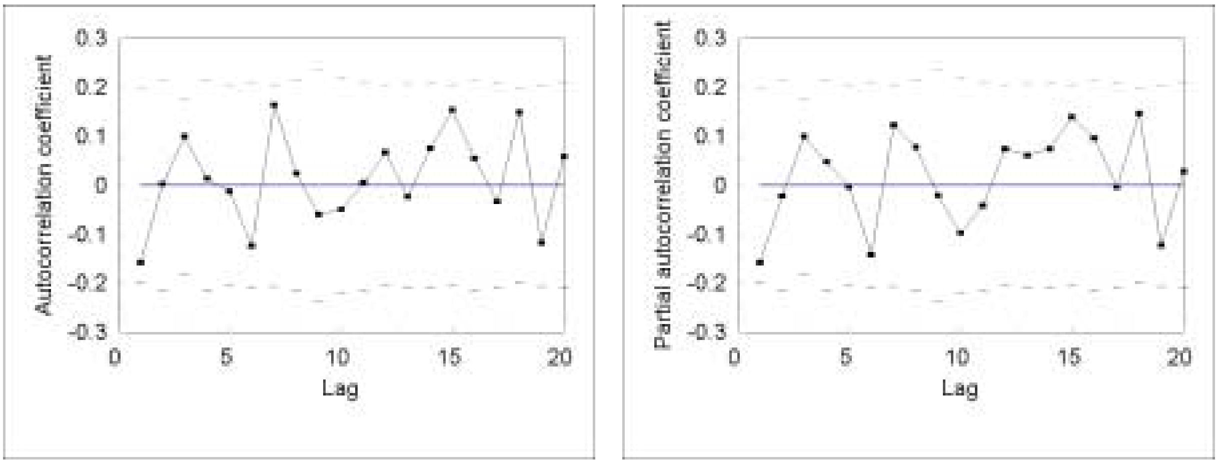


32. Tongala

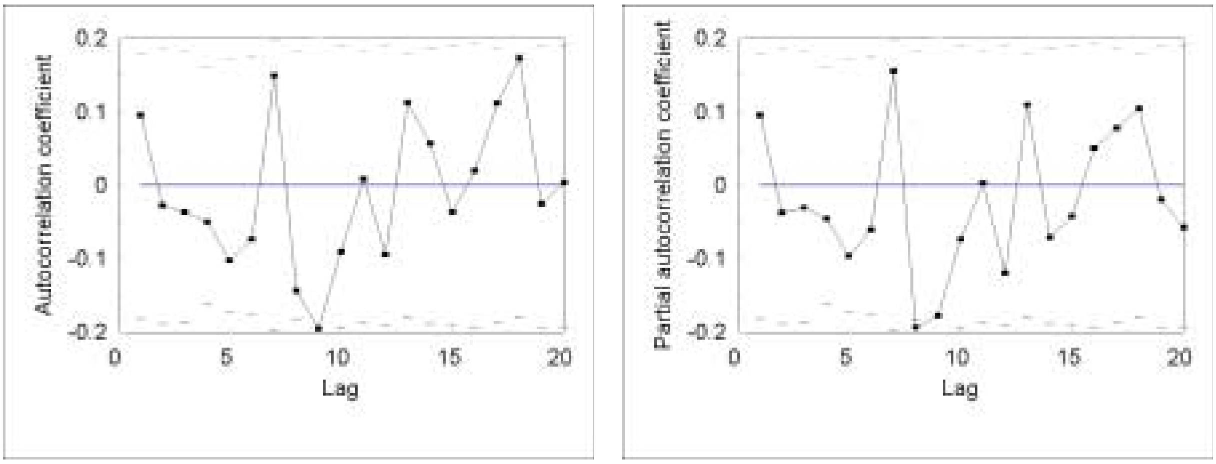


33. Caniambo

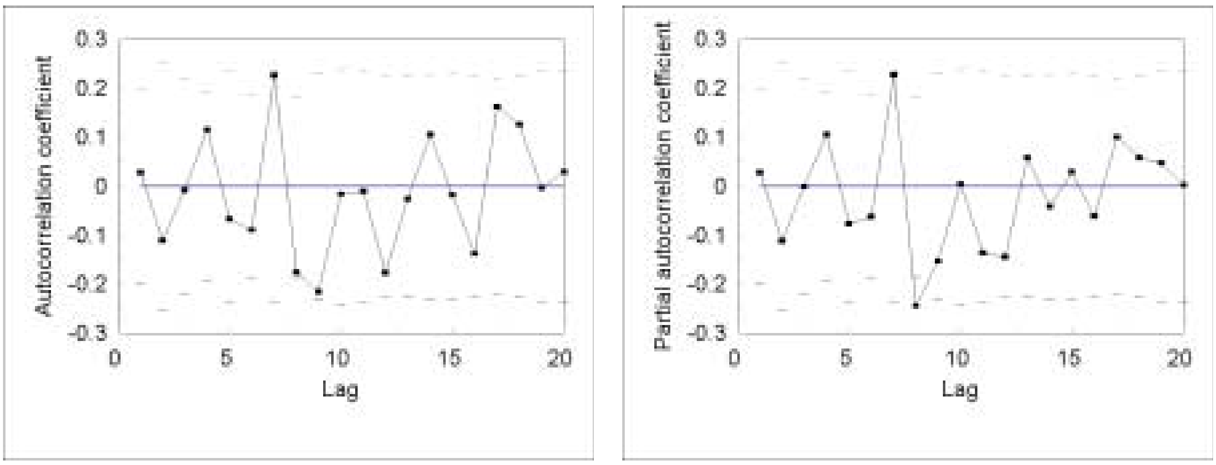
Figure A1. (Cont)



34. Orbost

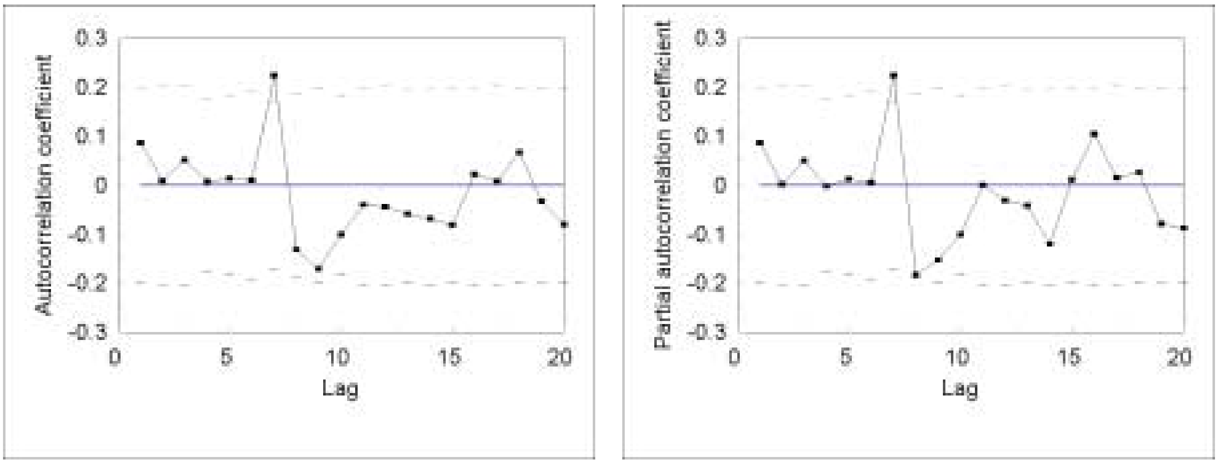


35. Melbourne

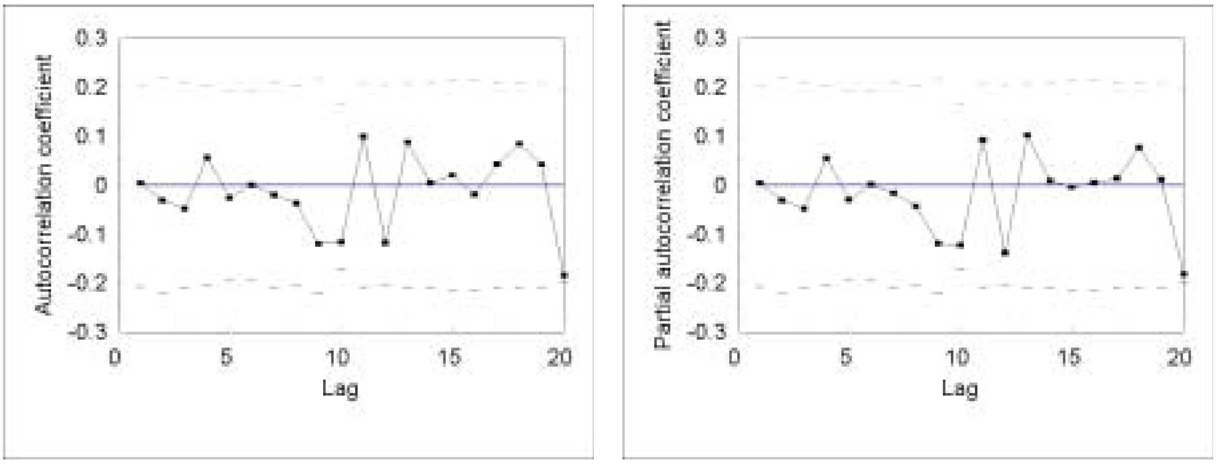


36. Toorourrong

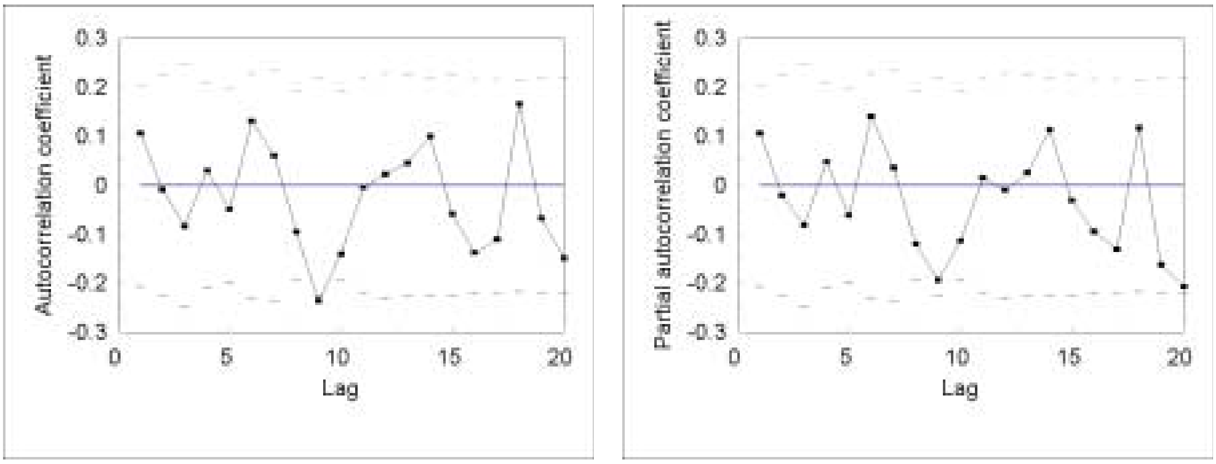
Figure A1. (Cont)



37. Meredith

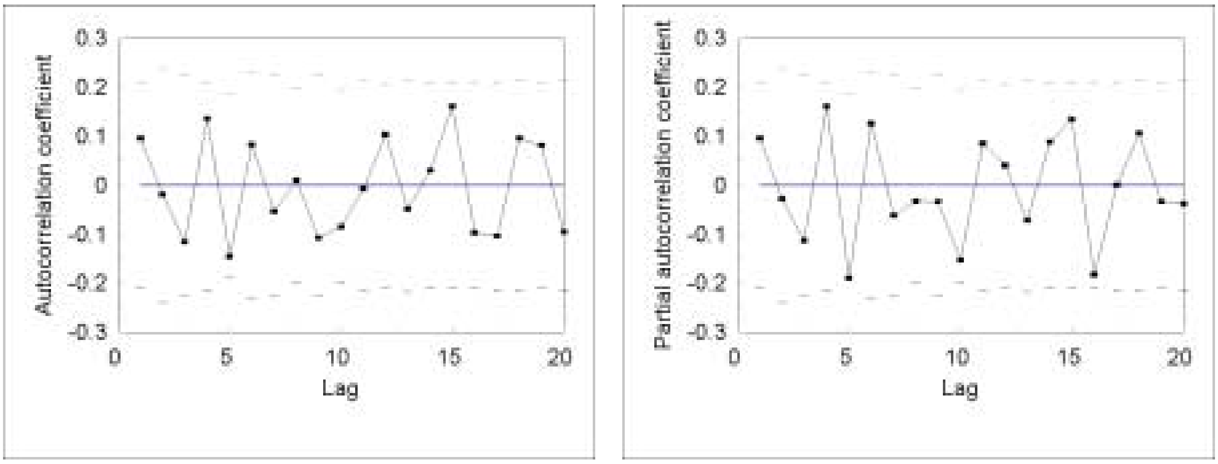


38. Frankford

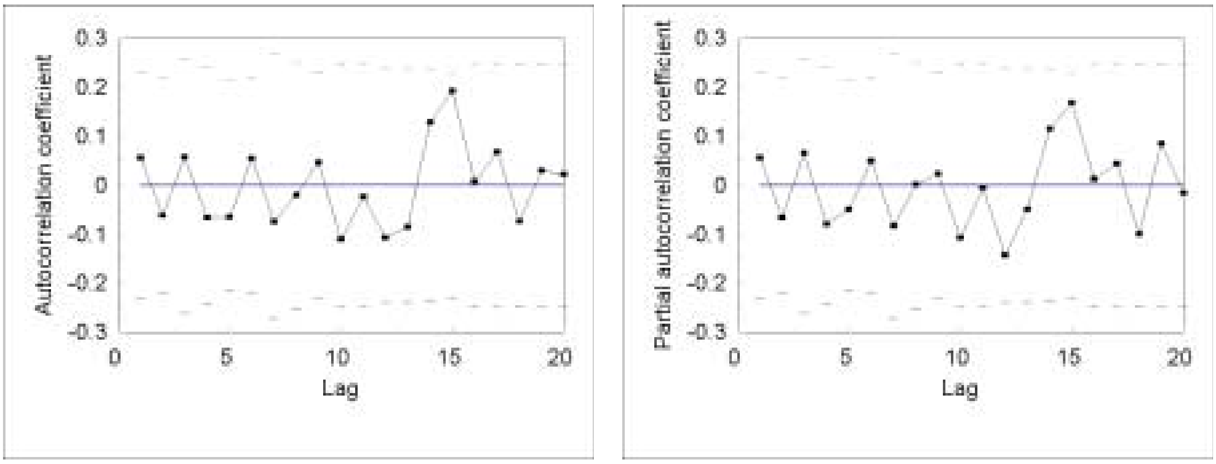


39. Fingal

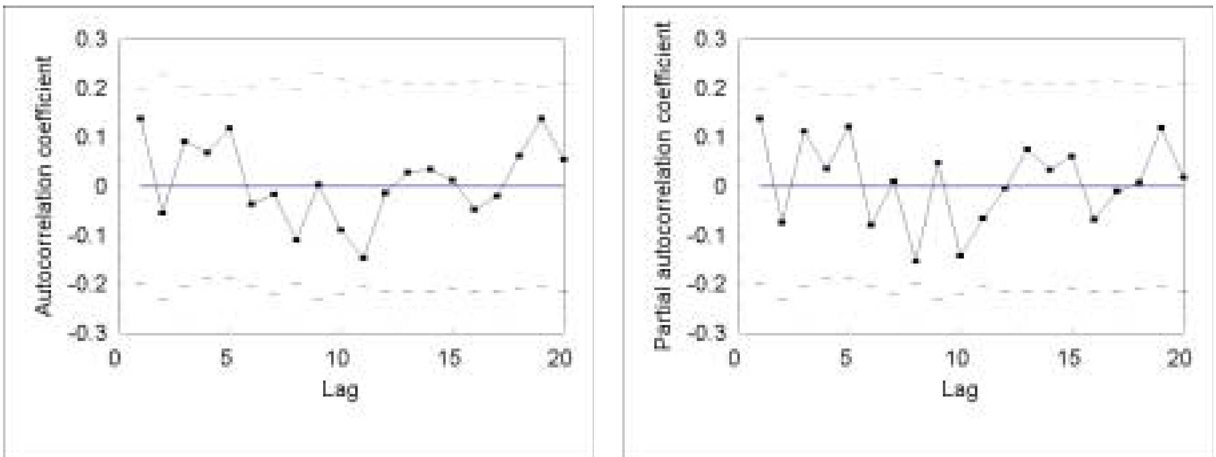
Figure A1. (Cont)



