A REVIEW OF THE METHODS FOR ESTIMATING AREAL REDUCTION FACTORS FOR DESIGN RAINFALLS

R. Srikanthan

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CATCHMENT HYDROLOGY

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PREFACE

A topic of considerable current interest and economic importance is the spillway adequacy of large dams. Recent changes to the means of estimating extreme rainfall (probable maximum precipitation) have led to the review of spillway design floods for dams throughout Australia.

CRC Project D3 "Probability and Risk of Extreme Floods" has a number of sub-projects which, collectively, have the potential for marked reductions in the uncertainties associated with extreme flood estimates. The overall approach is with rainfall-based methods of flood estimation, to take advantage of the longer periods of rainfall data available, compared with flow records.

In the application of design rainfalls, a correction factor (areal reduction factor) is applied to convert rainfalls for a point to the equivalent catchment average. This report, by Dr R Srikanthan of the Bureau of Meteorology, provides a review of current procedures and possibilities. As such, it is the necessary first step in the planning of a research project aimed at improving this aspect of the design flood process.

Project D3, like the others in the CRC, is a cooperative one, involving researchers from the Bureau of Meteorology, Rural Water Corporation, Monash University and the University of Melbourne. It is pleasing to acknowledge this contribution to the project from each of these groups.

Russell Mein
Program Leader, Flood Hydrology
Cooperative Research Centre for Catchment Hydrology

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SUMMARY

This report represents the first phase of Sub-Project 5 "Areal Reduction Factors for Victoria" of CRCCH Project D3 "Probability and Risk of Extreme Floods". It is a review of the existing methods for deriving areal reduction factors (ARFs). The aim of the review is to evaluate existing methods, to recommend a procedure for estimating ARFs and to identify further research areas.

The review starts with a brief discussion of storm-centred and fixed-area ARFs. As the former type is not relevant to design rainfalls of moderate to large average recurrence intervals (ARIs), only the fixed-area ARFs is considered. The available methods for deriving fixed-area ARFs are broadly classified into three categories, namely, empirical, analytical and analytical-empirical methods.

In empirical methods, recorded rainfall depths at a number of stations are used to derive empirically the ARF. Three methods, namely, the US Weather Bureau method, the UK method and the Bell's method, are available. The first two methods derive a single value of ARF for a given area and duration, while the third method derives ARF as a function of ARI.

The attraction of analytical methods is that they offer, at least theoretically, the potential for extending the ARF analysis to areas with limited coverage of pluviograph data. In analytical methods, a mathematical model is fitted to characterise the space-time variation of rainfall with simplifying assumptions. The ARF is then derived analytically from the properties of the fitted model. Four models, namely, the Roche method, the Rodriguez-Iturbe and Mejia method, the Meynink and Brady method and the statistical derivation of ARF fall under this category. All the methods assume that the rainfall process is stationary and isotropic. In addition, the first three methods assume that the data is normally distributed, while the last one assumes that it is log-normally distributed. Also the difficulty with the assumption of a zero mean process in practical application is illustrated.

Only one method, the Myers and Zehr method, falls under the analytical-empirical category. The ARFs recommended in "Australian Rainfall and Runoff" (Fig 2.6) for use over most of Australia are based on the application of this method with rainfall data from the Chicago area (US). Myers and Zehr developed a computerised model of the structure of annual maximum rain storms and a routine for computing ARFs from these storms. An Extreme Value Type I distribution was assumed in adjusting the data to a common length of 20 years and later in the analyses. This method requires a large amount of data and the process of deriving various statistical surfaces is an extremely time consuming process.

The above methods were reviewed in relation to their theoretical background and reported applications using Australian data. Based on the review of the above methods, the Bell's method is recommended for deriving ARFs where sufficient data is available. Further research is needed on the Rodriguez-Iturbe and Mejia method before it could be recommended for application in areas with less data.

1 INTRODUCTION

1.1 Background

In most hydrological applications, it is necessary to know the average rainfall over an area. Design rainfall data obtained from intensity-frequency-duration analysis is applicable to point locations and can be used for small areas e.g. 4 km^2 (page 38, Institution of Engineers, Australia, 1987). For application in large areas, the concept of an areal reduction factor (ARF) has been developed to obtain areal average rainfall frequency curves from point-rainfall frequency curves. To date little work has been done in Australia to derive areal reduction factors for use in different parts of the country. Australian Rainfall and Runoff (1987, hereafter referred to as ARR87) recommends the set of curves derived from a study in the Chicago area for all the zones except zone 5 (Figure 3.2 in ARR87) for any ARI up to 100 years. For use in zone 5, the area reduction factors were obtained from a study in the Arizona area. There is a concern in some sections of the hydrological community in Australia that the US results may not be appropriate for the Australian conditions. This concern was confirmed by the recent studies (Nittim, 1989; Avery, 1991; Masters, 1993; Masters and Irish, 1994; Meynink and Brady, 1993; Porter and Ladson, 1993) in which the authors found that the values from ARR87 were generally larger than those from their own study.

A survey of ARR87 users and the review of research needs by Jim Irish clearly identified ARFs as a high priority research area in flood estimation. Consequently, the CRCCH D3 Project agreement signed in March, 1994 included the ARF research as Sub-project 5.

1.2 Purpose of the Report

The report, which is a literature review, forms the important preparatory step for the CRCCH sub-project 5. The objectives of this report are:

- to describe the features of different existing methods for estimating ARFs
- to compare the results of reported applications of the methods using Australian data
- to evaluate the potential of selected methods to serve as a basis for routine estimation of design ARF values for different Australian Regions
- to indicate areas where further research is necessary to allow the development of a standard methodology for estimating ARFs.

1.3 Concept of ARF

The concept of an ARF is simple. The ARF essentially transforms rainfall depth at a point to an equivalent rainfall depth over an area with the same probability of exeedance as that of the point rainfall. This simple concept becomes complicated when one tries to apply this in practice to a catchment. The main problem is decide to "which location to choose for the point rainfall?". In practice, the point rainfall is taken at the centroid of a catchment, at a nearby point (Porter and Ladson, 1993) or an average of the point rainfalls at a number of points within the catchment (Avery, 1991). To some extent, the selection of the point rainfall should depend on the way the ARF is derived in the first place.

Another associated issue with the application of areal rainfall is its spatial distribution. In most studies with small to moderate average recurrence intervals (ARI), design rainfall is assumed to be uniform in space and vary only in time as given by the temporal patterns in ARR87. This contradicts with both the non-uniform areal pattern of actual storm events and the PMP design storm events. However, this spatial variablity of design storms is not discussed in this report.

Two types of areal reduction factors have been identified in the literature (US. Weather Bureau, 1960; Hershfield, 1962):

- storm-centred type
- fixed-area type

The storm-centred type is generally used with PMP storms while the fixed-area type is the one that is widely used in practice for designing drainage channels and hydraulics structures for flood control. For the sake of completeness, both types of areal reduction factors are reviewed in this report. However, more emphasis is given to the latter type because of its wider usage.

2 STORM-CENTRED AREAL REDUCTION FACTORS

The reference point for calculating the areal reduction factor is the centre of the storm as implied in the name "storm-centred". The location of the area of rainfall is not fixed and it changes with the storm. The areal reduction factor can be defined as:

$$K_c = P_a/P(0) \qquad (1)$$

where P_a areal average rainfall over an area, a, surrounding the storm centre
$P(0)$ rainfall at the storm centre

According to Eagleson (1970), the difference between the mean rainfall over an area and the rainfall at the storm centre, in general, has the following characteristics:

- Increases with decreasing rainfall depth
- Decreases with increasing duration
- Greater for convective and orographic precipitation than for cyclonic rainfall
- Increases with increasing area

For convective storms in Arizona, Woolhiser and Schwalen (1959) expressed the average areal rainfall as:

$$K_{c1} = P_a / P(0) = 1 - [0.14 / P(0)] A_s^{0.6} \qquad (2)$$

where $P(0)$ the rainfall depth at the storm centre
P_a the average depth over the circular area A_S (in square miles) surrounding the centre.

Eagleson (1967) gave the following structure for point precipitation as a function of radial distance, r, from the storm centre:

$$P(r) / P(0) = 1 - 0.72\,(r/r_0) \qquad (3)$$

where $r_0 = 1.73P(0)$ is the correlation radius in miles. Using this storm structure, one can obtain the areal reduction factor for a circular area as:

$$K_{c2} = 1 - 0.24\,(r/r_0) \qquad (4)$$

Fogel and Duckstein (1969) presented the following relationship for the areal distribution of rainfall within a storm:

$$P(r) = P(0) \exp(-\pi r^2 t) \qquad (5)$$

where t is a dispersion parameter given by:

$$t = 0.27e^{-0.67}\,P(0) \qquad (6)$$

The areal reduction factor corresponding to the above relationship can be derived as:

$$K_{c3} = [1 - \exp(-\pi r^2 t)] / \pi r^2 t \qquad (7)$$

For great cyclonic storms in the United States, Boyer (1957) obtained the following areal rainfall distribution:

$$P(r) = P(0)e^{-ar} \qquad (8)$$

Eagleson (1967) defined the parameter a in terms of r_0 as follows:

$$a = 1.68/r_0 \qquad (9)$$

There are a number of similar formulae in the literature which relate the areal average rainfall to the maximum rainfall at the storm centre. The expressions for ARF from these formulae are given in Table 1.

Table 1. Storm-centred areal reduction factors.

Storm-centred ARF	Area, A mi^2	Author
$\exp(-0.01A^{1/2})$	20 - 20 000	Horton (1924)
$2[1 - (1 + bv).e^{-bv}]/(bv)^2$ where b = 0.0235 and v is the distance of a point or an isohyet from the storm centre	> 100	Boyer (1957)
$\pi[1 - \exp(-Aab/\pi)] / Aab$ where parameters a and b define the scale and ellipticity of the area	Any	Court (1961)

For the sake of comparing the shape and position, four of the ARFs described above are plotted in Fig. 1 - 3. The storm centred areal reduction factors are no more than a description of the areal properties of the individual storms and are applicable to specific types of storm events. The above formulae are not very useful for design storms as the storm centre may not fall within such catchments. This type of areal distribution is useful only for the application of PMP to catchments.

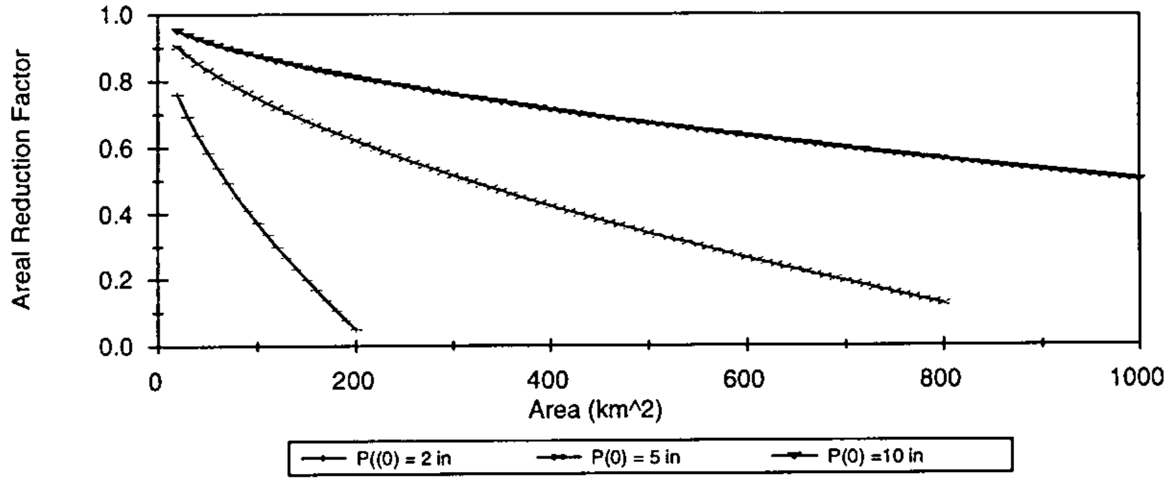

Figure 1 Variation of areal reduction factor K_{c1} with area for different values of P(0).

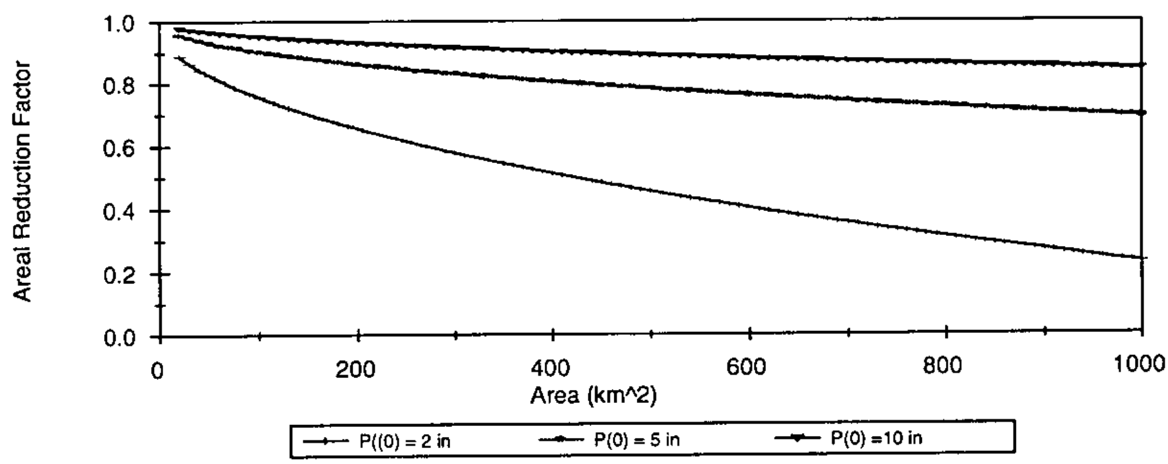

Figure 2 Variation of areal reduction factor K_{c2} with area for different values of P(0).