40. Sandford

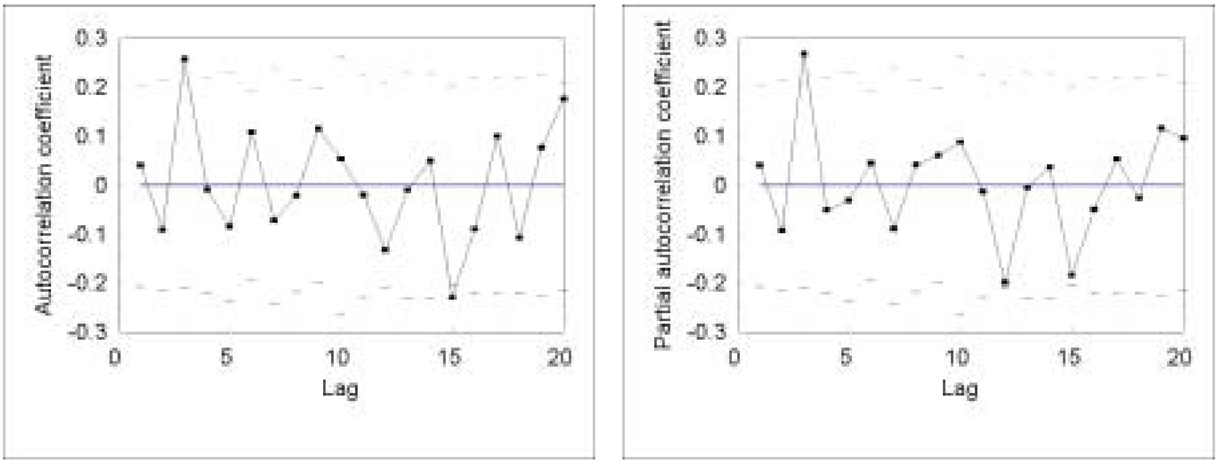


41. Wyndham Port

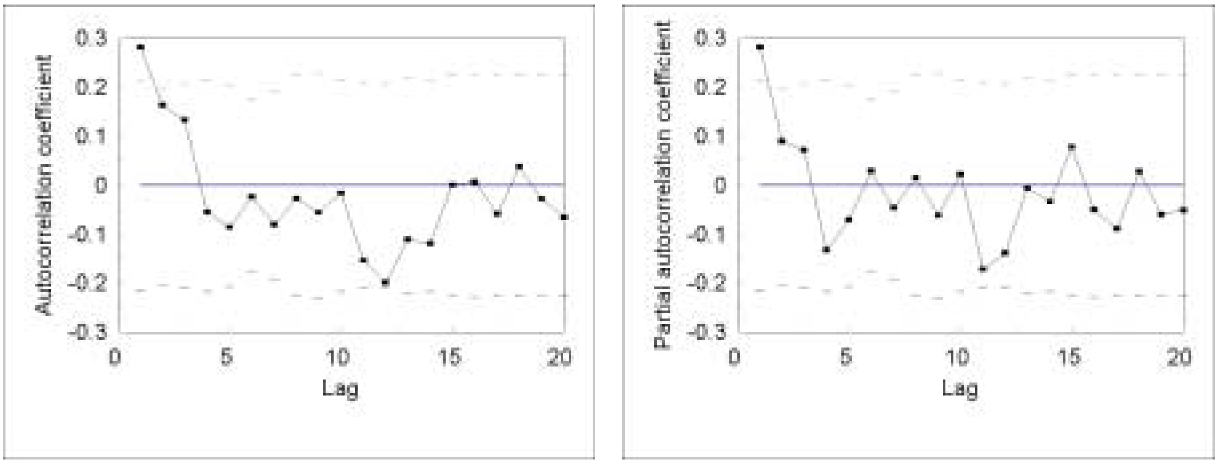


42. Lissadell

Figure A1. (Cont)



43. Katherine Council



44. Alice Springs

Figure A1. (Cont)

ARMA MODEL CALIBRATION RESULTS

The corrected Akaike Information Criterian (AICc) for ARMA(p,q) models where p, q = 0, 1, 2 is given first. The minimum value of AICc is shown in bold. The values of p and q for which AICc is minimum and the minimum value of AICc are then given. If the selected model is not ARMA(0,0) then the parameters of the selected model are also given.

1. Mardee

Model Order		q		
		0	1	2
p	0	**110.037736**	111.612973	113.557710
	1	111.566202	113.103859	115.104642
	2	113.431325	115.141501	115.930020

p & q of minimum AICc and parameters
0 0 110.04

2. Meedo

Model Order		q		
		0	1	2
p	0	96.043478	**91.663633**	92.500052
	1	93.129383	92.340923	94.492512
	2	93.888613	94.494864	97.297515

p & q of minimum AICc and parameters
0 1 91.66

phi .0000 .0000
theta -.3039 .0000

3. Perth

Model Order		q		
		0	1	2
p	0	117.035398	119.082629	120.654885
	1	119.079698	119.490108	120.640178
	2	120.792900	120.653968	**116.876088**

p & q of minimum AICc and parameters
2 2 116.88

phi .1103 .8581
theta -.0076 .9799

4. Cuttening

Model Order		q		
		0	1	2
p	0	**98.042553**	99.279261	100.868464
	1	99.410078	100.967455	103.197778
	2	100.939802	102.468416	104.390484

p & q of minimum AICc and parameters
0 0 98.04

5. Norseman

Model Order		q		
		0	1	2
p	0	**104.040000**	106.052396	108.142497
	1	106.049794	107.996332	110.313237
	2	108.134804	110.275915	111.680444

p & q of minimum AICc and parameters
0 0 104.04

6. Marree

Model Order		q		
		0	1	2
p	0	**113.036697**	114.520963	116.606564
	1	114.545260	116.619972	118.784508
	2	116.585675	118.746962	121.553540

p & q of minimum AICc and parameters
0 0 113.04

7. Orroroo

Model Order		q		
		0	1	2
p	0	**120.034483**	121.202979	123.292729
	1	121.164599	122.389147	124.391160
	2	123.167148	125.316886	124.191108

p & q of minimum AICc and parameters
0 0 120.03

8. Wallaroo

Model Order		q		
		0	1	2
p	0	**137.030075**	138.288944	139.418396
	1	138.435854	140.021108	137.382184
	2	139.317517	138.782182	138.456040

p & q of minimum AICc and parameters
0 0 137.03

9. Adelaide

Model Order		q		
		0	1	2
p	0	**141.029197**	141.804017	143.604504
	1	141.970396	143.806179	145.006218
	2	143.197043	145.135110	144.343431

p & q of minimum AICc and parameters
0 0 141.03

10. Eudunda

Model Order		q		
		0	1	2
p	0	120.034483	119.860100	**117.984845**
	1	120.734785	118.934599	119.846978
	2	119.116100	119.669041	126.102262

p & q of minimum AICc and parameters
0 2 117.98

phi .0000 .0000
theta -.1631 .1897

11. Palmerville

Model Order		q		
		0	1	2
p	0	**111.037383**	112.682321	114.686881
	1	112.708536	114.787764	115.934486
	2	114.733054	115.934783	116.853197

p & q of minimum AICc and parameters
0 0 111.04

12. Kalamia

Model Order		q		
		0	1	2
p	0	**114.036364**	116.014717	118.122672
	1	116.012648	117.359720	117.817906
	2	118.100217	119.173733	118.025358

p & q of minimum AICc and parameters
0 0 114.04

13. Emerald

Model Order		q		
		0	1	2
p	0	114.036364	113.839433	115.943709
	1	**113.827697**	115.936936	118.079431
	2	115.937262	117.899307	123.641155

p & q of minimum AICc and parameters
1 0 113.83
phi 0.1438 0.0000

14. Barcaldine

Model Order		q		
		0	1	2
p	0	114.036364	113.840007	115.943727
	1	113.827678	115.936973	118.076259
	2	115.937254	**112.450372**	122.476076

p & q of minimum AICc and parameters
2 1 112.45

phi 1.0833 -.2019
theta .9791 .0000

15. Cape Capricorn

Model Order		q		
		0	1	2
p	0	**89.047059**	89.736156	90.889567
	1	89.966004	90.308376	92.416352
	2	91.510336	92.405391	93.353931

p & q of minimum AICc and parameters
0 0 89.05

16. Rockhampton

Model Order		q		
		0	1	2
p	0	**98.042553**	99.921610	100.174134
	1	99.975704	99.765524	101.301756
	2	100.476090	101.346557	99.167862

p & q of minimum AICc and parameters
0 0 98.04

17. Cape Moreton

Model Order		q		
		0	1	2
p	0	**131.031496**	132.704429	134.765545
	1	132.717275	134.695634	136.921592
	2	134.786604	136.825790	139.745313

p & q of minimum AICc and parameters
0 0 131.03

18. Brisbane

Model Order		q		
		0	1	2
p	0	**135.030534**	136.591200	138.571284
	1	136.605989	137.061877	137.521082
	2	138.697473	137.417262	138.502485

p & q of minimum AICc and parameters
0 0 135.03

19. Pittsworth

Model Order		q		
		0	1	2
p	0	**112.037037**	112.907541	115.021777
	1	112.904235	115.019729	117.131555
	2	115.017289	116.951529	118.770592

p & q of minimum AICc and parameters
0 0 112.04

20. Miles

Model Order		q		
		0	1	2
p	0	116.035714	**115.312689**	116.907239
	1	115.845040	117.265758	116.713611
	2	116.007361	118.502653	122.226567

p & q of minimum AICc and parameters
0 1 115.31

phi .0000 .0000
theta .1703 .0000

21. Cunnamulla

Model Order		q		
		0	1	2
p	0	**122.033898**	123.180756	124.835287
	1	123.068652	124.381171	126.480286
	2	124.673112	126.483276	125.151345

p & q of minimum AICc and parameters
0 0 122.03

22. Wentworth

Model Order		q		
		0	1	2
p	0	**134.030769**	134.802655	136.888936
	1	134.784681	136.494958	138.585362
	2	136.874878	138.595144	138.868039

p & q of minimum AICc and parameters
0 0 134.03

23. Balranald

Model Order		q		
		0	1	2
p	0	**123.033613**	123.603196	125.628028
	1	123.515052	124.302652	126.385467
	2	125.483570	126.811026	127.086700

p & q of minimum AICc and parameters
0 0 123.03

24. Bingara

Model Order		q		
		0	1	2
p	0	115.036036	117.004092	115.936275
	1	117.033886	119.048986	113.891293
	2	117.261415	**113.124569**	116.710176

p & q of minimum AICc and parameters
2 1 113.12

phi -.6997 -.1799
theta -.7655 .0000

25. Mudgee

Model Order		q		
		0	1	2
p	0	124.033333	125.028333	126.898608
	1	125.086673	126.153976	**117.139207**
	2	127.158694	117.615917	117.287061

p & q of minimum AICc and parameters
1 2 117.14

phi -.7754 .0000
theta -.9486 .0399

26. Sydney

Model Order		q		
		0	1	2
p	0	142.028986	**141.337855**	141.527012
	1	141.942094	141.341798	143.167976
	2	142.668158	143.014843	145.304108

p & q of minimum AICc and parameters
0 1 141.34

phi .0000 .0000
theta -.1642 .0000

27. Moruya

Model Order		q		
		0	1	2
p	0	125.033058	119.534910	120.755238
	1	**118.250507**	119.463039	121.530355
	2	119.618850	121.567569	123.018101

p & q of minimum AICc and parameters
1 0 118.25

phi .2670 .0000

28. Adelong

Model Order		q		
		0	1	2
p	0	117.0354	116.8881	**115.7688**
	1	117.5627	115.7803	117.2147
	2	117.3913	116.9606	123.7492

p & q of minimum AICc and parameters
0 2 115.77

phi .0000 .0000
theta -.1817 .1645

29. Tumut

Model Order		q		
		0	1	2
p	0	**115.0360**	116.0310	117.3502
	1	116.2198	117.3831	119.3550
	2	117.5615	119.3021	122.5667

p & q of minimum AICc and parameters
0 0 115.04

30. Hay

Model Order		q		
		0	1	2
p	0	120.0345	119.7368	121.4405
	1	119.9871	**109.4792**	115.6117
	2	121.7409	115.8133	115.5746

p & q of minimum AICc and parameters
1 1 109.48

phi -.7710 .0000
theta -.9876 .0000

31. Narrapot

Model Order		q		
		0	1	2
p	0	114.0364	**111.8265**	113.9438
	1	111.9413	113.8569	115.7146
	2	114.0104	115.5674	117.1106

p & q of minimum AICc and parameters
0 1 111.83

phi .0000 .0000
theta -.1996 .0000

32. Tongala

Model Order		q		
		0	1	2
p	0	71.0597	67.9582	69.6379
	1	**67.6578**	69.8076	71.8091
	2	69.7843	69.4780	72.2292

p & q of minimum AICc and parameters
1 0 67.66

phi .2852 .0000

33. Caniambo

Model Order		q		
		0	1	2
p	0	97.0430	99.0197	98.7008
	1	99.0530	**96.5324**	100.9694
	2	99.0243	100.9513	98.7425

p & q of minimum AICc and parameters
1 1 96.53

phi .8798 .0000
theta .9942 .0000

34. Orbost

Model Order		q		
		0	1	2
p	0	117.0354	116.4680	118.4295
	1	**116.4476**	118.5261	118.5631
	2	118.4813	119.2332	128.8572

p & q of minimum AICc and parameters
1 0 116.45

phi -.1510 .0000

35. Melbourne

Model Order		q		
		0	1	2
p	0	**145.0284**	145.2688	147.2805
	1	145.3769	147.3009	149.3473
	2	147.2490	145.2866	148.0208

p & q of minimum AICc and parameters
0 0 145.03

36. Toorourrong

Model Order		q		
		0	1	2
p	0	**108.0385**	109.7630	111.3645
	1	109.8194	110.5376	113.5044
	2	111.2129	113.3755	114.5875

p & q of minimum AICc and parameters
0 0 108.04

37. Meredith

Model Order		q		
		0	1	2
p	0	**126.0328**	126.8384	128.9240
	1	126.8063	128.7223	130.7833
	2	128.8846	126.2736	127.2700

p & q of minimum AICc and parameters
0 0 126.03

38. Frankford

Model Order		q		
		0	1	2
p	0	**108.0385**	110.0196	111.9374
	1	110.0279	110.0068	112.0372
	2	111.9660	114.2955	113.1051

p & q of minimum AICc and parameters
0 0 108.04

39. Fingal

Model Order		q		
		0	1	2
p	0	**111.0374**	111.4699	113.5828
	1	111.5758	113.5847	111.3936
	2	113.1395	113.2700	117.0935

p & q of minimum AICc and parameters
0 0 111.04

40. Sandford

Model Order		q		
		0	1	2
p	0	113.0367	114.1183	116.1644
	1	114.1061	116.2144	**111.8697**
	2	116.1975	114.6065	118.3915

p & q of minimum AICc and parameters
1 2 111.87

phi -.8749 .0000
theta -1.0980 -.3136

41. Wyndham

Model Order		q		
		0	1	2
p	0	**81.0519**	82.0842	83.4038
	1	82.2793	83.2040	85.3022
	2	83.8430	85.2729	86.2183

p & q of minimum AICc and parameters
0 0 81.05

42. Lissadell

Model Order		q		
		0	1	2
p	0	**107.0388**	108.0125	109.6348
	1	108.1571	110.1763	109.0579
	2	109.9289	109.0712	111.5578

p & q of minimum AICc and parameters
0 0 107.04

43. Katherine

Model Order		q		
		0	1	2
p	0	**114.0364**	115.4934	117.2559
	1	115.5424	116.6806	116.1536
	2	117.5941	116.1553	117.5702

p & q of minimum AICc and parameters
0 0 114.04

44. Alice Springs

Model Order		q		
		0	1	2
p	0	114.0364	110.7688	112.3290
	1	109.8409	110.5695	112.3651
	2	110.9206	112.5848	114.7754

p & q of minimum AICc and parameters
1 0 109.84

phi .2390 .0000

APPENDIX B – DERIVATION OF SKEWNESS FOR RANDOM NUMBERS

B1. Moving average process

A moving average process of order q [MA(q)] is given by

$$z_t = a_t - \theta_1 a_{t-1} - \ldots - \theta_q a_{t-q} \tag{B1}$$

where z_t is a zero mean MA process and $\theta_1, \theta_2, \ldots, \theta_q$ are the MA parameters.

Squaring both sides of Eq (B1) and taking expectations, the variance of z_t is given by

$$\sigma_z^2 = (1 + \theta_1^2 + \theta_2^2 + \ldots + \theta_q^2)\sigma_a^2 \tag{B2}$$

Cubing both sides of Eq (B1) and taking expectations results in

$$E[z_t^3] = E[a_t^3] + \theta_1^3 E[a_{t-1}^3] + \ldots + \theta_q^3 E[a_{t-q}^3] \tag{B3}$$

Dividing both sides of Eq (B3) by from Eq (B2),

$$\gamma_z = \frac{1 + \theta_1^3 + \ldots + \theta_q^3}{(1 + \theta_1^2 + \ldots + \theta_q^2)^{3/2}} \gamma_a \tag{B4}$$

The skewness of the random numbers for a MA(q) process is

$$\gamma_a = \frac{(1 + \theta_1^2 + \ldots + \theta_q^2)^{3/2}}{1 + \theta_1^3 + \ldots + \theta_q^3} \gamma_z \tag{B5}$$

B2. Autoregressive process

Since the autoregressive (AR) terms are dependent on the past events, it is quite complicated to derive the skewness for the random numbers for an AR process of order p. The skewness of the random numbers for AR(1) is well known. The skewness of the random numbers for AR(2) is obtained from the residuals (a_t).

An AR(2) process is given by

$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + a_t \tag{B6}$$

where z_t is a zero mean AR process, ϕ_1 and ϕ_2 the AR parameters and a_t the noise term with zero mean and variance σ_a^2.

The variance of the random numbers is given by

$$\sigma_a^2 = (1-\rho_1\phi_1 - \rho_2\phi_2)\sigma_z^2$$
$$= \left(\frac{1+\phi_2}{1-\phi_2}\right)\{(1-\phi_2)^2 - \phi_1^2\}\sigma_z^2 \quad \text{(B7)}$$

The residual series can be obtained by rearranging Eq (B6).

$$a_t = z_t - \phi_1 z_{t-1} - \phi_2 z_{t-2} \quad \text{(B8)}$$

B3. Autoregressive moving average process

Since the autoregressive terms are dependent on the past events, it is also quite complicated to derive the skewness for the random numbers for an ARMA process of order (p,q). Only, the skewness of the random numbers for an ARMA(1,1) process is derived below:

An ARMA(1,1) process is given by

$$z_t = \phi_1 z_{t-1} + a_t - \theta_1 a_{t-1} \quad \text{(B9)}$$

Squaring both sides of Eq (B8) and taking expectations

$$\sigma_a^2 = \frac{1-\phi_1^2}{1+\theta_1^2 - 2\phi_1\theta_1}\sigma_z^2 \quad \text{(B10)}$$

Cubing Squaring both sides of Eq(B8) and taking expectations

$$E[z_t^3] = \phi_1^3 E[z_{t-1}^3] + E[a_t^3] - \theta_1^3 E[a_{t-1}^3] - 3\phi_1^2\theta_1 E[z_{t-1}^2 a_{t-1}] + 3\phi_1\theta_1^2 E[z_{t-1} a_{t-1}^2] \quad \text{(B11)}$$

For time t = t-1, the ARMA(1,1) process is

$$z_{t-1} = \phi_1 z_{t-2} + a_{t-1} - \theta_1 a_{t-2} \quad \text{(B12)}$$

Multiplying both sides of Eq (B12) by a_{t-1}^2 and taking expectations

$$E[z_{t-1} a_{t-1}^2] = E[a_{t-1}^3] \quad \text{(B13)}$$

Squaring both sides of Eq (B12), multiplying by a_{t-1} and taking expectations

$$E[z_{t-1}^2 a_{t-1}] = E[a_{t-1}^3] \quad \text{(B14)}$$

Substituting Eq (B12) and (B13) in Eq (B10),

$$(1-\phi_1^3)E[z_t^3] = (1-\theta_1^3 - 3\phi_1^2\theta_1 + 3\phi_1\theta_1^2)E[a_t^3] \quad \text{(B15)}$$

Dividing both sides by σ_a^3 and rearranging,

$$\gamma_a = \left\{\frac{1-\phi_1^3}{1-\theta_1^3 - 3\phi_1^2\theta_1 + 3\phi_1\theta_1^2}\right\}\left\{\frac{1+\theta_1^2 - 2\phi\theta}{1-\phi_1^2}\right\}^{3/2}\gamma_z \qquad \text{(B16)}$$

For an ARMA(1,2) process

$$z_t = -\phi_1 z_{t-1} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} \qquad \text{(B17)}$$

The variance of a_t is given by

$$\sigma_a^2 = \frac{1-\phi_1^2}{1+\theta_1^2 + \theta_2^2 - 2\phi_1\theta_1 - 2\phi_1^2\theta_2 + 2\phi_1\theta_1\theta_2}\sigma_z^2 \qquad \text{(B18)}$$

The variance of a_t for an ARMA(2,1) is

$$\sigma_a^2 = \quad = \left(\frac{1+\phi_2}{1-\phi_2}\right)\left(\frac{(1-\phi_2)^2 - \phi_1^2}{1+\theta_1^2 - 2\phi_1\theta_1}\right)\sigma_z^2 \qquad \text{(B19)}$$

The variance of a_t for an ARMA(2,2) process is

$$\sigma_a^2 = \quad = \left(\frac{1+\phi_2}{1-\phi_2}\right)\left(\frac{(1-\phi_2)^2 - \phi_1^2}{1+\theta_1^2 + \theta_2^2 - 2\phi_1\theta_1 - 2\phi_2\theta_2 - 2\phi_1^2\theta_2 + 2\phi_1\theta_1\theta_2}\right)\sigma_z^2 \qquad \text{(B20)}$$

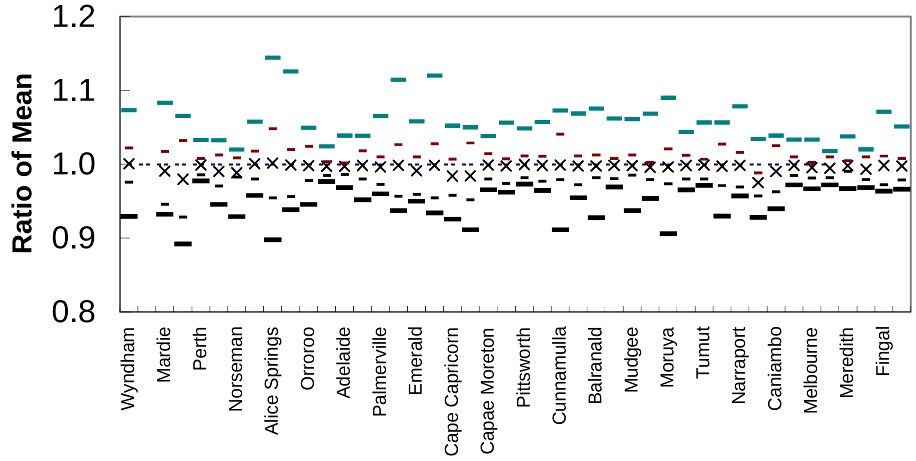
The skewness of a_t can be obtained empirically from the residual series which is obtained from

$$a_t = z_t - \phi_1 z_{t-1} - \ldots - \phi_p z_{t-p} + \theta_1 a_{t-1} + \ldots + \theta_q a_{t-q} \qquad \text{(B21)}$$

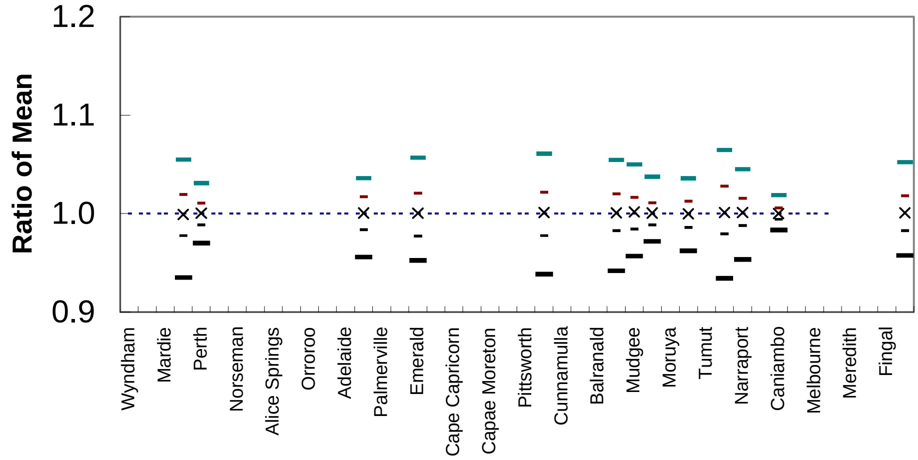
APPENDIX C – MODEL EVALUATION

Table C1. Comparison of the historical and generated mean annual rainfall (mm).

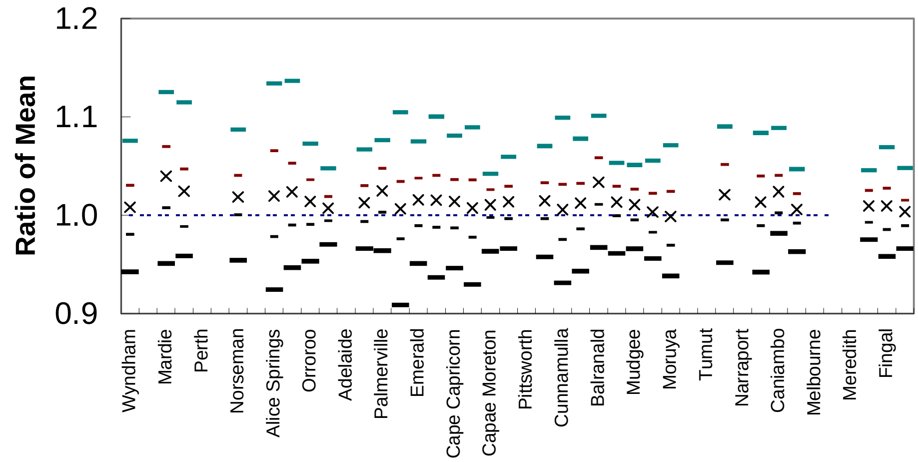
Station	Historical	ARM	ARMA	HSM
Wyndham	695	696		701
Lissadell	616	616		
Mardie	276	273		287
Meedo	216	211	215	221
Perth	868	867	869	
Cuttening	312	309		
Norseman	287	283		292
Katherine	974	974		
Alice Springs	280	280		285
Marree	164	164		168
Orroroo	341	340		346
Wallaroo	360	359		363
Adelaide	530	528		
Eudunda	446	446	447	452
Palmerville	1034	1030		1060
Kalamia	1085	1083		1092
Emerald	642	636	642	652
Barcaldine	496	496		504
Cape Capricorn	801	789		813
Rockhampton	946	932		954
Cape Moreton	1550	1548		1566
Brisbane	1154	1152		1170
Pittsworth	703	702		
Miles	661	659	661	670
Cunnamulla	374	374		376
Wentworth	288	288		292
Balranald	322	321		333
Bingara	745	744	745	755
Mudgee	670	669	671	677
Sydney	1226	1220	1227	1230
Moruya Heads	972	969		971
Adelong	795	794	795	
Tumut	822	821		
Hay	369	368	369	377
Narraport	354	354	355	
Tongala	443	432		449
Caniambo	524	519	524	537
Orbost	855	854		860
Melbourne	657	654		
Toorourrong	804	800		
Meredith	685	683		
Frankford	1069	1062		1079
Fingal	611	610		617
Sandford	578	577	579	580



(a) AR(1)