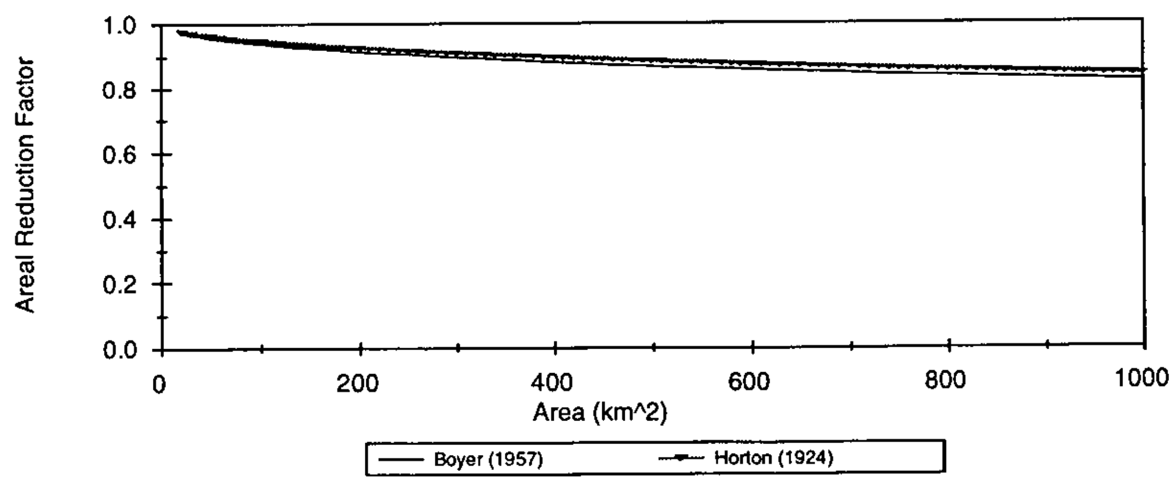

Figure 3 Variation of areal reduction factors given by Horton and Boyer with area.

3 FIXED-AREA REDUCTION FACTORS

In practical applications, one needs to know the design rainfall over a catchment which is fixed in space. Hence, from the design point of view, fixed-area reduction factors are necessary to convert the point rainfall estimates to corresponding areal estimates. A number of methods are available to compute ARFs and these can be broadly grouped into three categories.

- Empirical methods
- Analytical methods
- Analytical-Empirical methods

A number of different methods are available under each category and these are reviewed below.

3.1 Emprical Methods

In these methods, recorded rainfall depths at a number of stations are used to derive empirically the ARF. In some of the methods, a single value is derived for ARF to cover all the ARIs while the other methods derive ARFs as function of ARI.

3.1.1 US Weather Bureau Method

Using data from networks covering areas from 250 - 1000 km², the US Weather Bureau (1957) used the following expression to derive the areal reduction factor:

$$K_1 = N\sum_{j=1}^{n}\sum_{i=1}^{N} w_i R'_{ij} \Big/ \sum_{j=1}^{n}\sum_{i=1}^{N} R_{ij} \qquad (10)$$

where w_i Thiessen weighting factor for station i

R_{ij} annual maximum point rainfall of the chosen duration for year j at station i

R'_{ij} point rainfall for station i on the day the annual maximum areal rainfall occurs in year j

N number of stations within the area

n length of record in years

Using Thiessen (1911) weights, the areal rainfall for each event of the chosen duration is calculated and the highest of these in each year of record is selected. The sum of the resulting annual series multiplied by the number of stations constitutes the numerator in Eq. (10). The next step is to select the highest point rainfalls at each station in each year and the grand sum of all these over all stations and all years of the record is the denominator in Eq. (10).

Most of the areas are located to the east of the Mississippi River. Data period ranges from 7 to 15 years. The average number of recording rain gauges is 6. Only annual series of maximum point and areal rainfalls of the duration of interest were used and the resulting ARFs are shown graphically in Fig. 4.

Leclerc and Shaake (1972) expressed the areal reduction factor derived by the US Weather Bureau as:

$$K'_1 = 1 - \exp(-1.1\, t^{1/4}) + \exp(-1.1\, t^{1/4} - 0.01A) \quad (11)$$

where t storm duration in hours
A area in square miles.

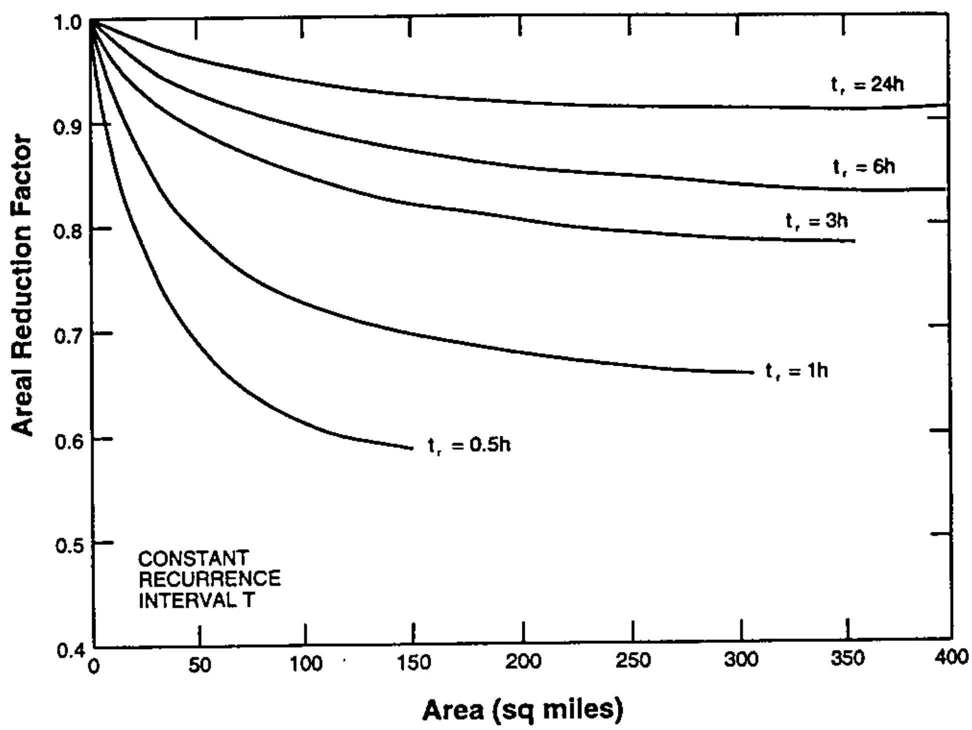

Figure 4 Areal reduction factor according to US Weather Bureau (1957).

3.1.2 Institute of Hydrology, Wallingford, UK Method

In this method (National Environment Research Council, 1975) , areal rainfall is computed to identify the time the annual maximum event occurs and the point values, R'_{ij}, are noted. The maximum point values, R_{ij}, at each station in the same year are also identified. The ratio of R'_{ij} to R_{ij} at each station in the year is found. The grand mean of these ratios over all stations and all years of record is taken as the ARF and is expressed as:

$$K_2 = \frac{1}{nN} \sum_{i=1}^{N} \sum_{j=1}^{n} \left(R'_{ij} / R_{ij}\right) \quad (12)$$

From a nationwide study of UK rainfall records, the Flood Studies Report (NERC, 1975) produced a composite diagram giving ARFs for a range of areas and durations. For the sake of comparison with the US Weather Bureau curves, ARFs derived for UK are plotted in Fig. 5 in the same manner as the former set of curves.

The US curves generally lie below the corresponding UK curves, but the gap between the curves decreases with increasing duration and area.

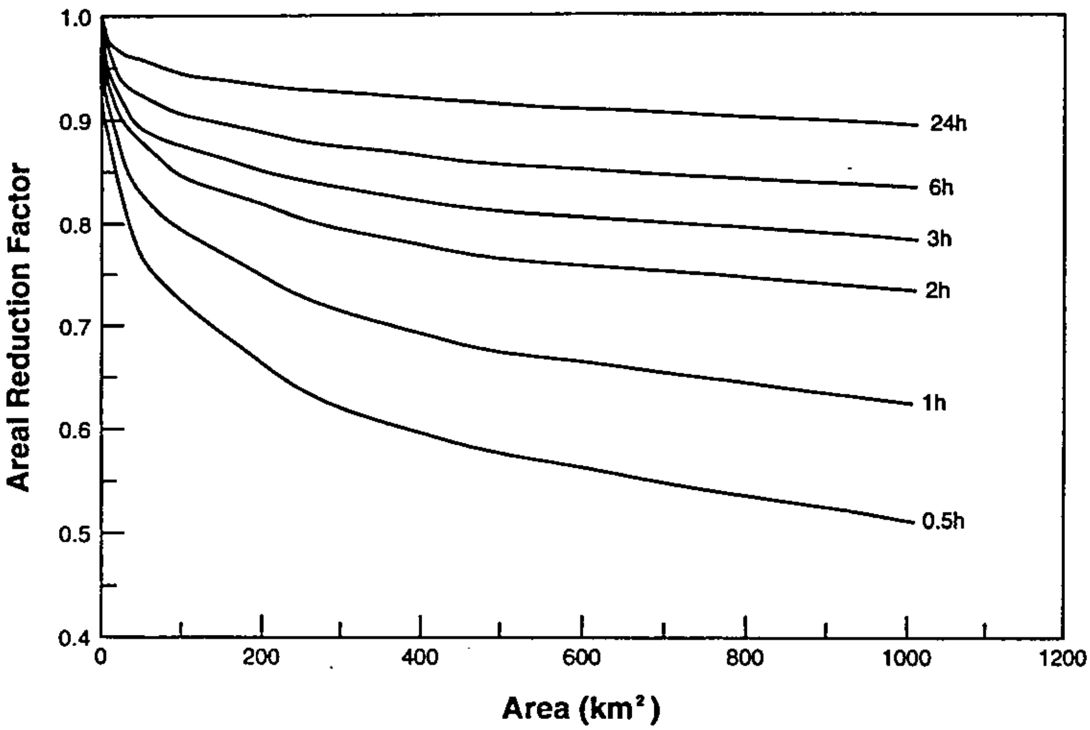

Figure 5. Areal reduction factors derived for UK.

3.1.3 Bell's Method

The annual maximum areal rainfall series and the annual maximum point rainfall series at each station are ranked. The Thiessen weighted mean of point rainfalls of the same rank is then computed resulting in an annual series of weighted maximum point rainfalls. The areal reduction factor K_m, of rank m, is the ratio of the areal rainfall of rank m to the weighted average of point rainfall of the same rank.

$$K_m = \left(\sum_{i=1}^{N} w_i R'_{ij} \right)_m \Big/ \sum_{i=1}^{N} w_i R_{im} \quad (13)$$

If areal reduction factors are assumed to be independent of return period, then the mean of K_m over all ranks gives the best estimate of the factor for the location.

$$K_3 = \frac{1}{n} \sum_{m=1}^{n} K_m \quad (14)$$

3.1.4 Variation of Bell's Method

In one variation of this method (Avery, 1991), point rainfall at various recurrence intervals is obtained from ARR87 instead of the rainfall data which was used to obtain the areal rainfall. In other words, the numerator in Eq (13) is obtained from the rainfall data while the denominator is obtained from ARR87. Since the ARR87 values have been smoothed for spatial consistency, the resulting ARF values might not be consistent. In addition, problems with converting from restricted to unrestricted intervals also arise.

In another variation of the method, only point rainfall from a nearby rainfall station has been used in the denominator of Eq. (13) (Porter and Ladson, 1993).

3.1.5 Summary

All empirical methods derive ARF as a ratio of areal average rainfall to point rainfall but with subtle differences. In the US Weather Bureau method, Thiessen weights are used to obtain areal average rainfall while simple arithmetic average is used for the point rainfall. In Bell's method, Thiessen weights are used for both the areal and point rainfall. In the UK method, areal rainfall is not used in deriving ARF. Instead, point rainfall ratios are averaged, based on the days of occurrence of the annual maximum point and areal rainfalls. Because of these subtle differences in the methods, they will in general give different values for the ARF. The important issue is here to select the appropriate one. Since the true value of ARF is not known and only estimates from different methods are available, one has to choose the method which is robust and closely resembles the way in which ARF is used in practice. Another point to note here is the dependence of ARF on ARI. Bell's method takes ARI into account while the other two methods could be modified to take ARI into account in calculating ARF.

3.2 Analytical Methods

In these methods, a mathematical model is fitted to characterise the space-time variation of rainfall with simplifying assumptions. From the properties of the fitted model, ARFs are derived analytically.

3.2.1 Roche Method

A general method with a theoretical base for the transformation of point rainfall to areal rainfall was developed by Roche (1963). It is based on deriving analytically the average rainfall over a surface for the same level of probability as that of the point rainfall at an arbitrary point on the surface. The rainfall process is assumed isotropic in the sense that the rainfall at any point on the surface follows the same probability law.

Let S be a surface with a homogeneous distribution of points 1, 2, .., n which are associated with the rainfall depths h_1, h_2, ..., h_n. These depths are random variables assumed to follow the probability density $p(h_1, h_2, ..., h_n)$. The probability that the average rainfall depth over the area S is larger than a value h is given by

$$\int_1\int_2 \cdots \int_n p(h_1, h_2, \ldots, h_n) dh_1, dh_2, \ldots, dh_n$$

with the condition

$$(h_1 + h_2 + \ldots + h_n) / n \geq h$$

Roche (1963) starts the analysis by considering two points, 1 and 2, on the surface separated by a distance X_{12}. Let $p(h_1, h_2)$ be the density function for the couple $(h_1 + h_2)$ and let $z = (h_1 + h_2)/2$ be the average rainfall over the two points in consideration. Curves are now drawn to represent lines of constant probability (Figure 6). Draw the line $h_1 + h_2 = 2z$ and the area above this line enclosed by the curves represents the probability that the average rainfall over points 1 and 2 is larger than z.

$$\iint p(h_1, h_2) dh_1 dh_2 \qquad h_1 + h_2 \geq 2z$$

The above analysis is repeated for several values of z to obtain the variation of z as a function of the probability as shown in Figure 7. By varying the distance between the points, the variation of z as a function of distance among two points is obtained (Figure 8).

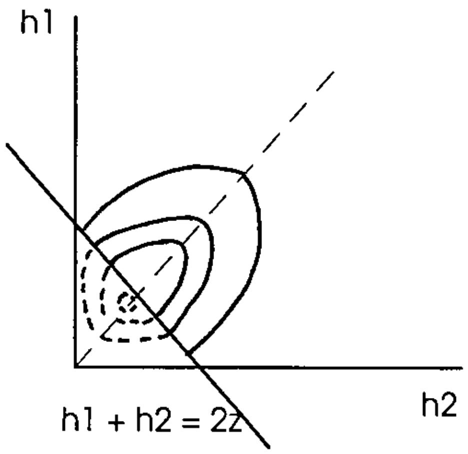

Figure 6. Curves of equal probability density $p(h_1,h_2)$ for the pair (h_1,h_2) (Roche, 1963)

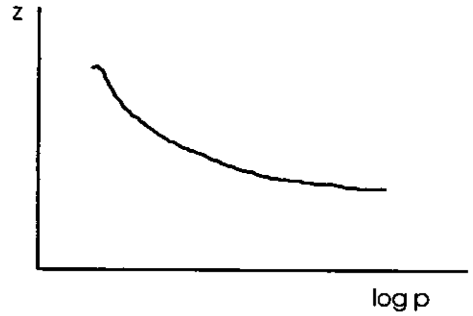

Figure 7. Average rainfall depth over two points at a fixed distance as a function of the probability level (Roche, 1963).

Figure 8. Average rainfall depth over two points as a function of the distance between them and the probability level (Roche, 1963).