(b) ARMA

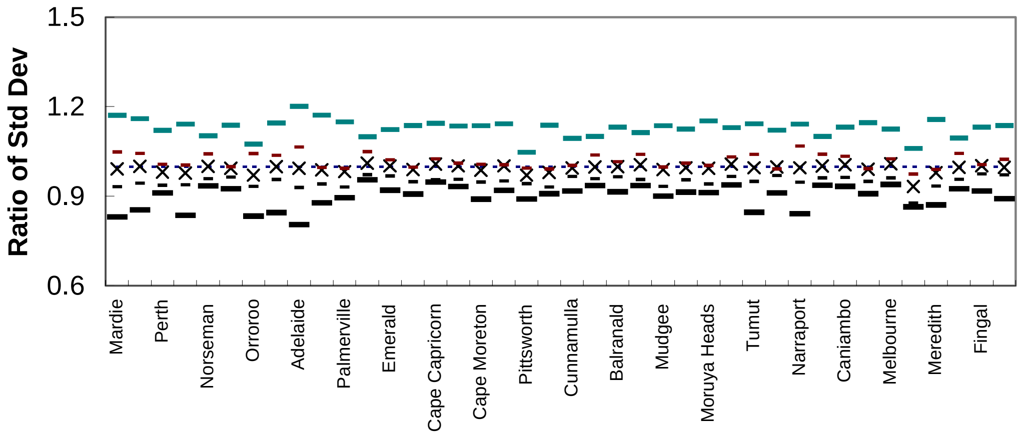


(c) HSM

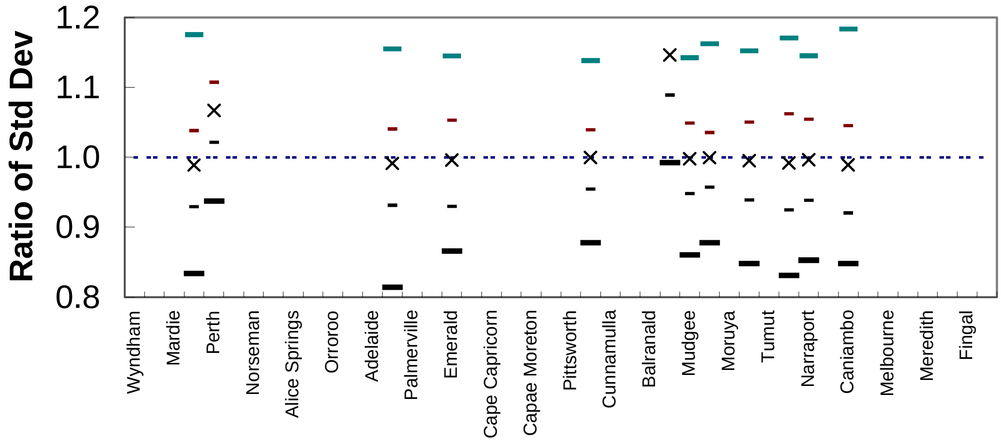
Figure C1. Comparison of the historical and generated mean annual rainfall.

Table C2. Comparison of the historical and generated standard deviation of annual rainfall (mm).

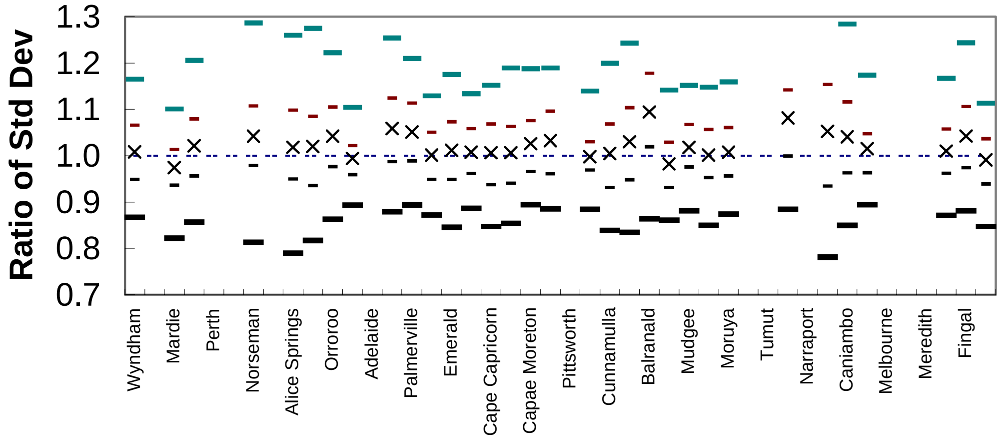
Station	Historical	ARM	ARMA	HSM
Wyndham	220	218		222
Lissadell	182	182		
Mardie	172	168		167
Meedo	90	88	89	91
Perth	158	158	169	
Cuttening	76	75		
Norseman	91	88		94
Katherine	269	268		
Alice Springs	141	140		143
Marree	84	83		85
Orroroo	110	108		115
Wallaroo	83	84		83
Adelaide	106	106		
Eudunda	114	112	113	120
Palmerville	316	318		333
Kalamia	502	502		502
Emerald	212	209	211	215
Barcaldine	214	215		216
Cape Capricorn	267	259		268
Rockhampton	360	353		363
Cape Moreton	395	392		405
Brisbane	369	368		381
Pittsworth	163	163		
Miles	214	215	214	213
Cunnamulla	158	156		158
Wentworth	100	100		103
Balranald	109	108		119
Bingara	201	203	231	198
Mudgee	174	173	174	177
Sydney	335	334	334	335
Moruya	313	312		316
Adelong	191	191	190	
Tumut	191	192		
Hay	127	125	126	137
Narraport	103	103	102	
Tongala	122	114		129
Caniambo	140	137	138	145
Orbost	201	200		204
Melbourne	122	122		
Toorourrong	159	158		
Meredith	130	131		
Frankford	232	230		234
Fingal	167	166		174
Sandford	126	126	126	125



(a) AR(1)



(b) ARMA

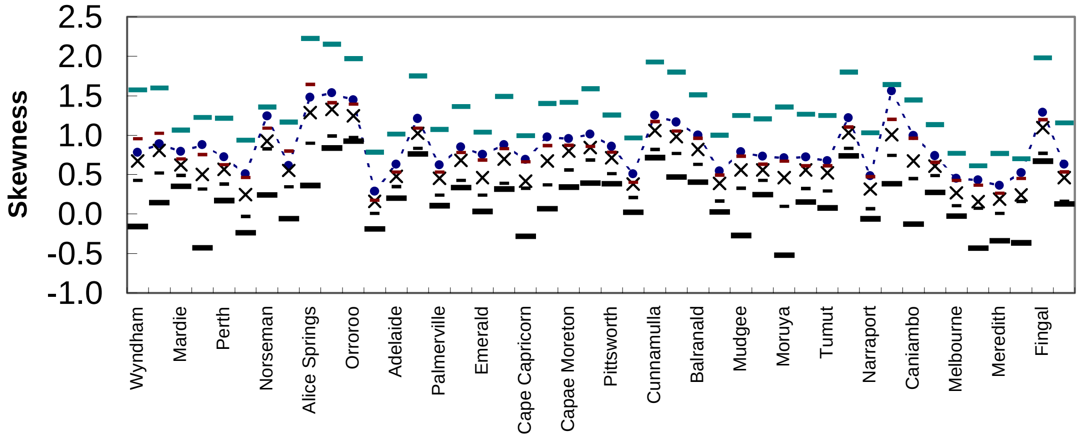


(c) HSM

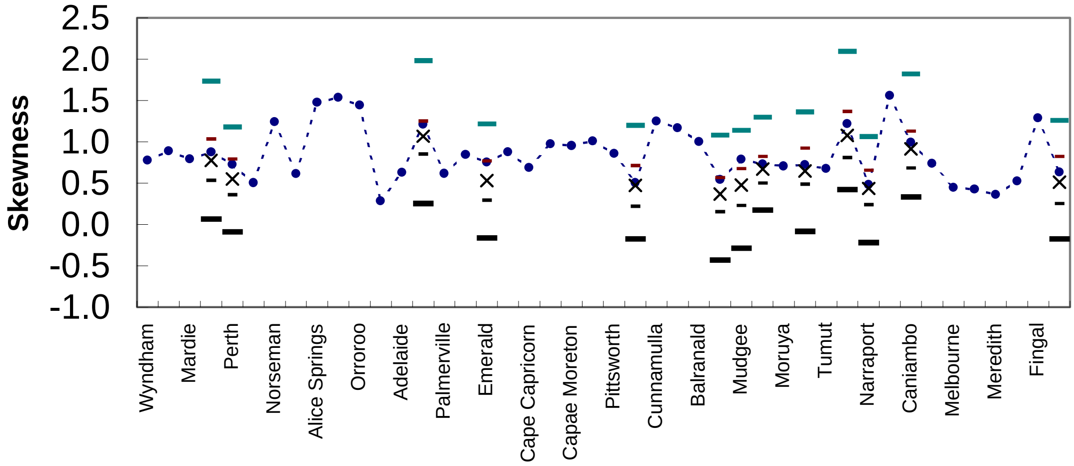
Figure C2. Comparison of the historical and generated standard deviation of annual rainfall.

Table C3. Comparison of the historical and generated skewness of annual rainfall.

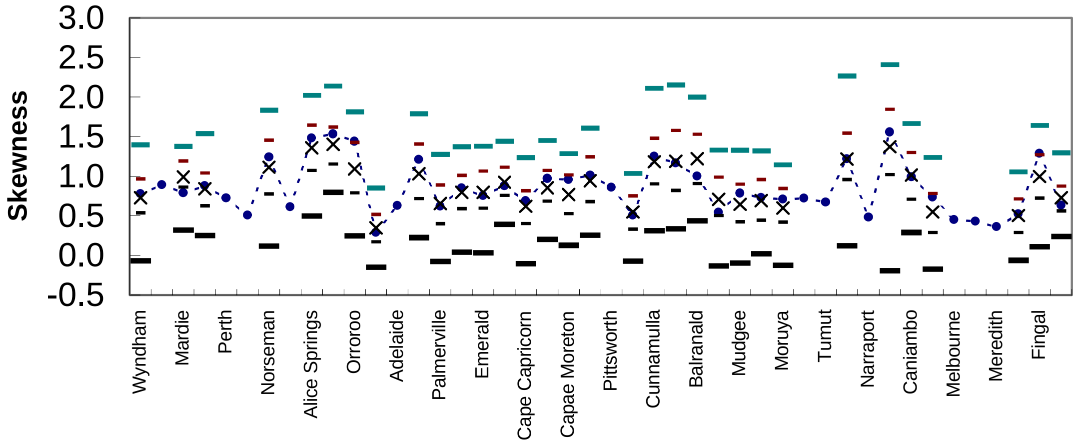
Station	Historical	ARM	ARMA	HSM
Wyndham	0.777	0.675		0.726
Lissadell	0.889	0.804		
Mardie	0.791	0.622		0.989
Meedo	0.876	0.500	0.777	0.842
Perth	0.724	0.565	0.552	
Cuttening	0.506	0.248		
Norseman	1.241	0.922		1.115
Katherine	0.610	0.553		
Alice Springs	1.478	1.286		1.357
Marree	1.533	1.324		1.403
Orroroo	1.443	1.244		1.093
Wallaroo	0.285	0.160		0.347
Adelaide	0.628	0.475		
Eudunda	1.211	1.025	1.069	1.031
Palmerville	0.617	0.451		0.654
Kalamia	0.847	0.679		0.796
Emerald	0.753	0.463	0.532	0.798
Barcaldine	0.876	0.699		0.924
Cape Capricorn	0.688	0.416		0.621
Rockhampton	0.972	0.674		0.852
Cape Moreton	0.954	0.799		0.769
Brisbane	1.009	0.843		0.942
Pittsworth	0.857	0.713		
Miles	0.506	0.379	0.472	0.547
Cunnamulla	1.249	1.060		1.183
Wentworth	1.167	0.982		1.191
Balranald	1.001	0.819		1.22
Bingara	0.541	0.391	0.365	0.707
Mudgee	0.785	0.560	0.477	0.644
Sydney	0.729	0.559	0.672	0.692
Moruya	0.708	0.461		0.6
Adelong	0.721	0.558	0.649	
Tumut	0.674	0.521		
Hay	1.217	1.029	1.079	1.218
Narraport	0.482	0.318	0.437	
Tongala	1.557	1.007		1.372
Caniambo	0.991	0.673	0.917	1.013
Orbost	0.738	0.607		0.549
Melbourne	0.447	0.270		
Toorourrong	0.427	0.159		
Meredith	0.359	0.189		
Frankford	0.522	0.239		0.505
Fingal	1.286	1.095		0.997
Sandford	0.630	0.464	0.511	0.726



(a) AR(1)



(b) ARMA

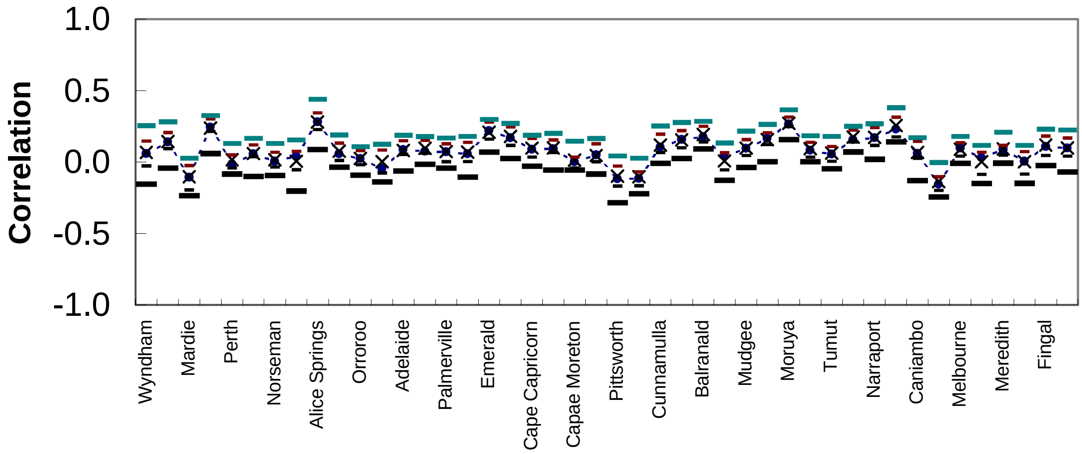


(c) HSM

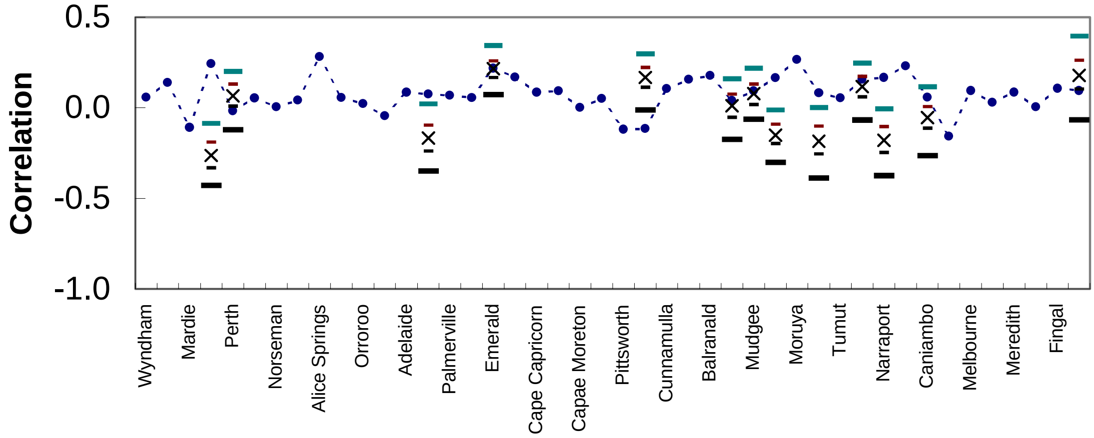
Figure C3. Comparison of the historical and generated skewness of annual rainfall.

Table C4. Comparison of the historical and generated lag one autocorrelation coefficient of annual rainfall.

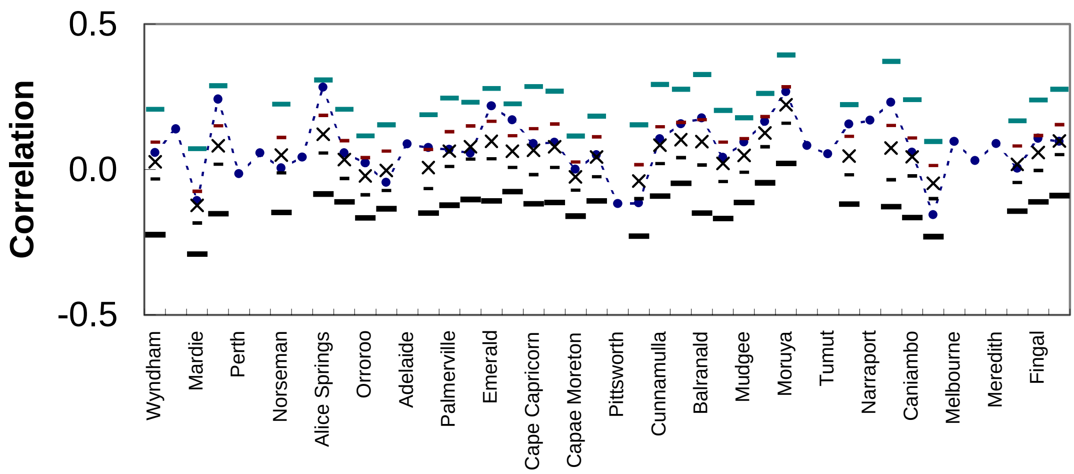
Station	Historical	ARM	ARMA	HSM
Wyndham	0.057	0.067		0.026
Lissadell	0.139	0.143		
Mardie	-0.108	-0.104		-0.124
Meedo	0.242	0.237	-0.263	0.081
Perth	-0.016	0.015	0.066	
Cuttening	0.055	0.060		
Norseman	0.005	0.015		0.049
Katherine	0.041	0.006		
Alice Springs	0.282	0.281		0.121
Marree	0.056	0.076		0.034
Orroroo	0.022	0.024		-0.023
Wallaroo	-0.045	0.001		-0.004
Adelaide	0.086	0.081		
Eudunda	0.074	0.095	-0.166	0.007
Palmerville	0.068	0.072		0.063
Kalamia	0.056	0.068		0.078
Emerald	0.218	0.208	0.216	0.097
Barcaldine	0.169	0.179		0.062
Cape Capricorn	0.087	0.097		0.066
Rockhampton	0.092	0.100		0.078
Cape Moreton	0.000	0.013		-0.025
Brisbane	0.050	0.046		0.042
Pittsworth	-0.118	-0.099		
Miles	-0.116	-0.103	0.168	-0.040
Cunnamulla	0.104	0.118		0.084
Wentworth	0.156	0.155		0.102
Balranald	0.176	0.194		0.096
Bingara	0.040	0.007	0.011	0.021
Mudgee	0.093	0.097	0.079	0.048
Sydney	0.164	0.156	-0.151	0.125
Moruya	0.266	0.272		0.222
Adelong	0.081	0.093	-0.184	
Tumut	0.053	0.063		
Hay	0.155	0.174	0.118	0.046
Narraport	0.168	0.173	-0.179	
Tongala	0.230	0.256		0.073
Caniambo	0.058	0.066	-0.054	0.043
Orbost	-0.157	-0.137		-0.048
Melbourne	0.096	0.089		
Toorourrong	0.029	0.001		
Meredith	0.087	0.091		
Frankford	0.005	0.002		0.017
Fingal	0.107	0.118		0.058
Sandford	0.096	0.100	0.179	0.098



(a) AR(1)



(b) ARMA

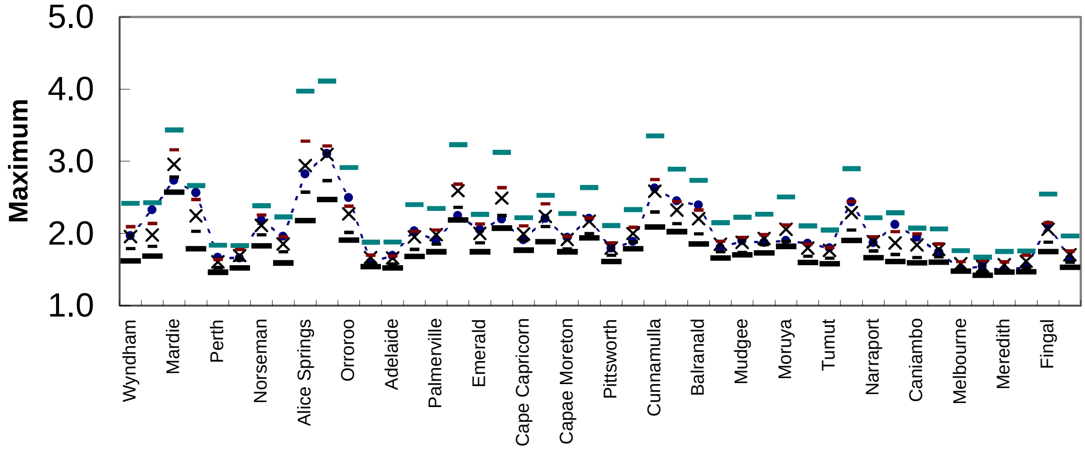


(c) HSM

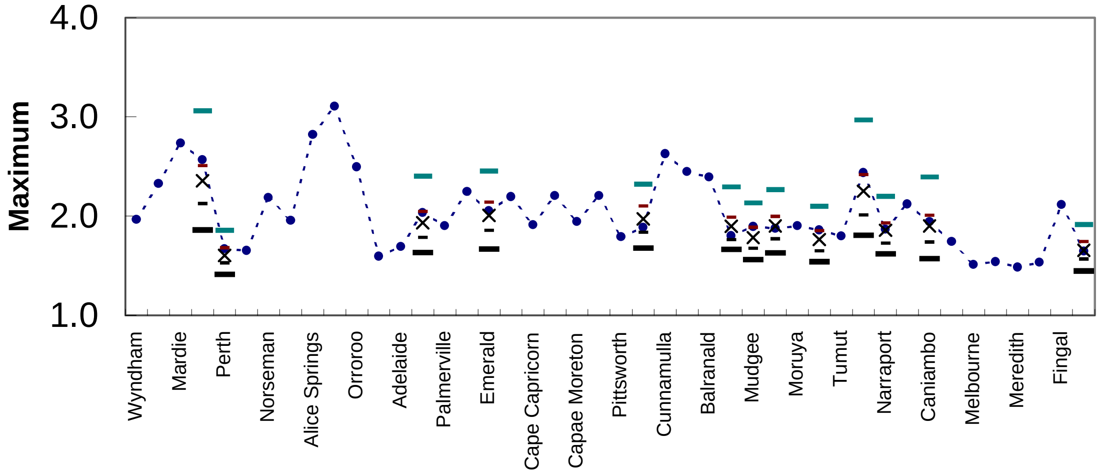
Figure C4. Comparison of the historical and generated lag one autocorrelation coefficient of annual rainfall.

Table C5. Comparison of the historical and generated maximum annual rainfall.

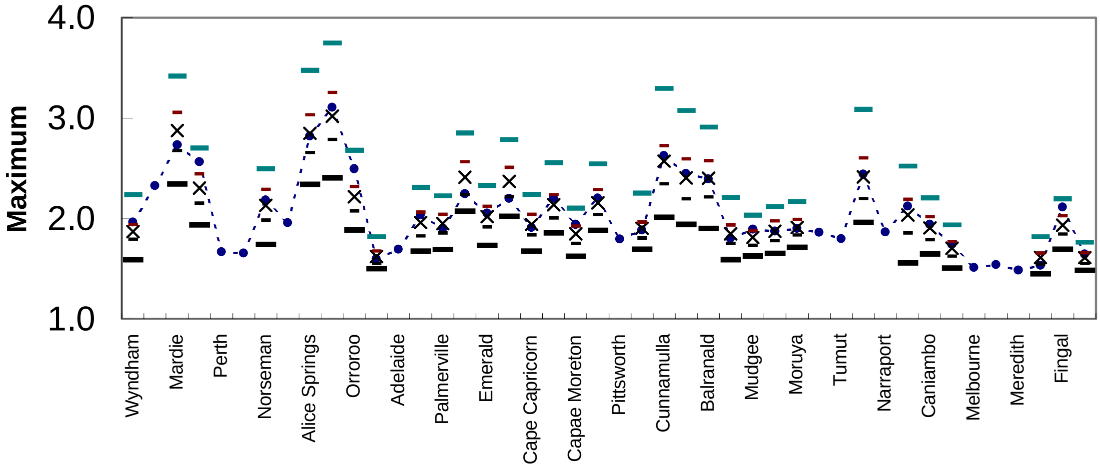
Station	Historical	ARM	ARMA	HSM
Wyndham	1.963	1.956		1.872
Lissadell	2.324	1.979		
Mardie	2.732	2.958		2.876
Meedo	2.564	2.243	2.356	2.303
Perth	1.667	1.606	1.603	
Cuttening	1.654	1.693		
Norseman	2.185	2.113		2.133
Katherine	1.957	1.855		
Alice Springs	2.820	2.941		2.850
Marree	3.104	3.094		3.020
Orroroo	2.495	2.269		2.217
Wallaroo	1.596	1.666		1.621
Adelaide	1.691	1.656		
Eudunda	2.033	1.951	1.931	1.960
Palmerville	1.901	1.984		1.950
Kalamia	2.245	2.593		2.412
Emerald	2.051	2.001	2.005	2.021
Barcaldine	2.196	2.491		2.372
Cape Capricorn	1.909	1.991		1.942
Rockhampton	2.204	2.235		2.138
Cape Moreton	1.941	1.915		1.848
Brisbane	2.203	2.168		2.157
Pittsworth	1.792	1.798		
Miles	1.883	1.998	1.973	1.903
Cunnamulla	2.627	2.583		2.571
Wentworth	2.447	2.323		2.406
Balranald	2.392	2.203		2.400
Bingara	1.802	1.850	1.898	1.848
Mudgee	1.894	1.879	1.783	1.813
Sydney	1.873	1.931	1.901	1.875
Moruya	1.900	2.057		1.913
Adelong	1.860	1.798	1.762	
Tumut	1.797	1.763		
Hay	2.438	2.290	2.25	2.415
Narraport	1.864	1.884	1.856	
Tongala	2.120	1.864		2.038
Caniambo	1.941	1.839	1.9	1.906
Orbost	1.740	1.778		1.706
Melbourne	1.511	1.580		
Toorourrong	1.541	1.545		
Meredith	1.486	1.566		
Frankford	1.533	1.614		1.613
Fingal	2.112	2.058		1.934
Sandford	1.646	1.702	1.657	1.611



(a) AR(1)



(b) ARMA

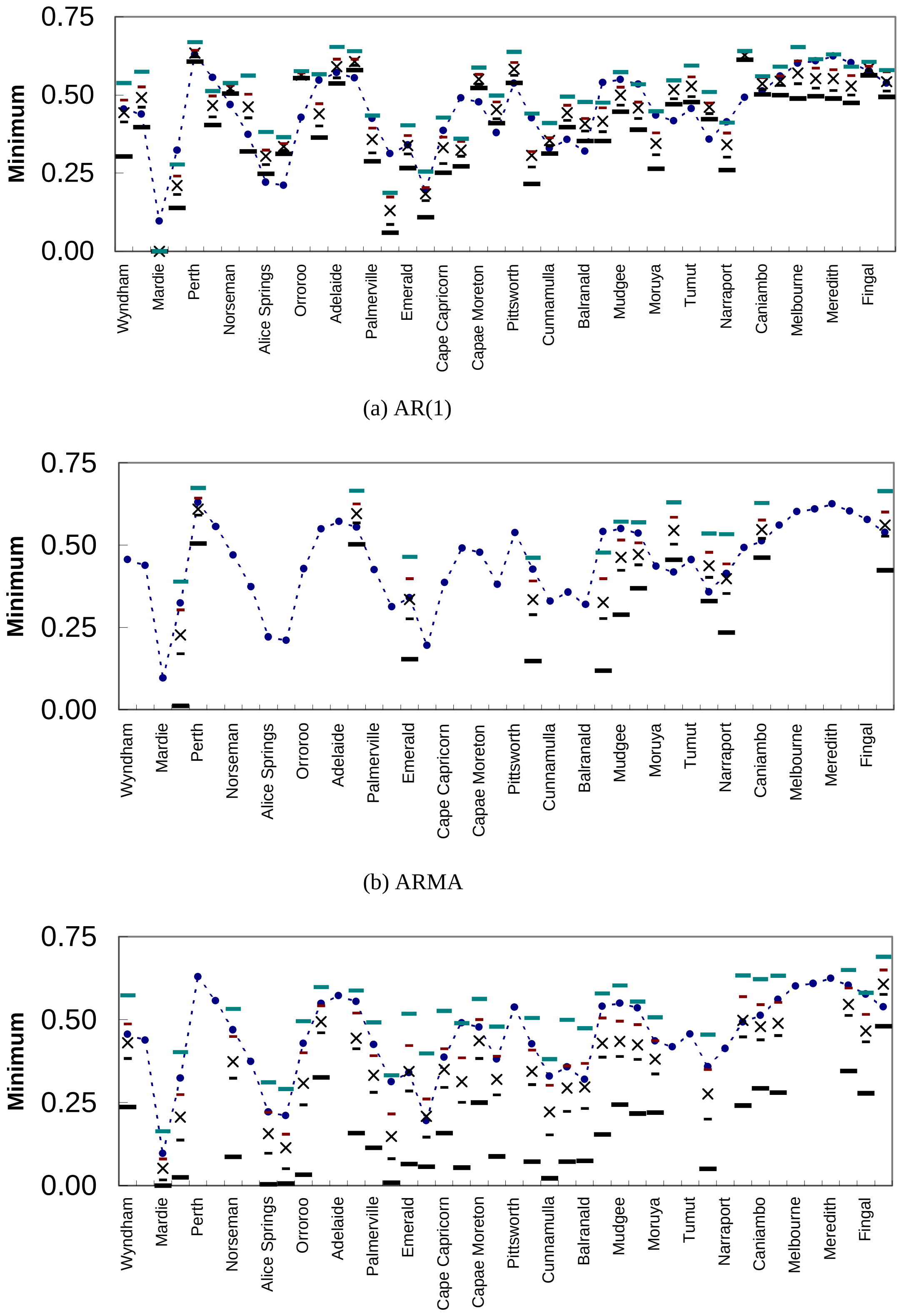


(c) HSM

Figure C4. Comparison of observed maximum rainfall

Table C6. Comparison of the historical and generated minimum annual rainfall.