To transform point rainfall to area rainfall, consider a rectangle of length L and width l. The average rainfall over the surface is:

$$h_m = \frac{1}{lL}\int_0^L dx \int_x^{x+l} z(y)dy = \frac{1}{lL}\int_0^L \left[\int_0^{x+l} z(x)dx - \int_0^x z(x)dx\right]dx \tag{15}$$

The above equation can be integrated graphically from the curves shown in Figure 4 for different levels of probability.

3.2.2 Rodriguez-Iturbe and Mejia Method

Rodriguez-Iturbe and Mejia (1974b) developed a multidimensional method to obtain areal reduction factors. Let f(x,t) represent the point precipitation intensity at point x during time t. It is assumed that the process is stationary, isotropic and the correlation function can be factored into spatial and temporal parts (Rodriguez-Iturbe and Mejia, 1974a):

$$r(v,\tau) = r_1(v)r_2(\tau) \tag{16}$$

where v the distance between two points in space

τ the time difference

Consider the process f(x,t) with zero mean and variance σ_p^2. The rainfall volume for a fixed period centred around time t over a given area A is

$$f'(t) = \int_A f(x,t)dx \tag{17}$$

The mean of f'(t) is zero and its covariance structure in the time domain is given by

$$\begin{aligned} E\left[\int_A f(x_1,t_1)dx_1 \int_A f(x_2,t_2)dx_2\right] &= \int_A\int_A E[f(x_1,t_1)f(x_2,t_1+\tau)dx_1dx_2 \\ &= \sigma_p^2 r_2(\tau)\int_A\int_A r_1(x_1-x_2)dx_1dx_2 \\ &= A^2\sigma_p^2 r_2(\tau)\bar{r}(x_1-x_2|A) \end{aligned} \tag{18}$$

where $\bar{r}(x_1 - x_2 | A)$ represents the expected correlation coefficient between two points randomly chosen in the area A.

The variance of the areal rainfall is $A^2\sigma_p^2\bar{r}(x_1 - x_2 | A)$. The areal reduction factor is then given by:

$$K_4 = [(\bar{r}(x_1 - x_2 | A)]^{1/2} \qquad (19)$$

The areal reduction factor is derived as the ratio of the standard deviations of the areal and point rainfall for a Gussian rainfall process. This is a geographically fixed areal reduction factor. For a given spatial correlation structure, this factor depends only on the characteristics of the area in question.

When the point process is Gaussian, the area process will also be Gaussian and the application of the reduction factor K_4 will relate identical frequencies or return periods. If the point process is non-Gaussian, the area process will tend towards Gaussian and there will be no exact correspondence between the frequencies of the two processes.

Two different types of spatial correlation structure have been identified as being relevant or analysable for the areal processes in hydrology (Rodriguez-Iturbe and Mejia, 1974a). The exponentially decaying structure is expressed by:

$$r_1(v) = e^{-\alpha|v|} \qquad (20)$$

where α is the decay parameter. The Bessel-type structure can be written as:

$$r_1(v) = vbK_1(vb) \qquad (21)$$

where $K_1(\ )$ is the modified Bessel function of the second kind and b is a parameter. The form of the two type correlation functions are shown in Figure 9.

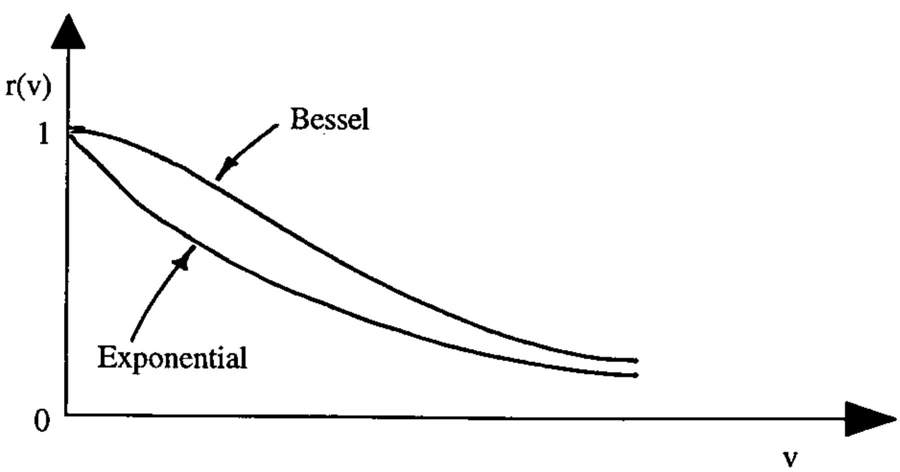

Figure 9. Exponential and Bessel functions.

The correlation parameter is estimated by fitting the correlation function so that it preserves the estimated correlation coefficient at a typical distance in the area under consideration. This distance is called the "characteristic correlation distance" which is the mean separation between two points randomly selected in the area.

Gosh (1951) derives the distribution of the distance between two points chosen at random in a plane convex region. Using this distribution, Matern (1960) derived the mean distance between two randomly chosen points in seven regions of unit area: circle, 0.5108; regular hexagon, 0.5126; square, 0.5214; equilateral triangle, 0.5544; rectangle with $\beta = 2$, 0.5691,; rectangle with $\beta = 4$, 0.7137; rectangle with $\beta = 16$, 1.3426 where β is the ratio of the longer to the shorter side of a rectangle.

The value of the expected correlation coefficient $\bar{r}(\mathbf{x}_1 - \mathbf{x}_2 | A)$ is the same when A and α or b vary as long as the product $A\alpha^2$ or Ab^2 remains constant (Rodriquez-Iturbe and Mejia, 1974a).

Figures 5 and 6 in Rodriquez-Iturbe and Mejia (1974b) give the values of K_4 as a function of $A\alpha^2$ and Ab^2 for the exponential and Bessel type correlation functions respectively.

Masters (1993) and Masters and Irish (1994) estimated the correlation parameter by taking expectations of Eq. (20) after log transformation.

$$\alpha = \frac{-E[\ln r(v)]}{E[v]} \tag{22}$$

where E[ ] is the expectation operator.

3.2.3 Meynink and Brady Method

Meynink and Brady (1993) used the Matalas and Langbein (1962) expression for the effective number of independent rainfall stations, n_e, to derive an areal reduction factor.

$$n_e = \frac{n}{1 + \bar{r}(n-1)} \tag{23}$$

where n — the number of rainfall stations
$\bar{r}$ — the mean interstation correlation

The mean interstation correlation is defined as the arithmetic mean of the correlation coefficients between all the station pairs in the area under consideration.

Since the possibility of two design rainfall events occurring simultaneously at two independent stations is extremely rare, only one of the n_e stations will contribute significantly to a flood. Hence, the effective areal reduction factor is given by the storm area as a fraction of the catchment area.

$$K_5 = \frac{1}{n_e} = \frac{1 + \bar{r}(n-1)}{n} = \bar{r} + \frac{1}{n}(1 - \bar{r}) \tag{24}$$

which for large n reduces to

$$K_5 \approx \bar{r} \tag{25}$$

Using the Walnut Gulch relationship between correlation coefficient and inter-gauge distance for thunderstorm data (Osborn et al, 1979), $\bar{r}$ was found to be 0.57 for a compact catchment and 0.49 for an elongated catchment, both approximately 45 km^2 in area.

3.2.4 Statistical Derivation of ARF

Assuming that the rainfall depths are log normally distributed in space, Omolayo (1989) derived an expression for ARF as:

$$K_6 = \exp\left\{K_T\sigma\left(\sqrt{\frac{1+(n-1)\rho}{n}} - 1\right)\right\} \tag{26}$$

where K_T frequency factor corresponding to return period T
σ standard deviation of rainfall depths in the log domain
ρ average correlation coefficient
n number of rainfall stations

ARF appears to vary directly with spatial correlation coefficient and inversely with return period, standard deviation and the number of rainfall stations.

If there is perfect correlation among all the stations ($\rho = 1$), then

$$K_6 = 1 \tag{27}$$

On the other hand, if there is zero spatial correlation, then

$$K_6 = \exp\ \{K_T\sigma(n^{-1/2} - 1)\} \tag{28}$$

In reality, ARF should be somewhere between the above extremes given by Eqs (27) and (28). The lower bound value of ARF for some typical combinations of T, n and σ are given below.

Table 2. Limiting areal reduction factors (Omolayo, 1989)

T	K	n = 3		n = 10		n = ∞	
		$\sigma = 0.15$	$\sigma = 0.40$	$\sigma = 0.15$	$\sigma = 0.40$	$\sigma = 0.15$	$\sigma = 0.40$
10	1.282	0.922	0.805	0.877	0.704	0.825	0.599
100	2.326	0.863	0.675	0.788	0.529	0.705	0.394

Unlike the other methods described above, this method depends on the number of stations used in the analysis in addition to the return period and the standard deviation of rainfall depths.

For large n, Eq (29) reduces to

$$K_6 = \exp\{K_T\sigma(\rho^{1/2} - 1)\} \tag{29}$$

Using the above approximation, ARF for some typical combinations of T, ρ and σ are given in Table 3.

Table 3. Derived ARF for typical combinations of T, ρ and σ (Omolayo, 1989).

T	K	ρ = 0.2		ρ = 0.4		ρ = 0.6	
		σ = 0.15	σ = 0.40	σ = 0.15	σ = 0.40	σ = 0.15	σ = 0.40
10	1.282	0.899	0.753	0.932	0.828	0.958	0.891
100	2.326	0.825	0.598	0.880	0.710	0.924	0.811

If we assume that the rainfall depths are normally distributed with zero mean and standard deviation σ, ARF can be obtained as

$$K_6 = \left(\frac{1+(n-1)\rho}{n}\right)^{1/2} \tag{30}$$

Under this assumption of normal distribution, ARF becomes independent of the return period and the standard deviation of rainfall depths. It only depends on the spatial correlation coefficient and the number of stations. For large n, this reduces to

$$K_6 = \rho^{1/2} \tag{31}$$

This is the same expression as that derived by Rodriquez-Iturbe and Mejia (1974b) except that the ρ here is the average spatial correlation coefficient.

3.2.5 Summary

All the analytical methods assume that the rainfall process is stationary and isotropic. In addition, normal distribution or log normal distributions are assumed. In reality, the distribution of rainfall might be different from these. However, this difference might not significantly affect the magnitudes of the derived ARFs. The effect of using a wrong distribution can be assessed by deriving ARFs from actual rainfall data and comparing with those derived empirically. Another point to note in the Rodriguez-Iturbe and Mejia method is the assumption of a zero mean process. Suppose we obtain an estimate of 0.8 for ARF from this method. We need areal rainfall for a given ARI. The point value for the same ARI is, say, 30 mm and the mean rainfall is 10 mm. An appropriate way of obtaining areal rainfall is as follows:

Point value for a zero mean case (30 -10) = 20 mm
Corresponding areal value (20 x 0.8) = 16 mm

For a stationary and isotropic process, the point mean and the areal mean are the same. Hence the areal rainfall corresponding to a point rainfall of 30 mm is 26 mm (10 + 16 = 26 mm)

On the other hand, if we multiply the point value (30 mm) by ARF (0.8), we get 24 mm which is different from the value obtained from a zero mean process. From this example, the appropriate ARF for a non-zero mean process is 26/30 = 0.867 and not 0.8. The issue of applying this ARF in practice needs further examination.

3.3 Analytical-Empirical Method

3.3.1 Myers and Zehr Method

Myers and Zehr (1980) developed a computerised model of the structure of annual maximum rain storms and a routine for computing areal reduction factors from these storms, with emphasis on extracting as much information as possible from data of pairs of recording gauges only. By definition, for a given frequency, f, duration, Δt, and area, A, the area areal reduction factor is

$$K_7 = \frac{X_A(f,\Delta t, A)}{X_A(f,\Delta t, 0)} \tag{32}$$

where $X_A(f,\Delta t,A)$ the areal rainfall
$X_A(f,\Delta t,0)$ the point rainfall

Myers and Zehr modified Eq (32) to:

$$K_7 = \frac{\overline{X}'_A(\Delta t, A, N) + K(f,N)\overline{S}'_A(\Delta t, A, N)cv(\Delta t)}{1 + K(f,N)cv(\Delta t)} \tag{33}$$

where $\overline{X}'_A(\Delta t, A, N)$ the mean of areal rainfall relative to zero distance value
$\overline{S}'_A(\Delta t, A, N)$ the standard deviation of areal rainfall relative to zero distance value
K(f,N) a frequency factor
cv(Δt) the coefficient of variation of annual maximum point rainfall
N length of data

The first and second moments of all annual series were adjusted for a common length (20 years) and then smoothed over area and duration by fitting exponential surfaces in area-duration space. An extreme value type I (EVI) distribution was assumed in adjusting the data and later in the analyses. Fitted surfaces were derived for five different station pair statistics: first and second moments of annual maximum 2 station average, first and second moment of rain at the second station simultaneous with the single station annual maximum at the first and covariance between station annual maxima and the simultaneous rain. These statistics were then used in various combinations to estimate upper and lower bounds of the first and second moments of annual maximum areal average precipitation. The placement of surface between the bounds is by calibration with data from a limited number of quasi-symmetric five station groups. Finally, the areal reduction factors were derived by processing the first and second areal moments through Chow's generalised frequency equation. The ARFs recommended in ARR87 (Fig 2.6) for use over most of Australia are based on the application of this method with rainfall data from the Chicago area (US).

4 COMPARISON OF AREAL REDUCTION FACTORS ESTIMATED BY DIFFERENT METHODS

Roche (1963) estimated the ARF for an area of 50 km^2 in the Flakoho Basin, Ivory Coast (a tropical region) as 0.86 by fitting a truncated normal distribution to the logarithms of point rainfalls. He also found out that the reduction factors are independent of the frequency of occurrence for the above area. Rodriguez-Iturbe and Mejia (1974b) obtained the same value of ARF for the above area by their method using both the exponential and Bessel correlation functions. However, the US Weather Bureau relationship gave a value of 0.985 for the same area which is considerably higher than the above estimates.

Nittim (1989) estimated areal reduction factors using data from two catchments of the Georges River in NSW (Fig. 10) for three durations. The ARF was derived as a ratio of design rainfall obtained from depth-area-duration (DAD) analysis and intensity-frequency-duration data in Australian Rainfall and Runoff. Average catchment daily rainfalls were determined from 8 and 10 stations and 100 years of record for the two catchments using weighting coefficients. The design areal rainfall was obtained by using a log-normal distribution. The average catchment rainfalls from the DAD analysis were from 9 am to 9 am and they were converted to unrestricted rainfalls as described by Pierrehumbert (1972). The areal reduction factors obtained for the two catchments are given in Table 4. The variation of ARF with area for different probabilities of exceedance is shown in Figures 11 to 13.

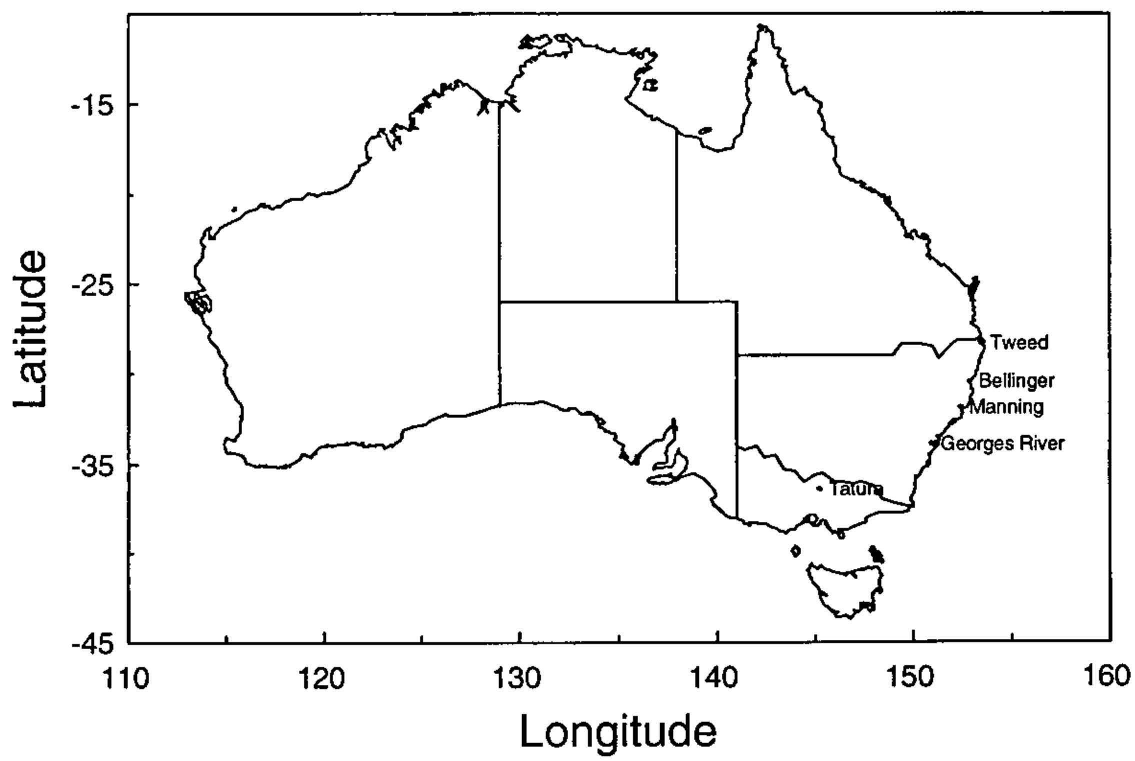

Figure 10. Location of the Australian study sites.

Table 4. Areal reduction factors for two catchments in NSW (Nittim, 1989)

(Note: ARR87 gives ARFs up to 24-hour duration)

Probability of Exceedance	Area 360 km^2 (Liverpool)			Area 600 km^2 (Picnic Point)		
	24 h	48 h	72 h	24 h	48 h	72 h
5 %	0.936	0.928	0.901	0.870	0.924	0.904
2 %	0.895	0.966	0.960	0.867	0.929	0.910
1 %	0.877	0.942	0.921	0.875	0.938	0.918
Average (Std Dev)	0.903 (0.025)	0.945 (0.016)	0.927 (0.025)	0.871 (0.003)	0.930 (0.006)	0.911 (0.006)
AR&R 1987	0.93	0.94	0.95	0.94	0.95	0.96
Difference(%)	2.8	-0.5	1.9	7.5	3.0	4.6

It can be seen from Table 4 that the estimated values varied from 0.87 to 0.94 while those given in ARR87 varied from 0.93 to 0.96. The areal reduction factors estimated are generally smaller than those published in ARR87. The differences are the greatest for the larger catchment. Even though the differences are large for the 24 hour storm on both catchments, the use of the unrestricted multiplier for Sydney 1.19, instead of the general multiplier 1.13, reduces the differences considerably to 3.3 % for the larger catchment and to almost zero for the smaller catchment. The uncertainty in the magnitude of the multiplier could have easily been avoided by estimating the point rainfalls from the same rainfall used in estimating the areal rainfalls. Except for one point in each of the Figures 11 to 13, all the other points are close together indicating that the variation in ARF with probability of exceedance is small.

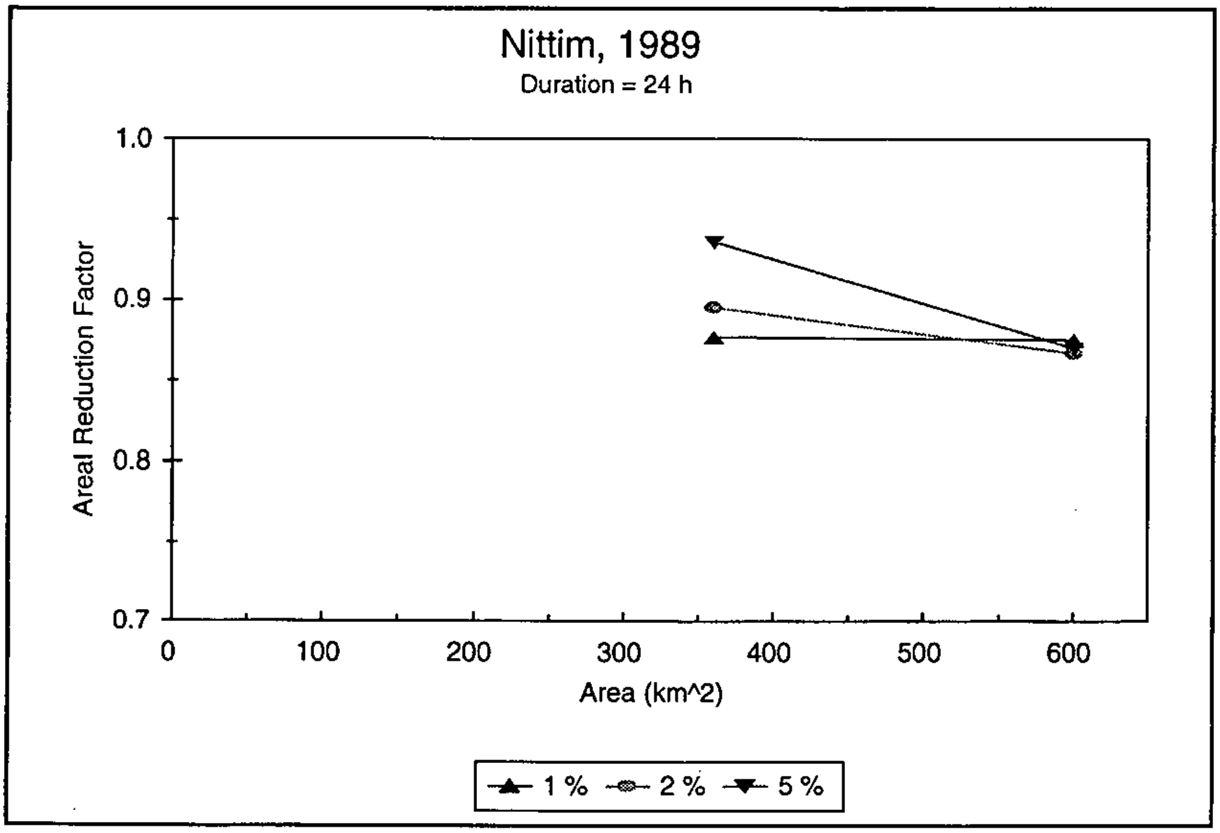

Figure 11. Variation of ARF with area for different probabilities (duration 24 h)

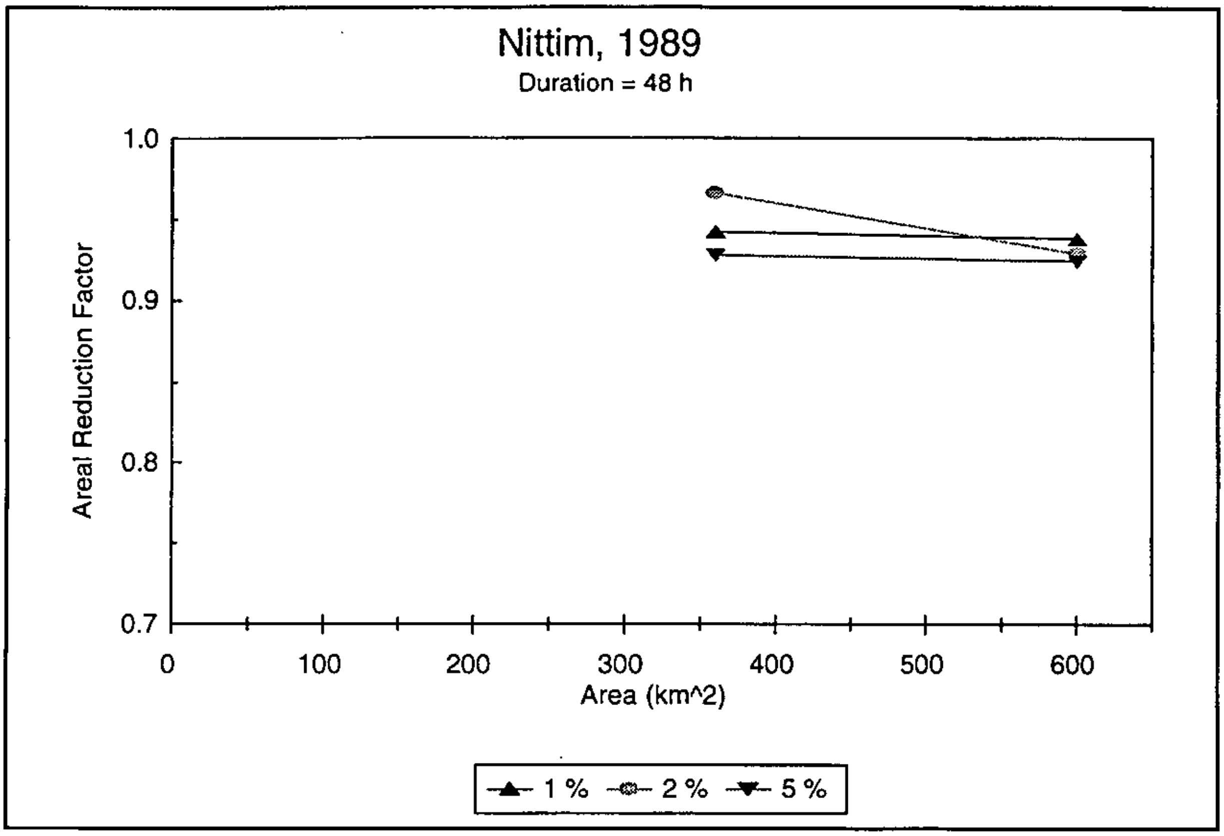

Figure 12. Variation of ARF with area for different probabilities (duration 48 h)

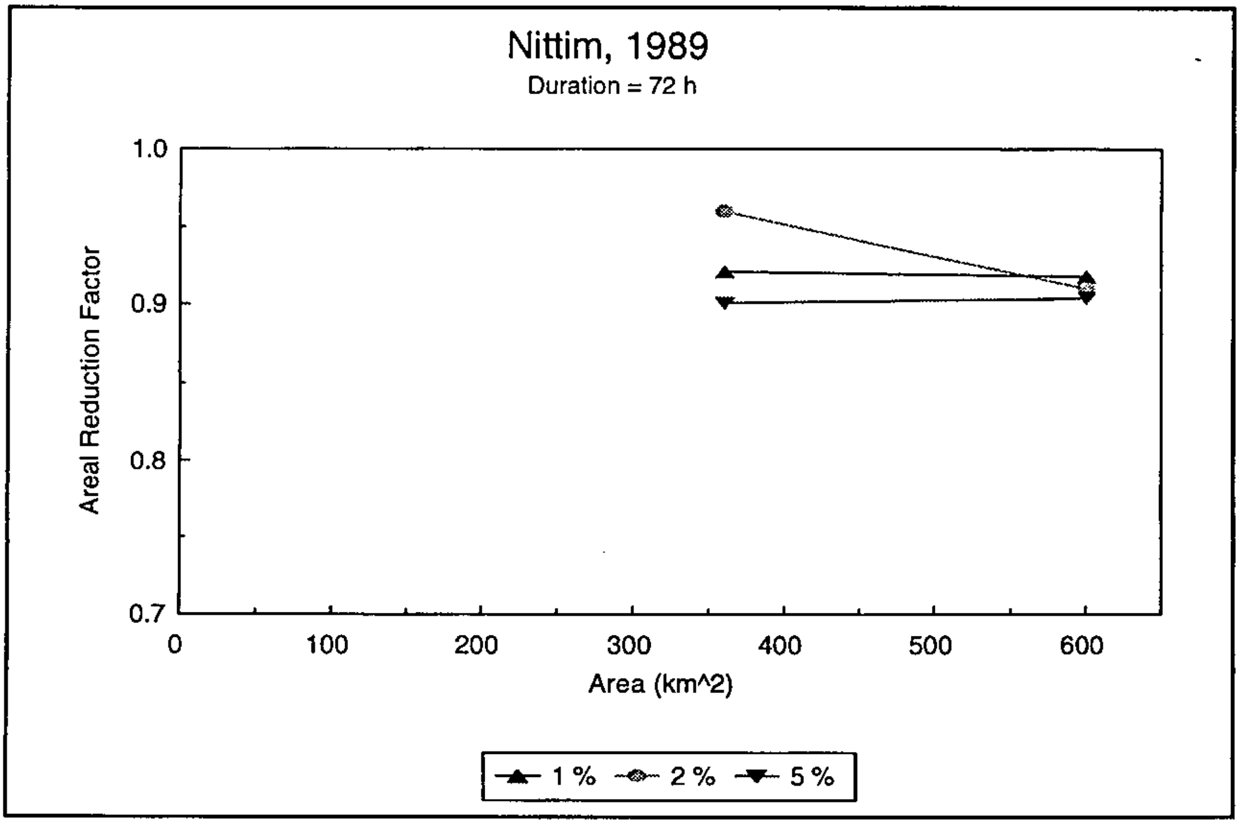

Figure 13. Variation of ARF with area for different probabilities (duration 72 h)

Avery (1991) calculated areal reduction factors for the Tweed, Bellinger and Manning river catchments by dividing the unrestricted areal average rainfalls by the corresponding Thiessen average of point IFD values from ARR87. Two durations and four probabilities of exceedance were considered. This is the same method as used by Nittim (1989) and the estimated ARFs are given in Table 5. The variation of ARF with area for different probabilities of exceedance is shown in Figures 14 and 15.