Station	Historical	ARM	ARMA	HSM
Wyndham	0.455	0.444		0.430
Lissadell	0.438	0.492		
Mardie	0.096	0.000		0.052
Meedo	0.323	0.210	0.227	0.206
Perth	0.629	0.634	0.609	
Cuttening	0.556	0.466		
Norseman	0.469	0.521		0.373
Katherine	0.373	0.462		
Alice Springs	0.221	0.304		0.156
Marree	0.210	0.335		0.114
Orroroo	0.428	0.564		0.308
Wallaroo	0.548	0.440		0.494
Adelaide	0.571	0.591		
Eudunda	0.554	0.607	0.595	0.444
Palmerville	0.425	0.358		0.332
Kalamia	0.312	0.130		0.148
Emerald	0.340	0.338	0.335	0.344
Barcaldine	0.195	0.184		0.209
Cape Capricorn	0.386	0.332		0.350
Rockhampton	0.490	0.324		0.313
Cape Moreton	0.477	0.551		0.436
Brisbane	0.380	0.453		0.320
Pittsworth	0.537	0.583		
Miles	0.426	0.307	0.334	0.344
Cunnamulla	0.329	0.354		0.222
Wentworth	0.357	0.443		0.294
Balranald	0.319	0.408		0.297
Bingara	0.540	0.416	0.326	0.429
Mudgee	0.549	0.500	0.463	0.434
Sydney	0.535	0.458	0.472	0.424
Moruya	0.435	0.345		0.381
Adelong	0.417	0.517	0.544	
Tumut	0.456	0.529		
Hay	0.358	0.461	0.437	0.276
Narraport	0.413	0.341	0.398	
Tongala	0.492	0.627		0.498
Caniambo	0.512	0.536	0.547	0.479
Orbost	0.560	0.544		0.489
Melbourne	0.601	0.570		
Toorourrong	0.609	0.552		
Meredith	0.624	0.552		
Frankford	0.603	0.529		0.546
Fingal	0.577	0.585		0.466
Sandford	0.538	0.543	0.561	0.607



(a) AR(1)

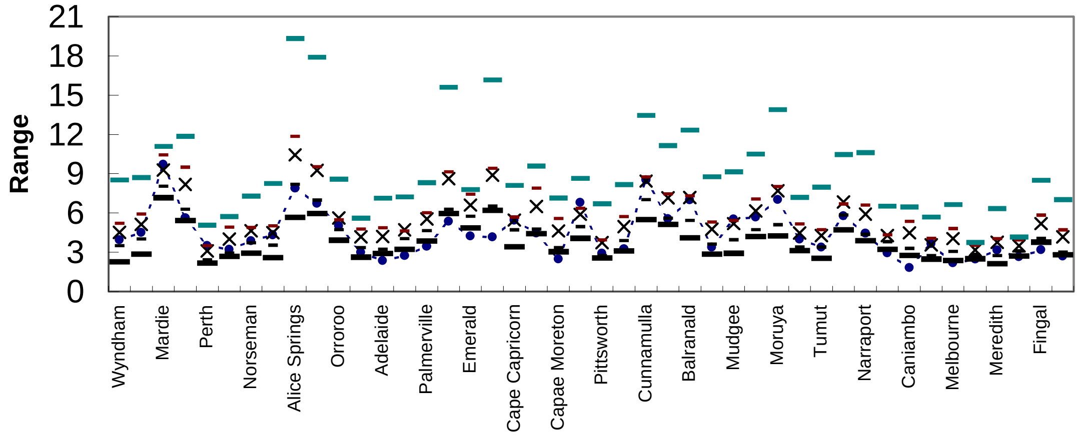
(b) ARMA

(c) HSM

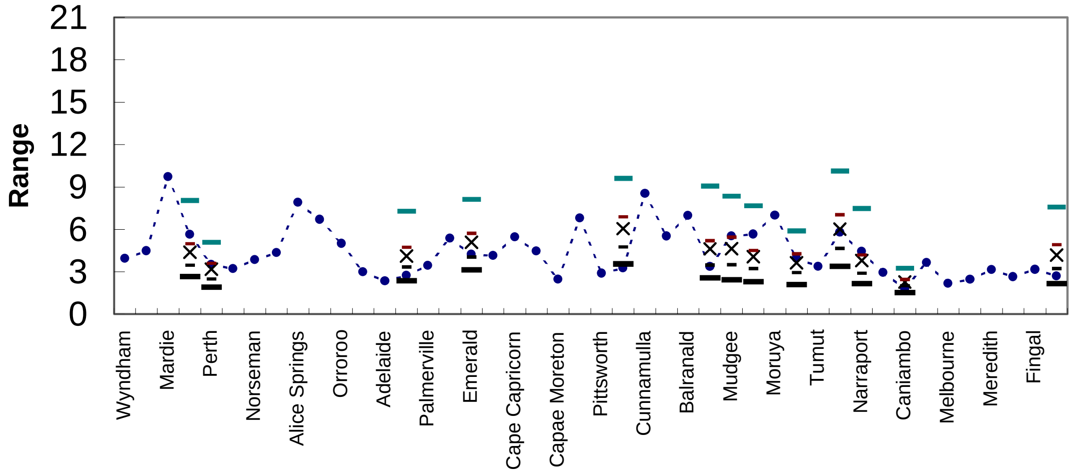
Figure C5. Comparison minimum annual rainfall

Table C7. Comparison of the historical and generated adjusted range of annual rainfall.

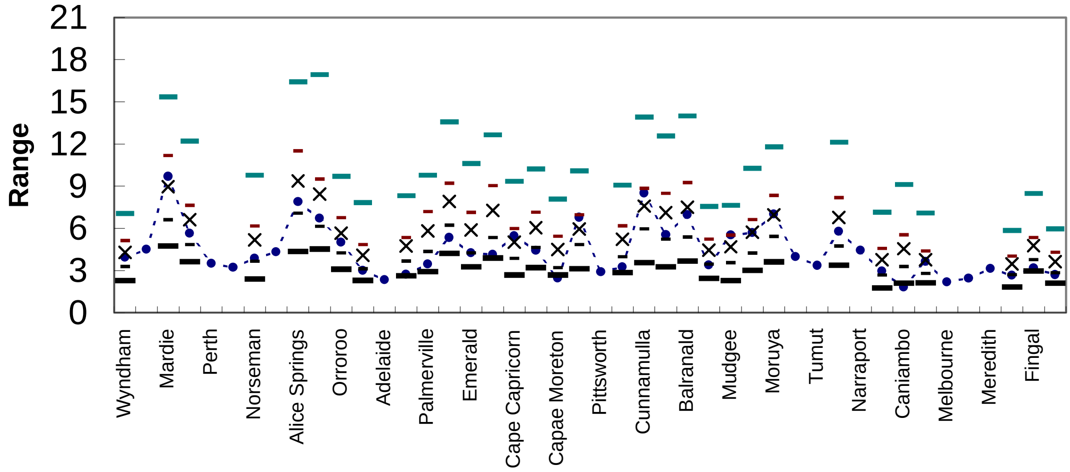
Station	Historical	ARM	ARMA	HSM
Wyndham	3.935	4.529		4.306
Lissadell	4.494	5.118		
Mardie	9.699	9.279		8.960
Meedo	5.635	8.179	4.390	6.608
Perth	3.499	3.111	3.188	
Cuttening	3.211	3.997		
Norseman	3.855	4.654		5.169
Katherine	4.325	4.490		
Alice Springs	7.892	10.442		9.378
Marree	6.704	9.260		8.443
Orroroo	4.999	5.623		5.661
Wallaroo	2.987	4.170		4.076
Adelaide	2.341	4.201		
Eudunda	2.730	4.716	4.128	4.750
Palmerville	3.439	5.546		5.823
Kalamia	5.348	8.631		7.917
Emerald	4.233	6.597	5.085	5.885
Barcaldine	4.147	8.896		7.280
Cape Capricorn	5.461	5.452		5.014
Rockhampton	4.443	6.493		6.046
Cape Moreton	2.453	4.622		4.487
Brisbane	6.783	5.904		5.947
Pittsworth	2.897	3.737		
Miles	3.255	4.957	6.061	5.240
Cunnamulla	8.518	8.441		7.576
Wentworth	5.534	7.143		7.107
Balranald	6.970	7.184		7.508
Bingara	3.374	4.766	4.619	4.450
Mudgee	5.516	5.197	4.641	4.679
Sydney	5.674	6.137	4.065	5.725
Moruya	7.000	7.691		6.965
Adelong	3.972	4.463	3.644	
Tumut	3.370	4.274		
Hay	5.774	6.826	6.009	6.768
Narraport	4.433	5.923	3.795	
Tongala	2.943	4.277		3.774
Caniambo	1.817	4.481	2.279	4.550
Orbost	3.645	3.564		3.779
Melbourne	2.172	4.035		
Toorourrong	2.450	3.171		
Meredith	3.137	3.755		
Frankford	2.652	3.497		3.470
Fingal	3.164	5.196		4.758
Sandford	2.694	4.162	4.184	3.643



(a) AR(1)



(b) ARMA

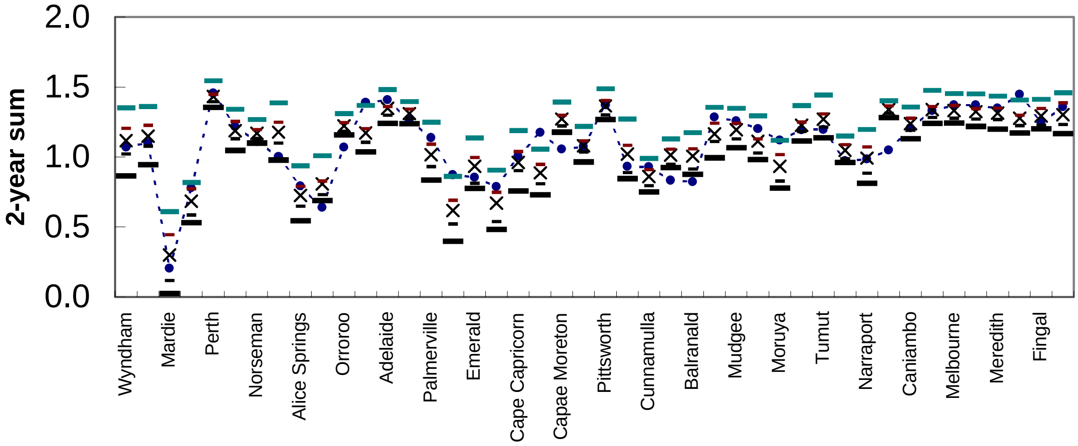


(c) HSM

Figure C6. Comparison of adjusted range of annual rainfall

Table C8. Comparison of the historical and generated minimum 2-year rainfall.

Station	Historical	ARM	ARMA	HSM
Wyndham	1.073	1.118		1.149
Lissadell	1.100	1.149		
Mardie	0.204	0.299		0.480
Meedo	0.777	0.684	1.033	0.858
Perth	1.458	1.434	1.409	
Cuttening	1.218	1.188		
Norseman	1.112	1.170		1.122
Katherine	1.001	1.177		
Alice Springs	0.791	0.725		0.730
Marree	0.638	0.809		0.660
Orroroo	1.069	1.222		1.032
Wallaroo	1.389	1.172		1.263
Adelaide	1.408	1.348		
Eudunda	1.263	1.311	1.446	1.216
Palmerville	1.138	1.016		1.050
Kalamia	0.874	0.617		0.741
Emerald	0.854	0.934	0.919	1.043
Barcaldine	0.788	0.669		0.820
Cape Capricorn	1.003	0.961		1.031
Rockhampton	1.174	0.887		0.976
Cape Moreton	1.054	1.271		1.207
Brisbane	1.075	1.081		1.021
Pittsworth	1.371	1.365		
Miles	0.932	1.021	0.935	1.059
Cunnamulla	0.929	0.861		0.841
Wentworth	0.834	1.014		0.972
Balranald	0.822	1.006		1.006
Bingara	1.283	1.165	1.025	1.204
Mudgee	1.258	1.196	1.188	1.197
Sydney	1.201	1.113	1.264	1.145
Moruya	1.119	0.933		1.044
Adelong	1.194	1.227	1.392	
Tumut	1.195	1.270		
Hay	0.972	1.048	1.161	0.980
Narraport	0.985	0.992	1.204	
Tongala	1.049	1.339		1.300
Caniambo	1.208	1.238	1.291	1.265
Orbost	1.330	1.340		1.278
Melbourne	1.372	1.332		
Toorourrong	1.372	1.324		
Meredith	1.347	1.315		
Frankford	1.447	1.278		1.343
Fingal	1.248	1.292		1.221
Sandford	1.358	1.302	1.310	1.406



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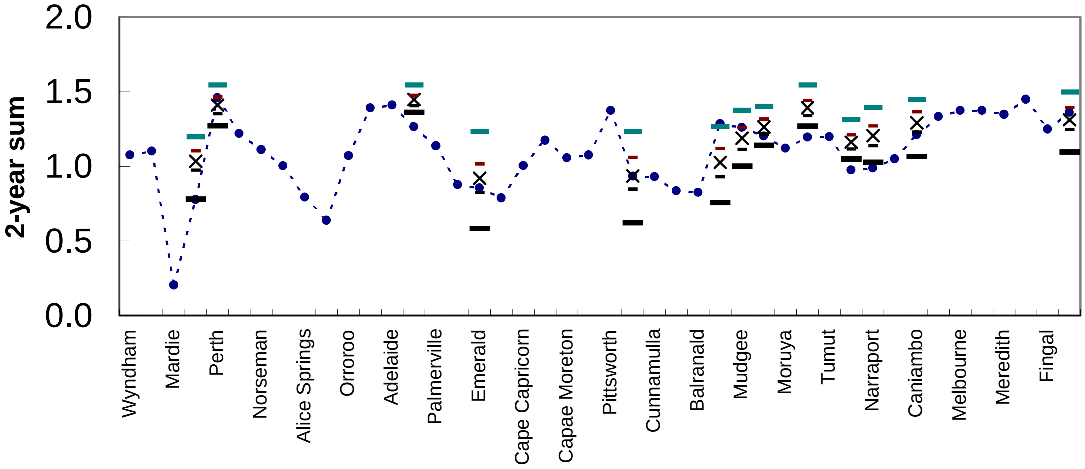
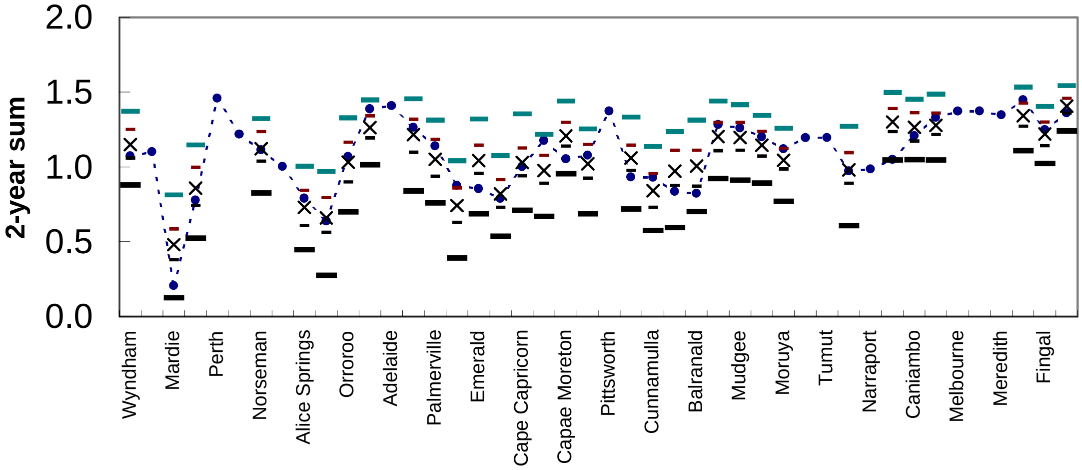


(b) ARMA

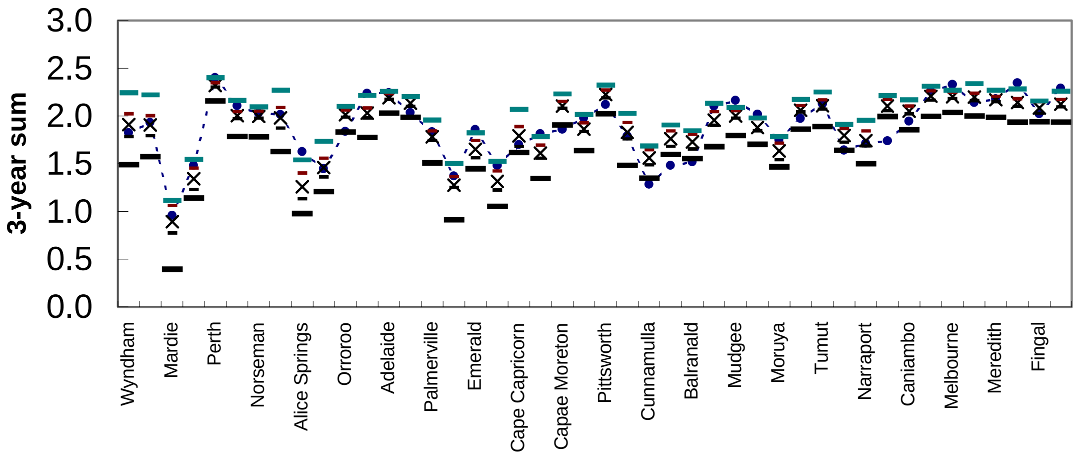


(c) HSM

Figure C7. Comparison of minimum 2-year rainfall

Table C9. Comparison of the historical and generated minimum 3-year rainfall.

Station	Historical	ARM	ARMA	HSM
Wyndham	1.823	1.909		1.950
Lissadell	1.929	1.906		
Mardie	0.957	0.893		1.069
Meedo	1.480	1.342	1.867	1.578
Perth	2.402	2.318	2.259	
Cuttening	2.105	2.005		
Norseman	2.028	1.994		1.917
Katherine	2.016	1.982		
Alice Springs	1.626	1.260		1.403
Marree	1.449	1.459		1.331
Orroroo	1.836	2.011		1.838
Wallaroo	2.235	2.029		2.089
Adelaide	2.242	2.189		
Eudunda	2.036	2.135	2.247	2.055
Palmerville	1.831	1.784		1.829
Kalamia	1.369	1.277		1.398
Emerald	1.854	1.653	1.719	1.819
Barcaldine	1.479	1.316		1.536
Cape Capricorn	1.698	1.792		1.817
Rockhampton	1.813	1.613		1.711
Cape Moreton	1.861	2.103		2.049
Brisbane	1.979	1.864		1.810
Pittsworth	2.118	2.223		
Miles	1.788	1.829	1.651	1.851
Cunnamulla	1.282	1.561		1.574
Wentworth	1.480	1.762		1.762
Balranald	1.518	1.728		1.778
Bingara	2.100	1.958	1.888	2.042
Mudgee	2.162	2.000	2.005	2.017
Sydney	2.019	1.882	2.092	1.955
Moruya	1.749	1.634		1.779
Adelong	1.974	2.062	2.214	
Tumut	2.131	2.107		
Hay	1.644	1.794	1.880	1.743
Narraport	1.712	1.744	2.048	
Tongala	1.738	2.105		2.156
Caniambo	1.943	2.050	2.110	2.110
Orbost	2.255	2.206		2.112
Melbourne	2.330	2.201		
Toorourrong	2.139	2.199		
Meredith	2.178	2.169		
Frankford	2.347	2.141		2.197
Fingal	2.021	2.085		2.038
Sandford	2.291	2.124	2.102	2.237



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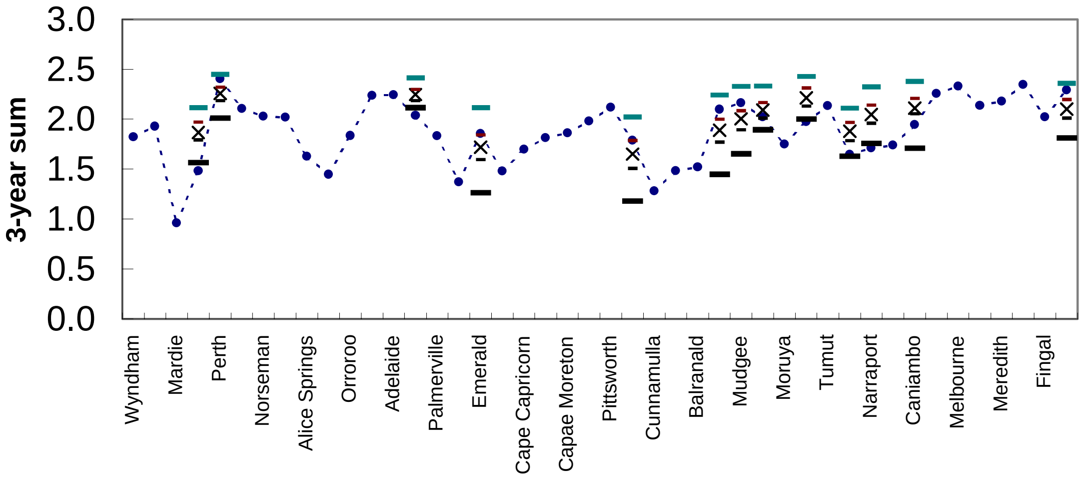

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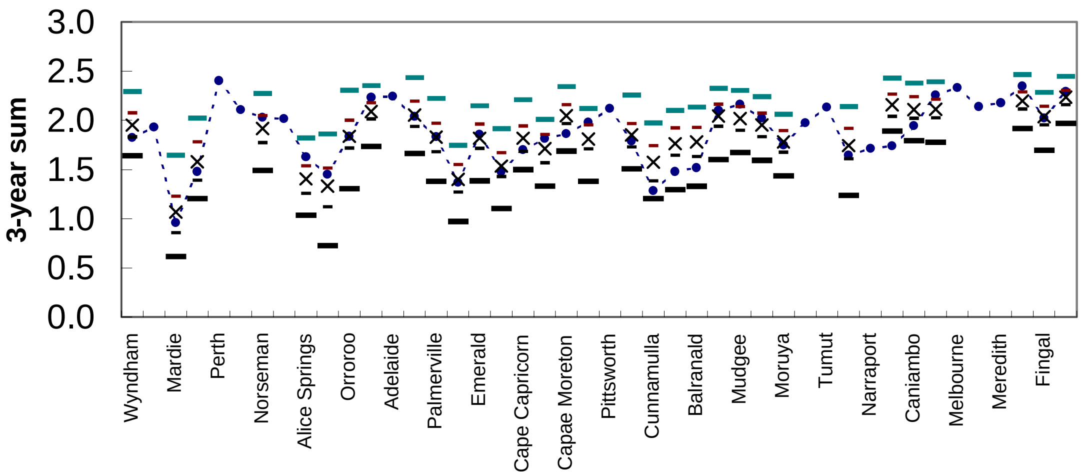
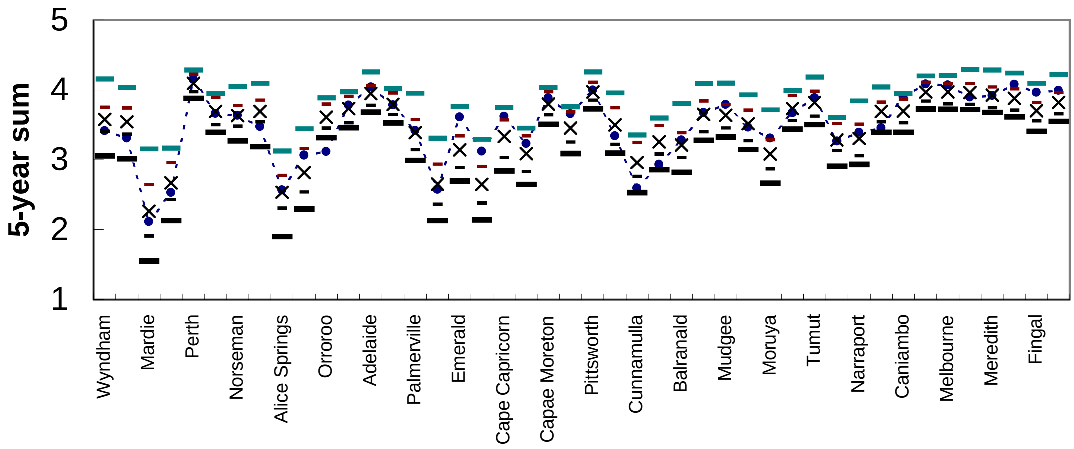


(c) HSM

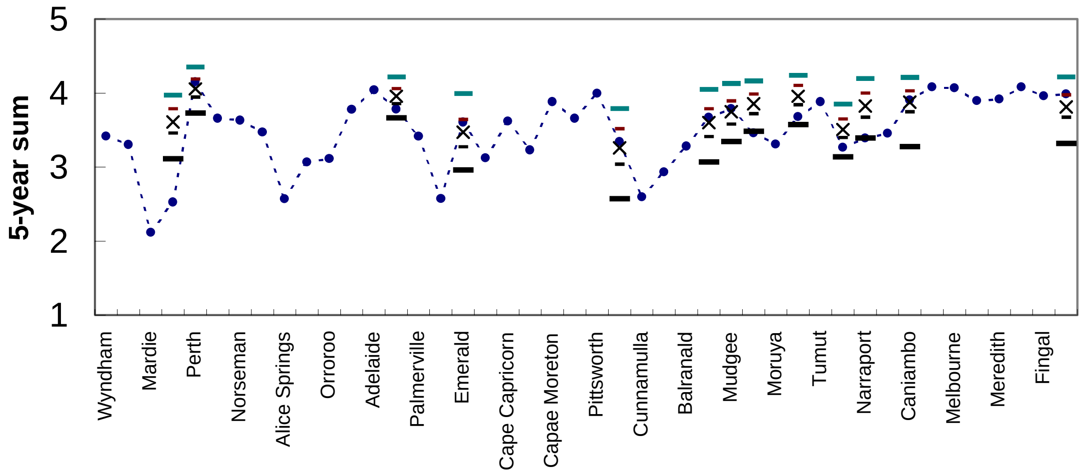
Figure C9. Comparison of minimum 3-year rainfall

Table C10. Comparison of the historical and generated minimum 5-year rainfall.

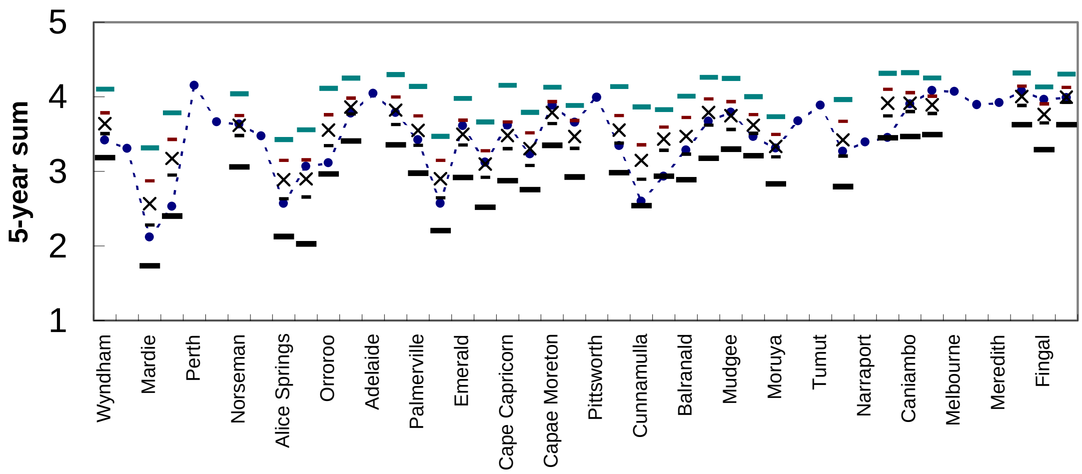
Station	Historical	ARM	ARMA	HSM
Wyndham	3.416	3.579		3.638
Lissadell	3.307	3.544		
Mardie	2.115	2.263		2.567
Meedo	2.529	2.672	3.612	3.175
Perth	4.151	4.094	4.057	
Cuttening	3.661	3.690		
Norseman	3.633	3.632		3.615
Katherine	3.473	3.691		
Alice Springs	2.568	2.536		2.889
Marree	3.065	2.818		2.898
Orroroo	3.113	3.611		3.553
Wallaroo	3.779	3.733		3.862
Adelaide	4.041	3.948		
Eudunda	3.784	3.792	3.959	3.821
Palmerville	3.418	3.391		3.552
Kalamia	2.574	2.652		2.903
Emerald	3.608	3.141	3.473	3.501
Barcaldine	3.121	2.650		3.100
Cape Capricorn	3.618	3.334		3.482
Rockhampton	3.232	3.084		3.304
Cape Moreton	3.883	3.797		3.788
Brisbane	3.658	3.453		3.466
Pittsworth	3.993	3.972		
Miles	3.341	3.500	3.263	3.556
Cunnamulla	2.595	2.959		3.149
Wentworth	2.932	3.257		3.432
Balranald	3.283	3.209		3.472
Bingara	3.671	3.651	3.599	3.789
Mudgee	3.790	3.635	3.748	3.747
Sydney	3.462	3.517	3.854	3.620
Moruya	3.310	3.082		3.336
Adelong	3.676	3.733	3.957	
Tumut	3.885	3.821		
Hay	3.266	3.286	3.509	3.422
Narraport	3.390	3.304	3.825	
Tongala	3.453	3.692		3.914
Caniambo	3.901	3.695	3.875	3.913
Orbost	4.081	3.970		3.890
Melbourne	4.067	3.971		
Toorourrong	3.896	3.959		
Meredith	3.918	3.925		
Frankford	4.079	3.877		4.004
Fingal	3.961	3.701		3.764
Sandford	3.987	3.822	3.815	3.998