Table 5. Areal reduction factors for three catchments in NSW (Avery, 1991)
(Note: ARR87 only gives ARFs up to 1000 km^2 and 24-hour duration)

Probability of Exceedance	Tweed (650 km^2)		Bellinger (640 km^2)		Manning (6560 km^2)	
	24 h	48 h	24 h	48 h	24 h	48 h
10 %	0.77	0.85	0.73	0.75	0.73	0.81
5 %	0.81	0.86	0.75	0.76	0.74	0.83
2 %	0.84	0.85	0.79	0.77	0.74	0.85
1 %	0.88	0.85	0.78	0.78	0.74	0.87
Average (Std Dev)	0.825 (0.040)	0.853 (0.004)	0.763 (0.024)	0.765 (0.011)	0.738 (0.004)	0.840 (0.022)
AR&R 1987	0.92	0.93	0.92	0.93	0.85	0.88
Difference(%)	10	8	17	15	13	5

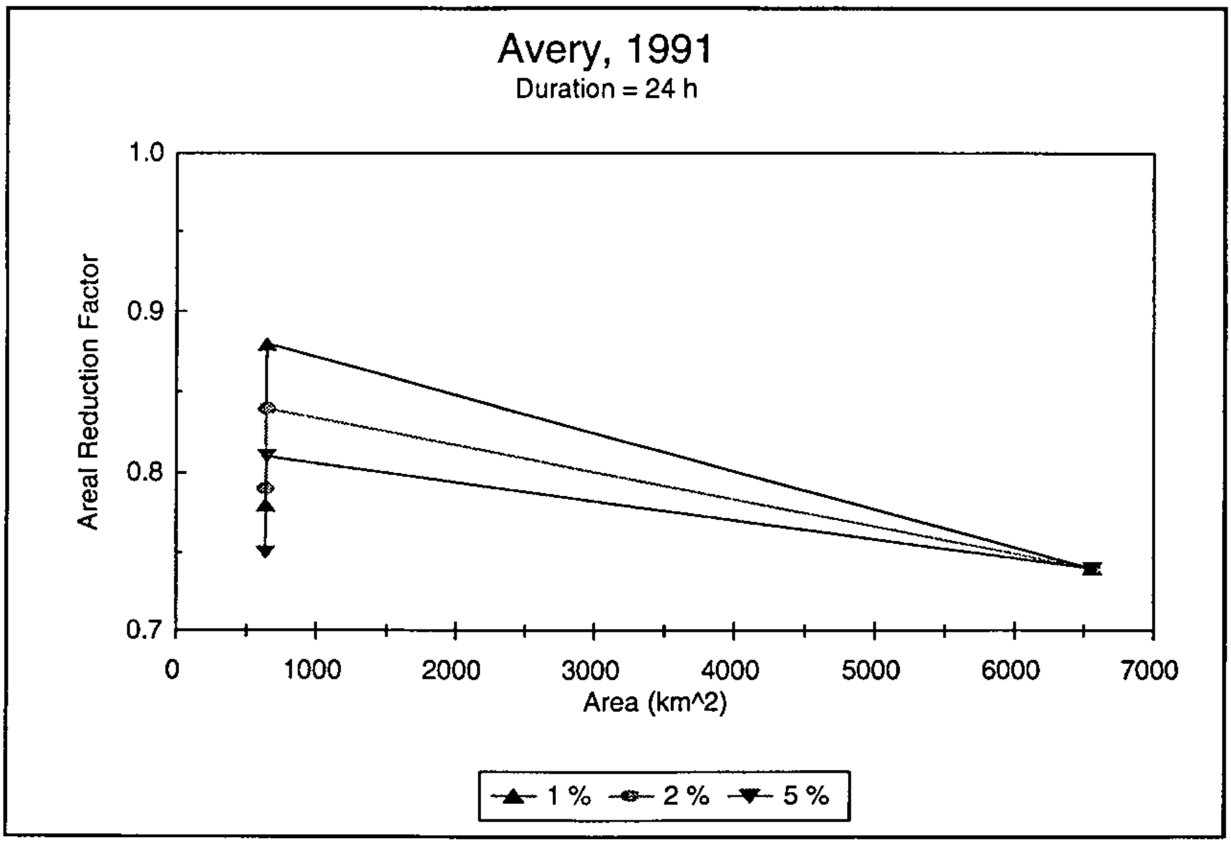

Figure 14. Variation of ARF with area for different probabilities (duration 24 h)

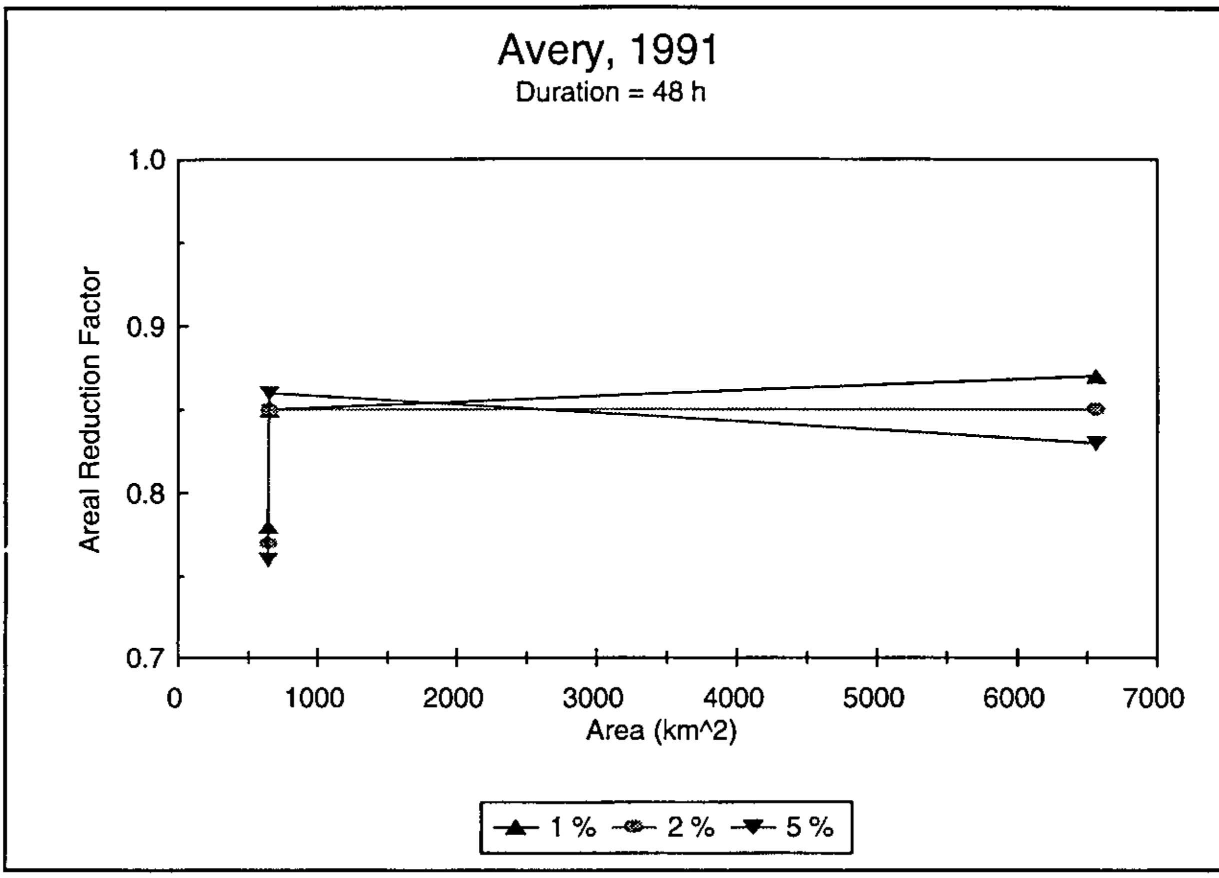

Figure 15. Variation of ARF with area for different probabilities (duration 48 h)

The variation in the magnitude of ARF with probability of exceedance is small for the larger catchment (Fig. 14 and 15) while for the smaller two catchments, considerable variation is observed. The magnitude of the estimated ARFs varies from 0.73 to 0.88. A slight tendency for the ARFs to increase with decreasing AEP is observed. However, the average estimated ARFs vary from 0.74 to 0.85 while those from ARR87 vary from 0.85 to 0.93 over both durations. The difference between the estimates from Avery (1991) and ARR87 is considerable and it is not mentioned in the paper how the unrestricted values were obtained. The use of a smaller multiplier might have resulted in large differences. As mentioned above, this complication could have been avoided by calculating the point rainfall also from the rainfall data instead of using the ARR87 values. It is interesting to note that the values obtained by Avery (1991) are lower than those obtained by Nittim (1989). This could be due to the use of different unrestricted multipliers but may indicate that ARFs are catchment specific.

Porter and Ladson (1993) applied the method proposed by Bell (1976) to estimate areal reduction factors for four fixed areas in the Deakin Main Drain region of northern Victoria (Tables 6 to 8). The point rainfall was obtained from ARR87 for four main centres in the area, namely, Kyabram, Tatura, Rushworth and Rochester. Since only a minor spatial variation (1.5 - 8 %) in rainfall was observed, they used the point rainfall values at Tatura to estimate the areal reduction factors.

Table 6. Fixed area rainfall reduction factors for 24-hour rainfall (Porter and Ladson, 1993)

Probability of Exceedance	Area 1 414 km^2	Area 2 460 km^2	Area 3 874 km^2	Area 4 1970 km^2
50 %	0.77	0.75	0.75	0.77
20 %	0.81	0.80	0.78	0.80
10 %	0.83	0.83	0.81	0.83
5 %	0.81	0.81	0.79	0.80
2 %	0.80	0.80	0.77	0.79
Average (Std Dev)	0.80 (0.020)	0.80 (0.026)	0.78 (0.020)	0.80 (0.019)
ARR87	0.94	0.94	0.92	

Table 7. Fixed area rainfall reduction factors for 48-hour rainfall (Porter and Ladson, 1993)

Probability of Exceedance	Area 1 414 km^2	Area 2 460 km^2	Area 3 874 km^2	Area 4 1970 km^2
50 %	0.76	0.78	0.74	0.77
20 %	0.84	0.81	0.81	0.83
10 %	0.89	0.85	0.85	0.88
5 %	0.89	0.84	0.85	0.87
2 %	0.89	0.83	0.85	0.86
Average (Std Dev)	0.85 (0.051)	0.82 (0.025)	0.82 (0.043)	0.84 (0.040)

Table 8. Fixed area rainfall reduction factors for 72-hour rainfall (Porter and Ladson, 1993)

Probability of Exceedance	Area 1 414 km^2	Area 2 460 km^2	Area 3 874 km^2	Area 4 1970 km^2
50 %	0.78	0.79	0.77	0.80
20 %	0.88	0.85	0.85	0.88
10 %	0.93	0.88	0.89	0.92
5 %	0.94	0.88	0.89	0.93
2 %	0.94	0.86	0.89	0.92
Average (Std Dev)	0.89 (0.061)	0.85 (0.033)	0.86 (0.047)	0.89 (0.048)

The ARF values ranged from 0.74 to 0.94 over all the durations and areas. The magnitude of the values initially increased with decreasing probability of exceedance and remained approximately constant for probability of exceedance less than 0.1 (Fig. 16 to 18). The variation in ARF with probability of exceedance is small as can be seen from the standard deviations given in Tables 6 - 8.

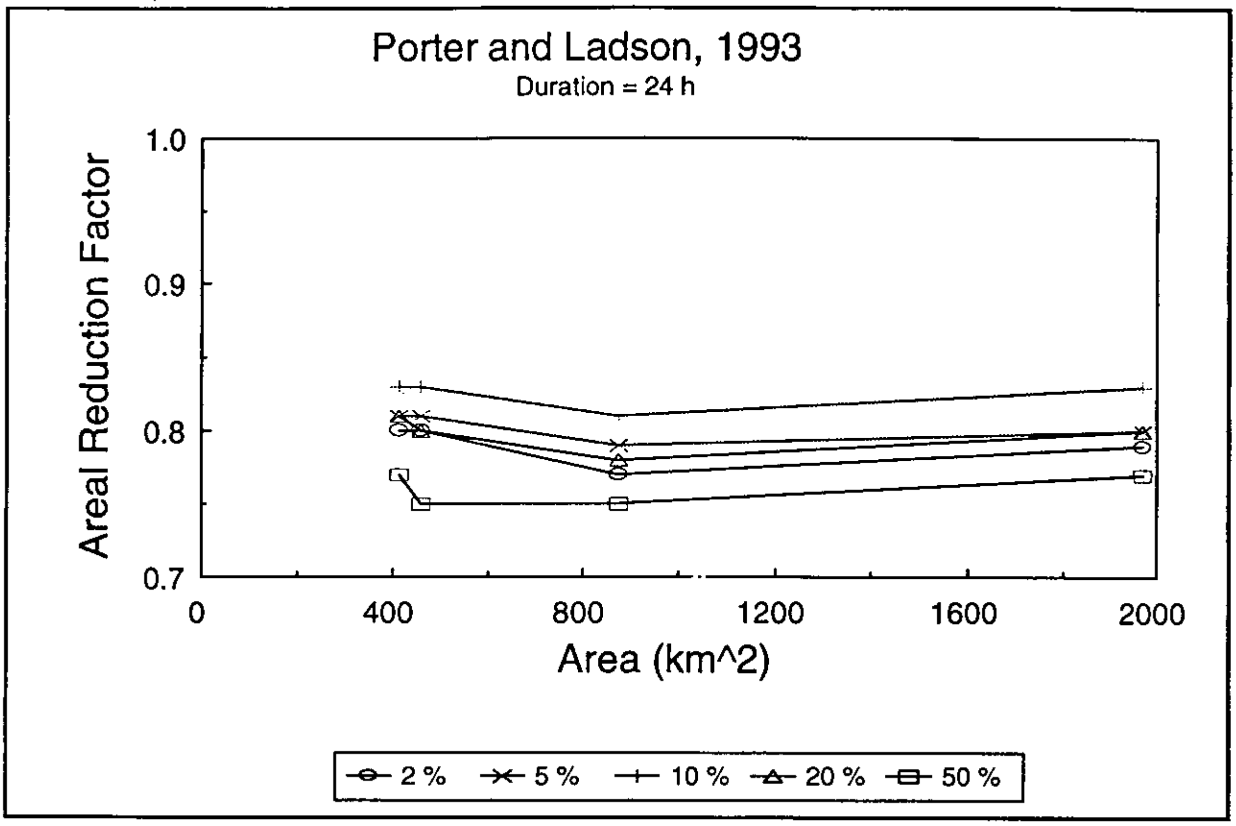

Figure 16. Variation of ARF with area for different probabilities (duration 24 h)