(a) AR(1)



(b) ARMA

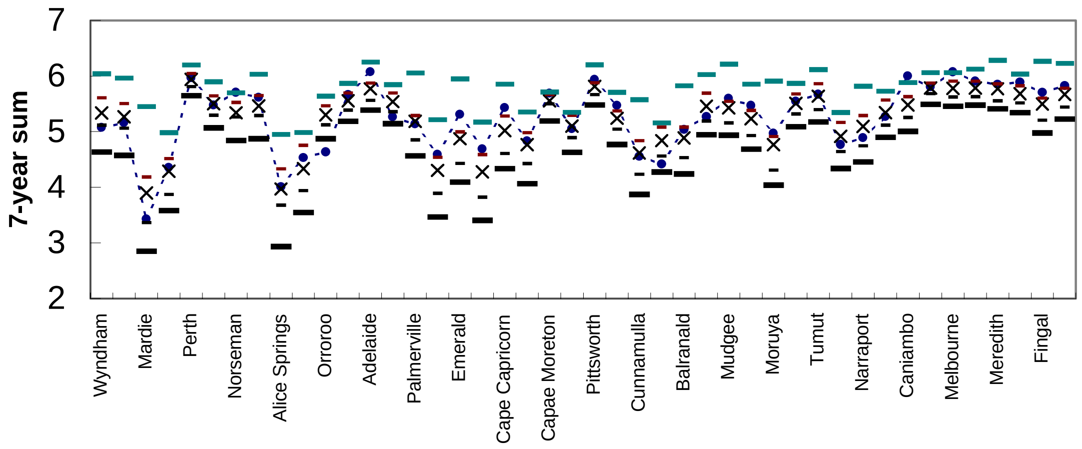


(c) HSM

Figure C10. Comparison of minimum 5-year rainfall

Table C11. Comparison of the historical and generated minimum 7-year rainfall.

Station	Historical	ARM	ARMA	HSM
Wyndham	5.069	5.334		5.452
Lissadell	5.157	5.266		
Mardie	3.423	3.897		4.189
Meedo	4.352	4.287	5.380	4.883
Perth	5.974	5.938	5.892	
Cuttening	5.477	5.504		
Norseman	5.709	5.342		5.387
Katherine	5.616	5.461		
Alice Springs	4.008	3.972		4.453
Marree	4.529	4.334		4.551
Orroroo	4.639	5.303		5.335
Wallaroo	5.662	5.557		5.734
Adelaide	6.070	5.772		
Eudunda	5.259	5.539	5.738	5.644
Palmerville	5.132	5.187		5.379
Kalamia	4.587	4.302		4.496
Emerald	5.310	4.873	5.214	5.235
Barcaldine	4.687	4.281		4.758
Cape Capricorn	5.429	5.023		5.249
Rockhampton	4.827	4.765		5.019
Cape Moreton	5.688	5.555		5.597
Brisbane	5.051	5.098		5.244
Pittsworth	5.940	5.815		
Miles	5.467	5.242	4.968	5.384
Cunnamulla	4.555	4.620		4.775
Wentworth	4.418	4.837		5.132
Balranald	5.034	4.890		5.237
Bingara	5.269	5.458	5.375	5.622
Mudgee	5.595	5.430	5.532	5.549
Sydney	5.472	5.233	5.662	5.374
Moruya	4.972	4.764		5.008
Adelong	5.546	5.511	5.758	
Tumut	5.675	5.638		
Hay	4.769	4.916	5.200	5.177
Narraport	4.889	5.093	5.650	
Tongala	5.268	5.345		5.777
Caniambo	5.996	5.482	5.717	5.741
Orbost	5.802	5.786		5.706
Melbourne	6.076	5.781		
Toorourrong	5.908	5.789		
Meredith	5.853	5.770		
Frankford	5.890	5.684		5.859
Fingal	5.704	5.498		5.546
Sandford	5.824	5.669	5.614	5.797



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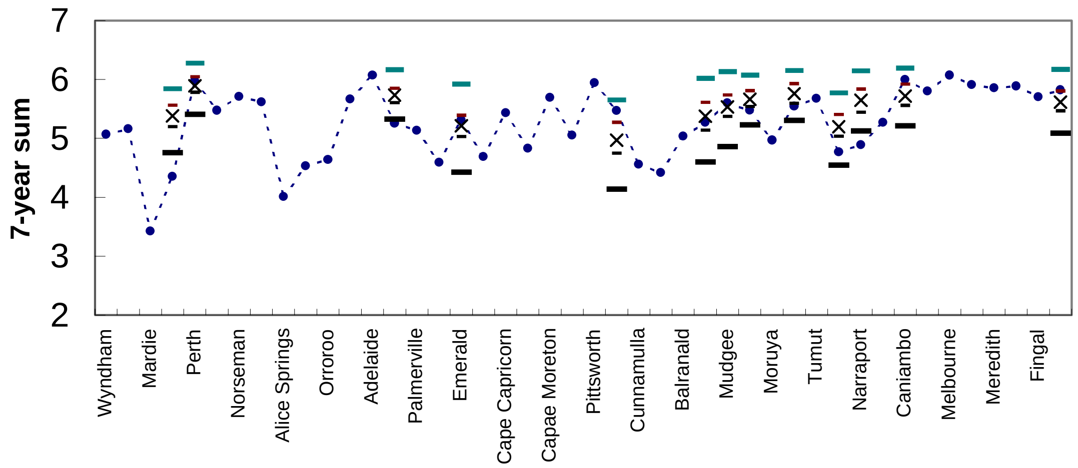

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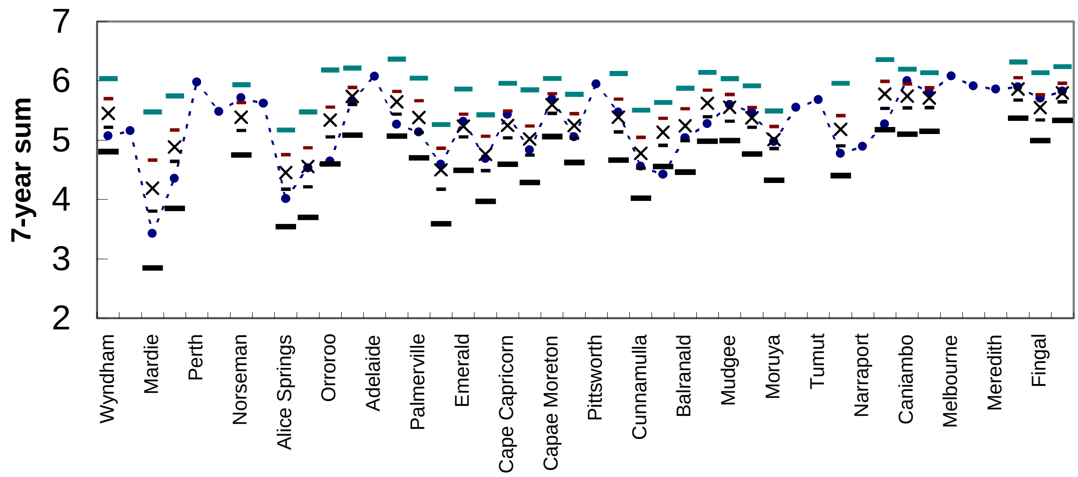
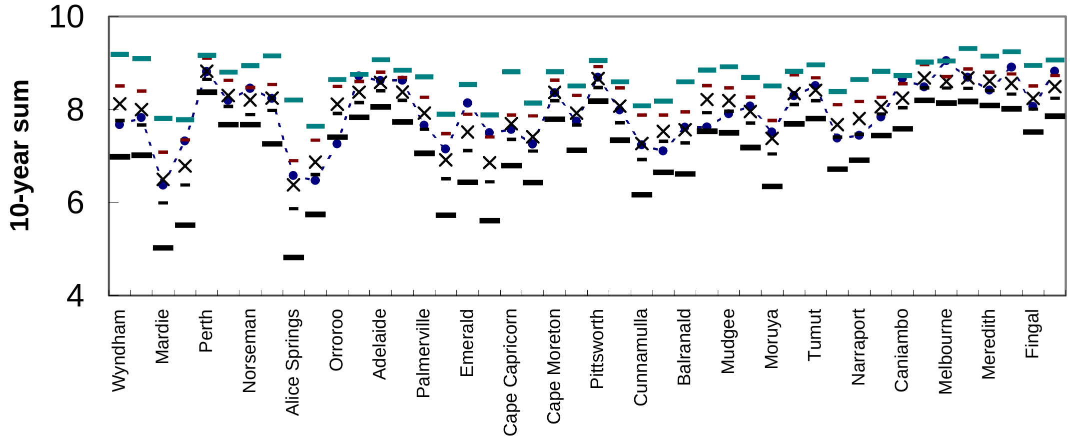


(c) HSM

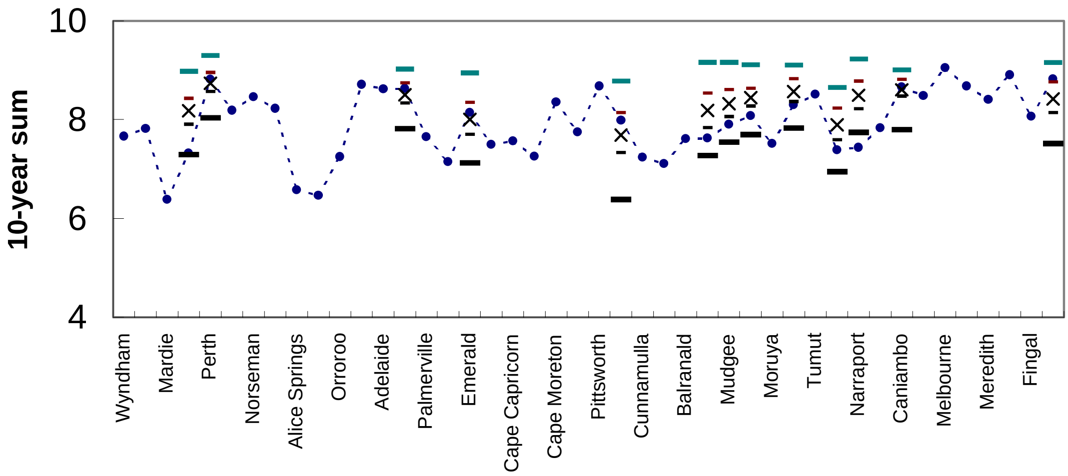
Figure C11. Comparison of minimum 7-year rainfall

Table C12. Comparison of the historical and generated minimum 10-year rainfall.

Station	Historical	ARM	ARMA	HSM
Wyndham	7.667	8.119		8.232
Lissadell	7.822	7.999		
Mardie	6.377	6.496		6.803
Meedo	7.318	6.789	8.181	7.574
Perth	8.813	8.828	8.737	
Cuttening	8.192	8.306		
Norseman	8.458	8.202		8.190
Katherine	8.23	8.247		
Alice Springs	6.571	6.382		7.028
Marree	6.469	6.868		7.177
Orroroo	7.250	8.117		8.090
Wallaroo	8.715	8.371		8.504
Adelaide	8.620	8.587		
Eudunda	8.627	8.381	8.506	8.454
Palmerville	7.651	7.920		8.181
Kalamia	7.144	6.919		7.036
Emerald	8.138	7.515	8.009	7.976
Barcaldine	7.495	6.858		7.401
Cape Capricorn	7.564	7.692		8.003
Rockhampton	7.257	7.411		7.720
Cape Moreton	8.356	8.368		8.384
Brisbane	7.748	7.920		7.950
Pittsworth	8.682	8.668		
Miles	7.988	8.076	7.685	8.183
Cunnamulla	7.234	7.271		7.382
Wentworth	7.107	7.528		7.810
Balranald	7.610	7.562		7.986
Bingara	7.623	8.214	8.185	8.421
Mudgee	7.902	8.189	8.320	8.374
Sydney	8.071	7.956	8.443	8.094
Moruya	7.510	7.384		7.563
Adelong	8.296	8.347	8.568	
Tumut	8.507	8.417		
Hay	7.382	7.675	7.896	7.900
Narraport	7.440	7.807	8.494	
Tongala	7.831	8.065		8.595
Caniambo	8.659	8.244	8.606	8.568
Orbost	8.480	8.677		8.560
Melbourne	9.044	8.602		
Toorourrong	8.683	8.678		
Meredith	8.407	8.607		
Frankford	8.906	8.560		8.713
Fingal	8.065	8.239		8.293
Sandford	8.821	8.489	8.423	8.583



(a) AR(1)



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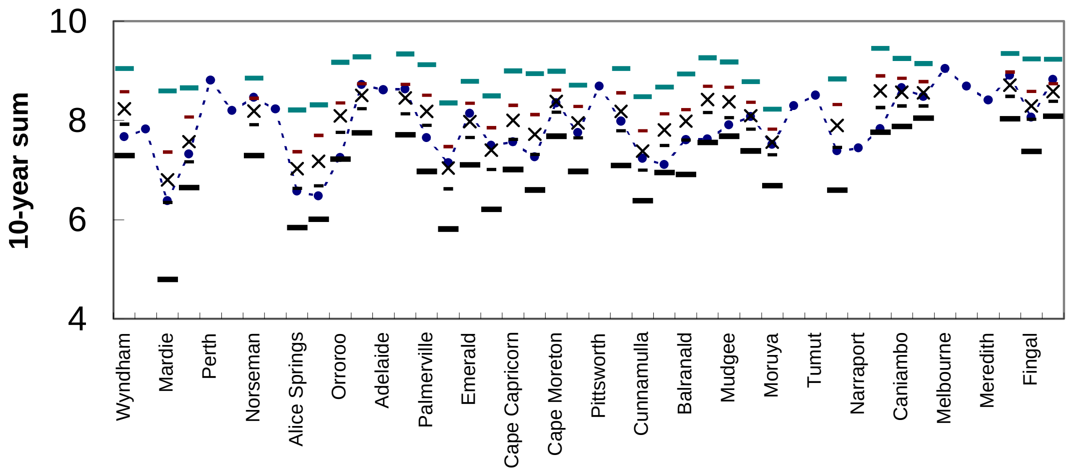

(c) HSM

Figure C12. Comparison of minimum 10-year rainfall

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- Brisbane City Council
- Bureau of Meteorology
- CSIRO Land and Water
- Department of Land and Water Conservation, NSW
- Department of Natural Resources and Environment, Vic
- Goulburn-Murray Water
- Griffith University
- Melbourne Water
- Monash University
- Murray-Darling Basin Commission
- Natural Resources and Mines, Qld
- Southern Rural Water
- The University of Melbourne
- Wimmera Mallee Water

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