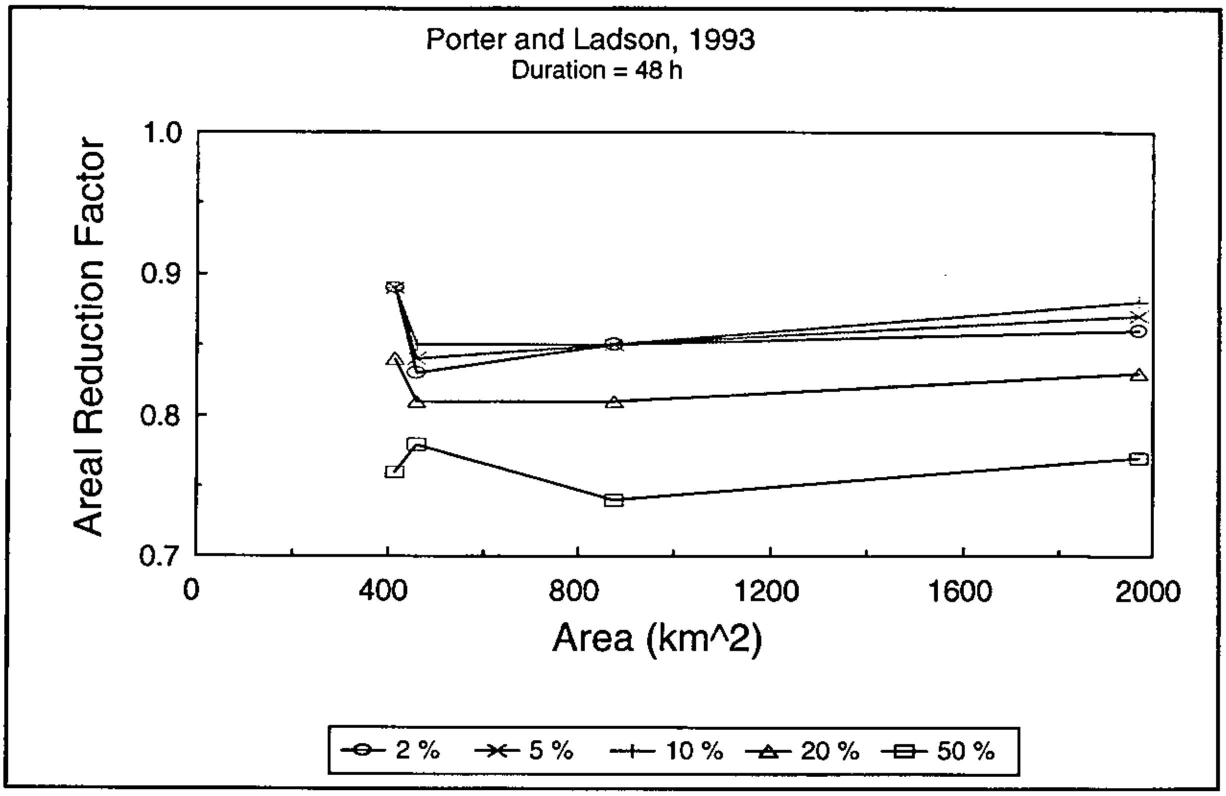

Figure 17. Variation of ARF with area for different probabilities (duration 48 h)

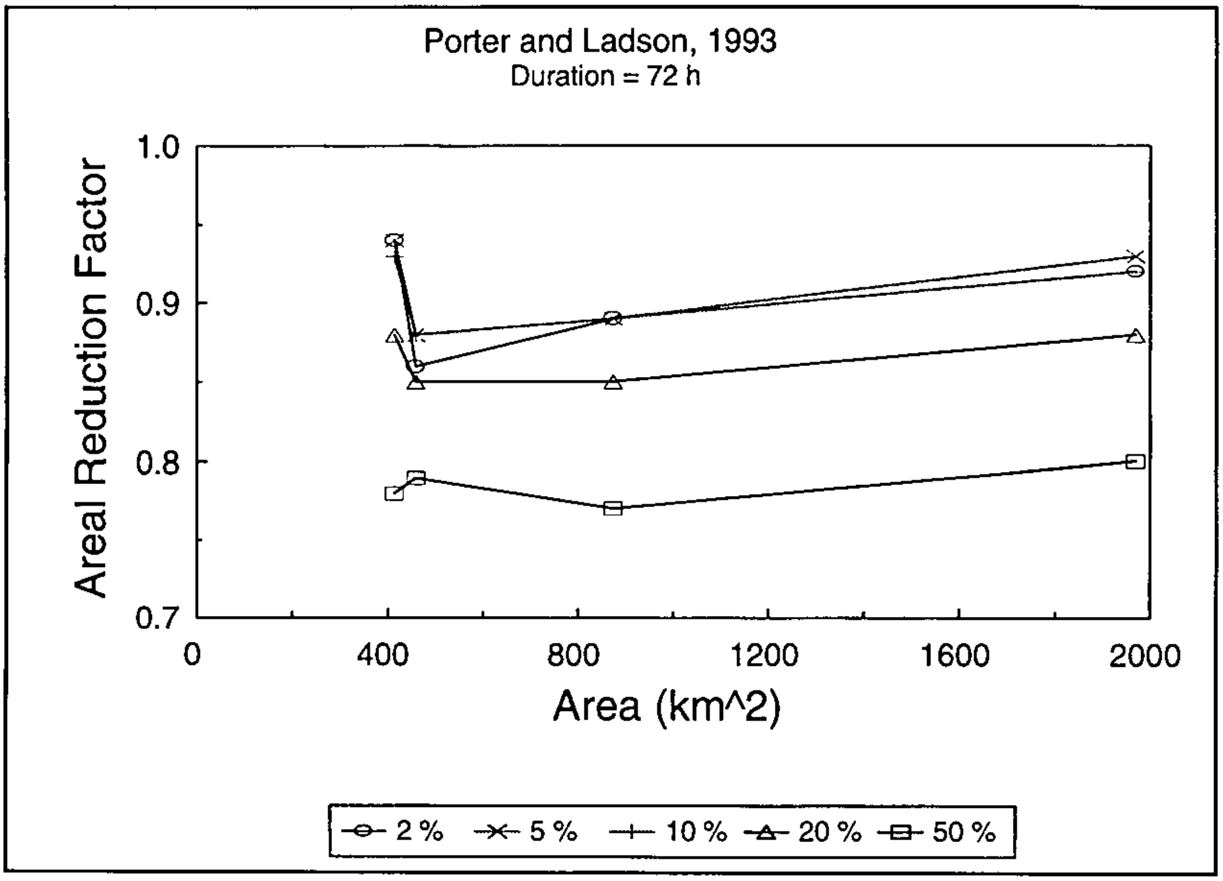

Figure 18. Variation of ARF with area for different probabilities (duration 72 h)

Omolayo (1993) estimated the ARFs for six Australian capital cities by considering circular areas of 100, 200, 250, 500 and 1000 km^2 within each city for one day rainfall. Due to lack of readily available data, the ARF for 1 hour rainfall was obtained for Melbourne only. He used four methods, namely, the US Weather Bureau method, UK method, Bell's method and Rodriguez-Iturbe and Mejia method and the results are given in Tables 9 to 13.

Table 9. 1 day areal reduction factors by the US Weather Bureau method (Omolayo, 1993)

City	Area km^2				
	100	200	250	500	1000
Adelaide	0.954	0.927	0.908	0.909	0.884
Brisbane	0.960	0.961	0.963	0.956	0.843
Hobart	0.869	0.920	0.908	0.844	0.879
Melbourne	0.939	0.927	0.935	0.917	0.901
Perth	0.957	0.964	0.982	0.882	0.865
Sydney	0.863	0.943	0.936	0.916	0.864
Average	0.924	0.940	0.939	0.904	0.873
US curves	0.970	0.950	0.940	0.920	0.910

The variation in ARF obtained by the US method with area is shown in Figure 19 for the six sites. Except for Brisbane and Melbourne, the ARF values tend to vary up and down with area size. For Brisbane, ARF stays relatively constant at about 0.96 up to 500 km² and falls rapidly to 0.84.

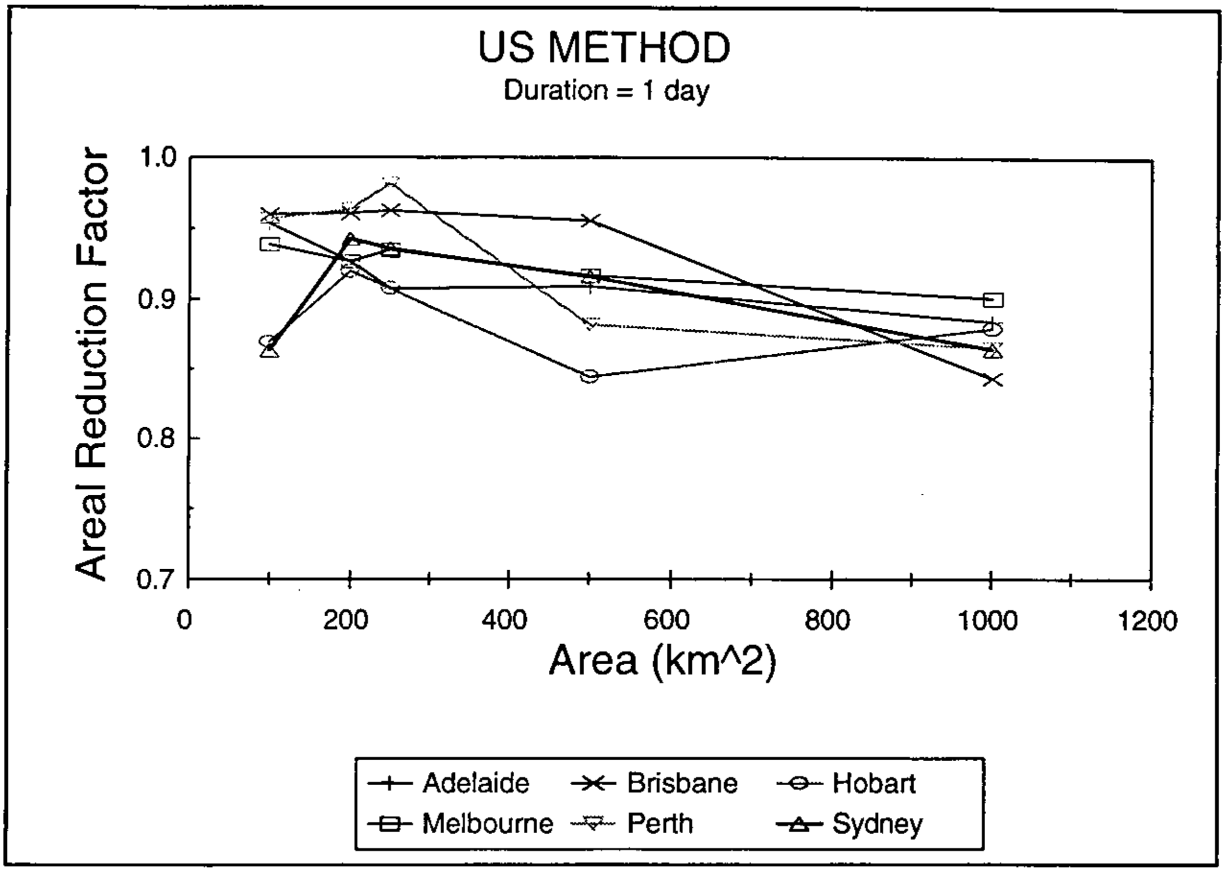

Figure 19. Variation ARF obtained by the US method with area (duration 1 day)

Table 10. 1 day areal reduction factors by the UK method (Omolayo, 1993)

City	Area km²				
	100	200	250	500	1000
Adelaide	0.930	0.928	0.905	0.855	0.834
Brisbane	0.938	0.922	0.932	0.920	0.908
Hobart	0.835	0.853	0.863	0.828	0.855
Melbourne	0.945	0.938	0.937	0.914	0.898
Perth	0.929	0.915	0.918	0.860	0.863
Sydney	0.926	0.902	0.920	0.895	0.881
Average	0.916	0.909	0.913	0.879	0.873
UK curves	0.940	0.930	0.920	0.910	0.890

The variation of ARF obtained by the UK method with area is shown in Figure 20. Except for Hobart and Sydney, for the four other sites the ARF decreases with area size. Most of the ARF values are in the range 0.85 to 0.95.

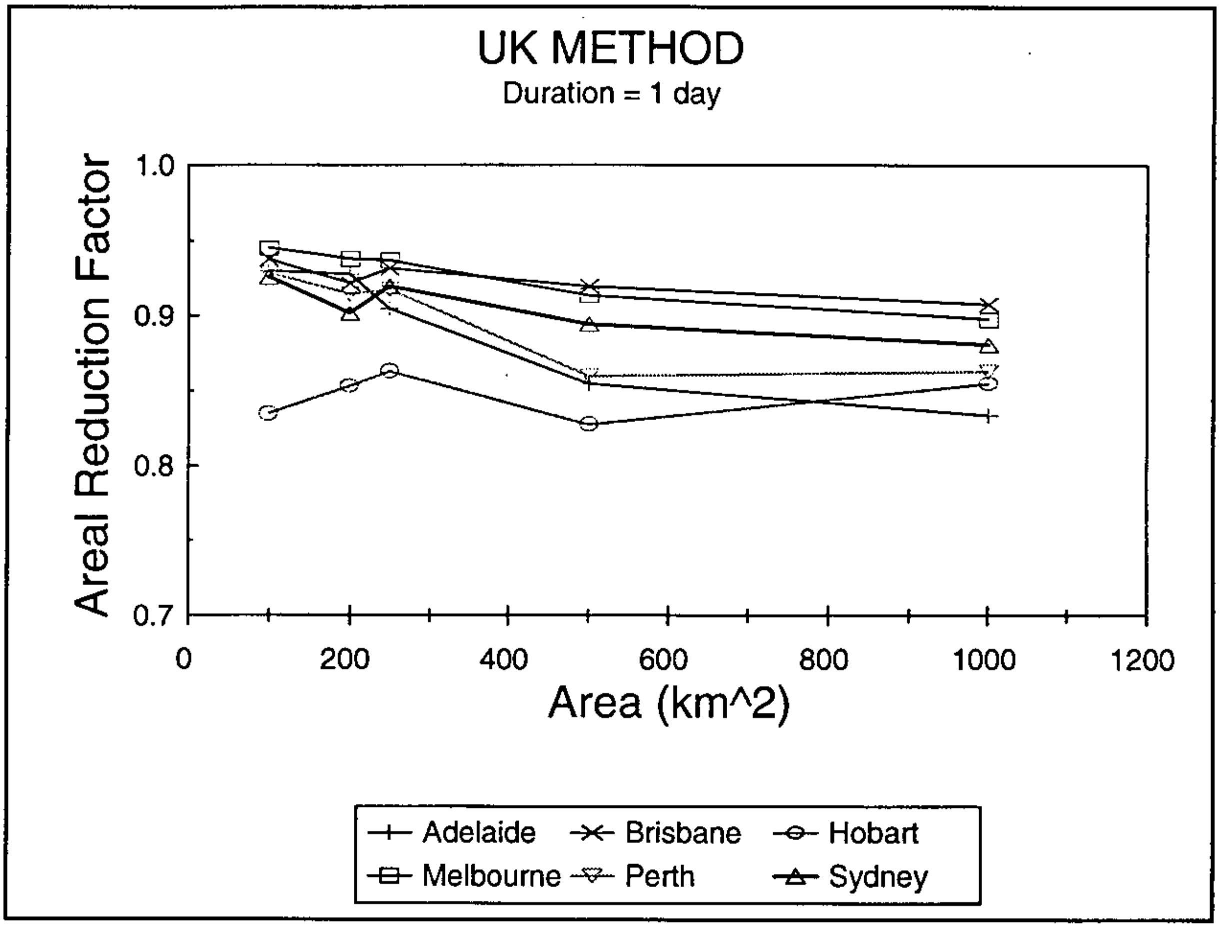

Figure 20. Variation ARF obtained by the UK method with area (duration 1 day)

Table 11 1 day areal reduction factors by the Bell's method (Omolayo, 1993)

City	Area km²				
	100	200	250	500	1000
Adelaide	0.949	0.948	0.945	0.889	0.856
Brisbane	0.964	0.948	0.953	0.947	0.930
Hobart	0.881	0.905	0.927	0.889	0.867
Melbourne	0.946	0.946	0.946	0.937	0.917
Perth	0.940	0.936	0.936	0.882	0.867
Sydney	0.939	0.932	0.929	0.923	0.907
Average	0.937	0.936	0.939	0.911	0.891

The variation of ARF obtained by Bell's method with area is shown in Figure 21. Except for Hobart, for all the other sites the ARF decreases with area size. For Hobart, ARF increases initially up to 250 km² and then decreases in the same manner as Adelaide and Perth.

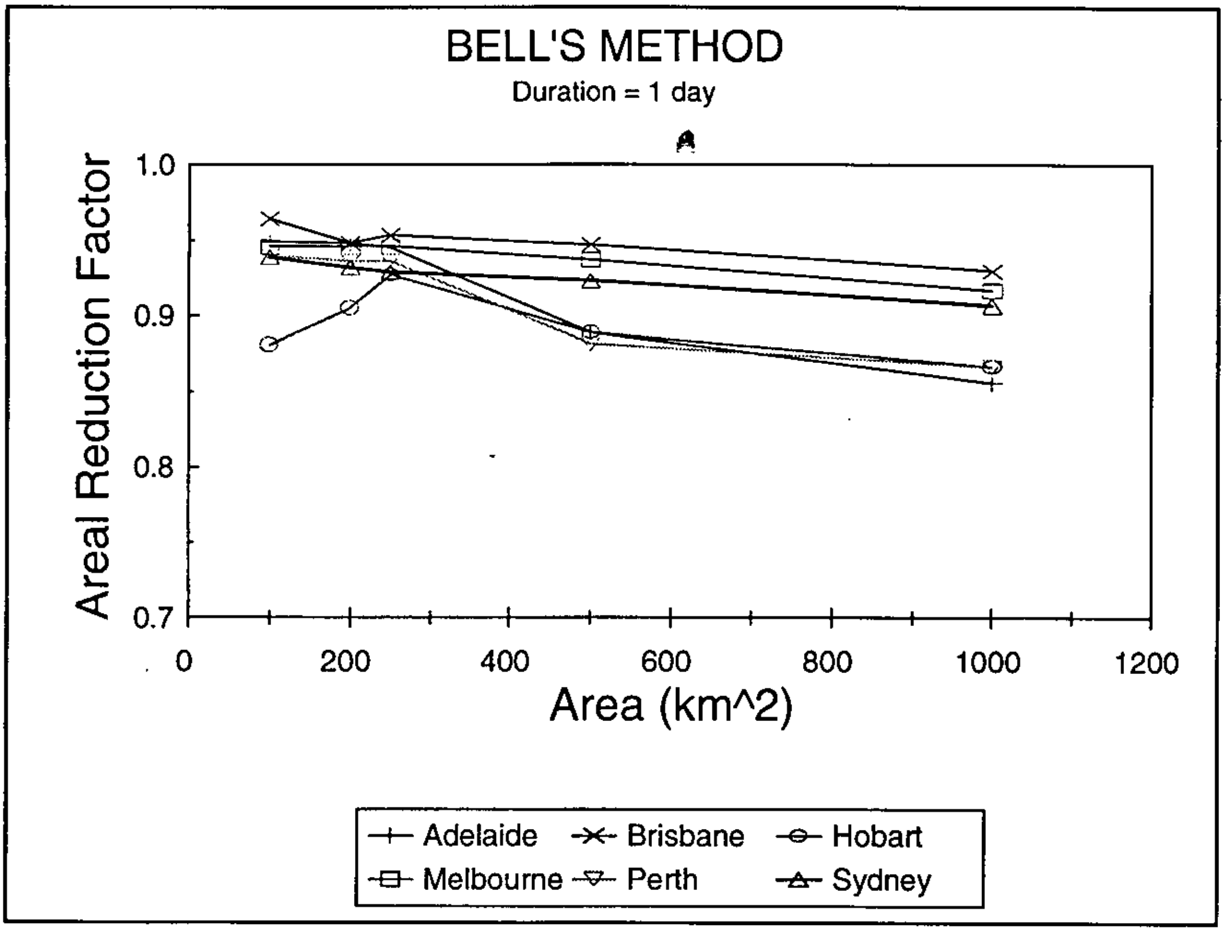

Figure 21. Variation ARF obtained by the Bell's method with area (duration 1 day)

Table 12. 1 day areal reduction factors by the Rodriguez-Iturbe and Mejia method (Omolayo, 1993)

City	Area km²				
	100	200	250	500	1000
Adelaide	0.967	0.792	0.846	0.920	0.768
Brisbane	0.946	0.840	0.875	0.832	0.869
Hobart	0.696	0.642	0.768	0.488	0.809
Melbourne	0.918	0.933	0.820	0.820	0.670
Perth	0.880	0.509	0.470	0.639	0.611
Sydney	0.779	0.893	0.881	0.875	0.549
Average	0.864	0.768	0.777	0.762	0.713

The variation of ARF obtained by the Rodriguez-Iturbe and Mejia method with area is shown in Figure 22. There is no consistent pattern in the variation of ARF with area size. Of the four methods considered, the UK and Bell's methods appeared to give consistent estimates for ARF in the sense that for most cases the values of ARF decreased with area. The values derived by the US Weather Bureau method also displayed this tendency for areas larger than 200 km².

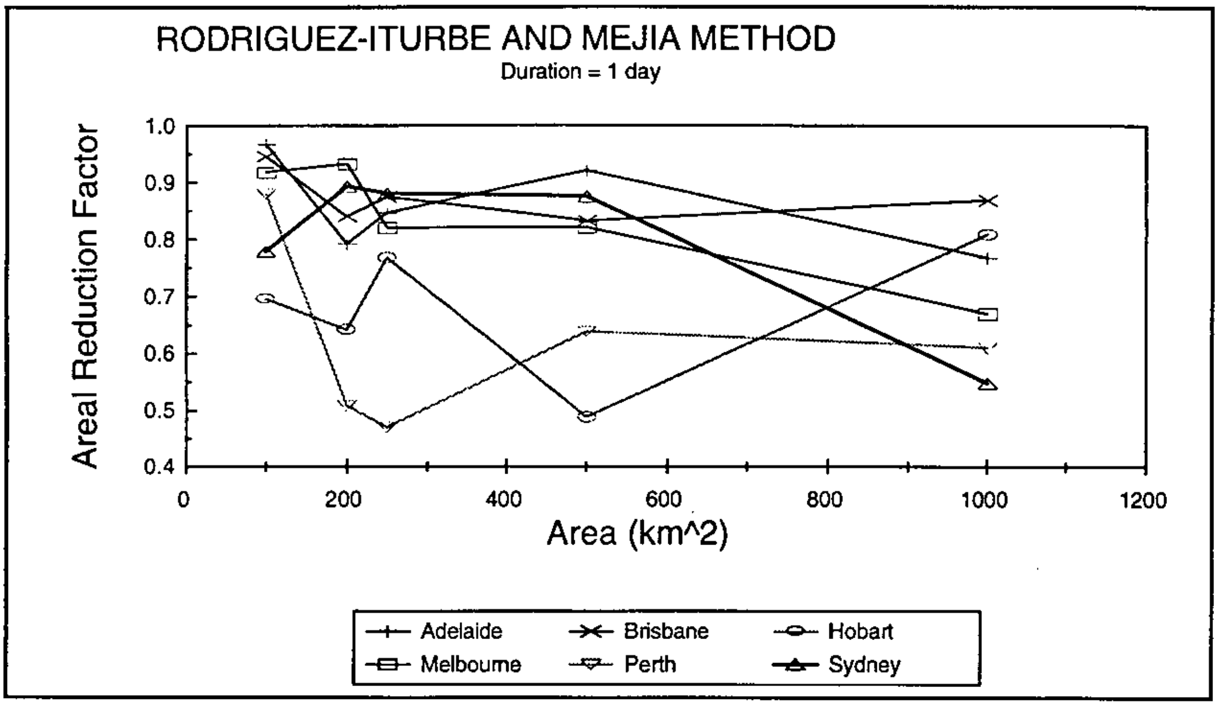

Figure 22. Variation ARF obtained by the Rodriguez-Iturbe and Mejia method with area (duration 1 day)

The estimates of ARF for six sites obtained by different methods (Omolayo, 1993) for 1 day rainfall are shown in Figures 23 to 28. An inconsistent variation of ARF obtained by the Rodriguez-Iturbe and Mejia method with area is observed for all the six sites. However, a consistent behaviour is observed in the estimates of ARF by the UK and Bell's method.

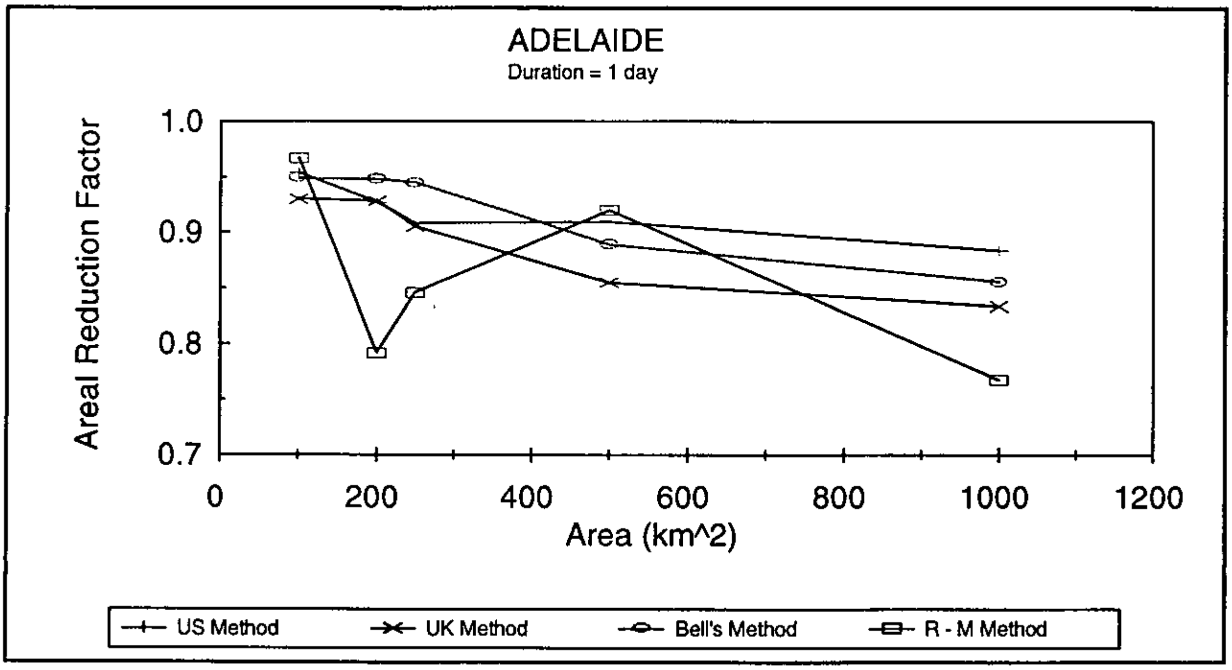

Figure 23. Comparison of ARFs obtained by different method for Adelaide (duration 1 day)

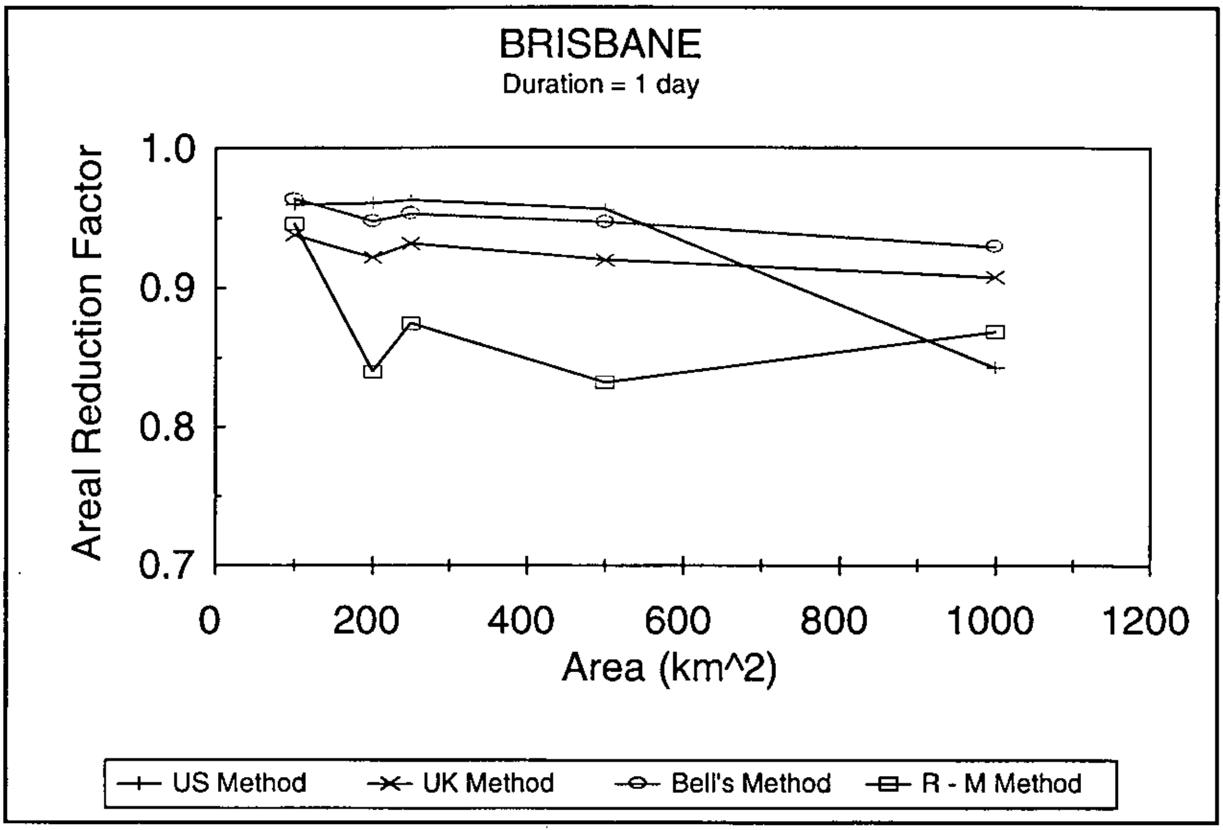

Figure 24. Comparison of ARFs obtained by different method for Brisbane (duration 1 day)

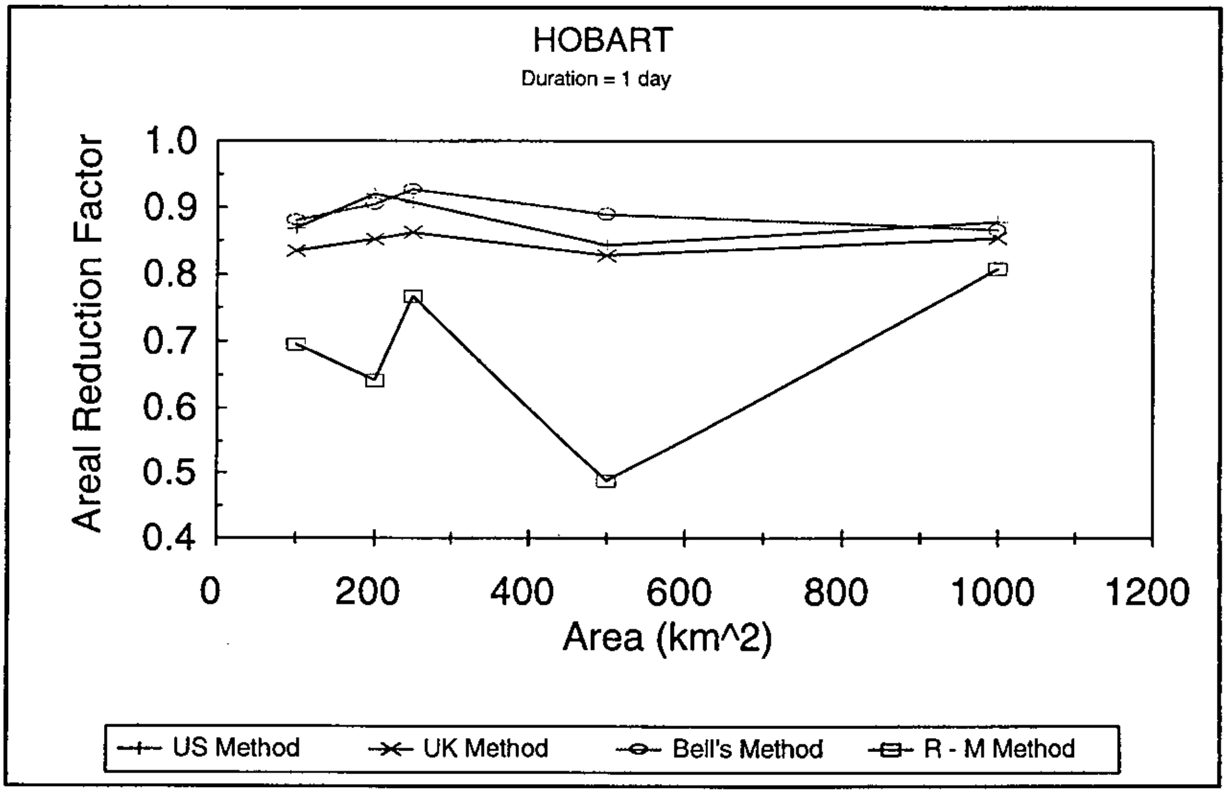

Figure 25. Comparison of ARFs obtained by different method for Hobart (duration 1 day)

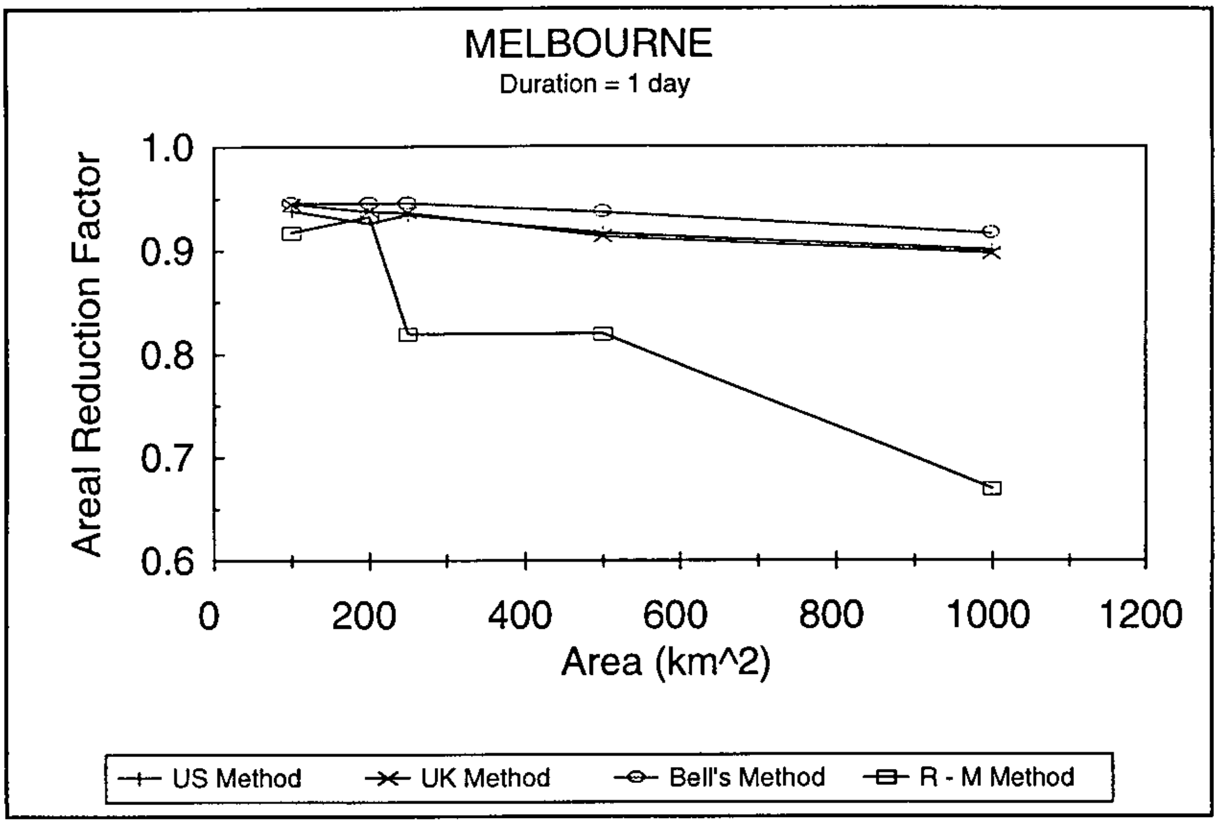

Figure 26. Comparison of ARFs obtained by different method for Melbourne (duration 1 day)

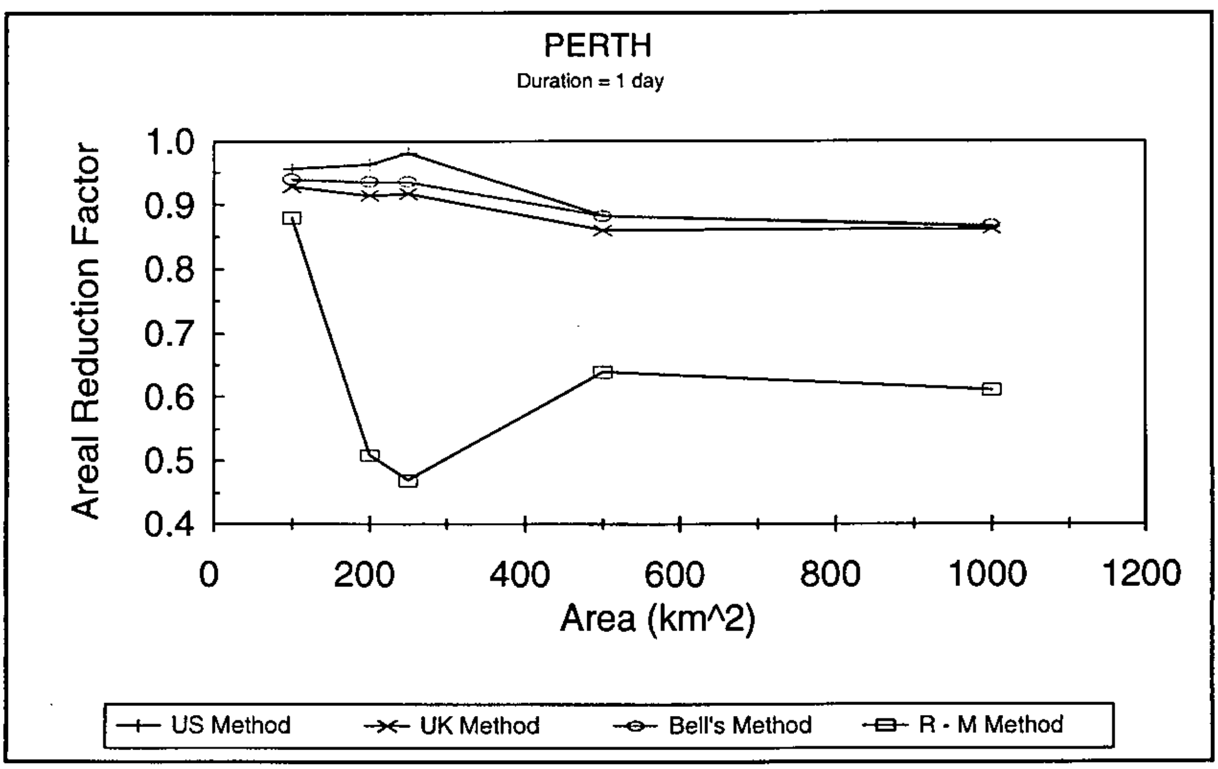

Figure 27. Comparison of ARFs obtained by different method for Perth (duration 1 day)

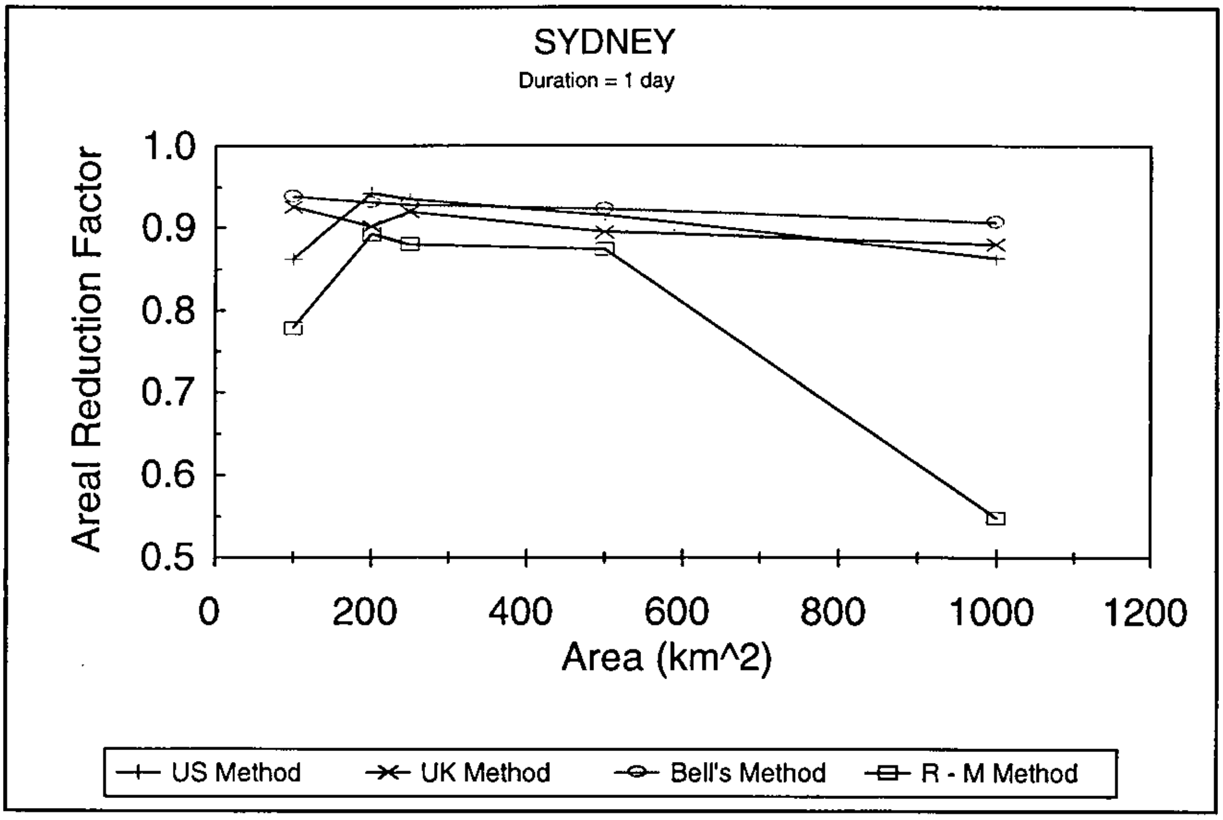

Figure 28. Comparison of ARFs obtained by different method for Sydney (duration 1 day)

Due to lack of suitable data, Omolaya (1993) derived ARF values for 1 hour rainfall for Melbourne only and the results are given in Table 13. The variation of ARF with area is shown graphically in Figure 29. Estimates obtained from the US Weather Bureau method decrease initially up to 250 km² as expected but then increase with area. Both the UK and Bell's methods show very little variation in ARF with area.

Table 13. 1 hour areal reduction factors for Melbourne (Omolayo, 1993)

Method	Area km²				
	100	200	250	500	1000
US method	0.808	0.800	0.750	0.778	0.790
UK method	0.767	0.768	0.756	0.763	0.769
Bell's method	0.749	0.748	0.749	0.745	0.756
Rod/Mejia	0.797	0.815	0.815	0.834	0.773
US curves	0.820	0.750	0.730	0.680	-
UK curves	0.790	0.750	0.730	0.680	0.620

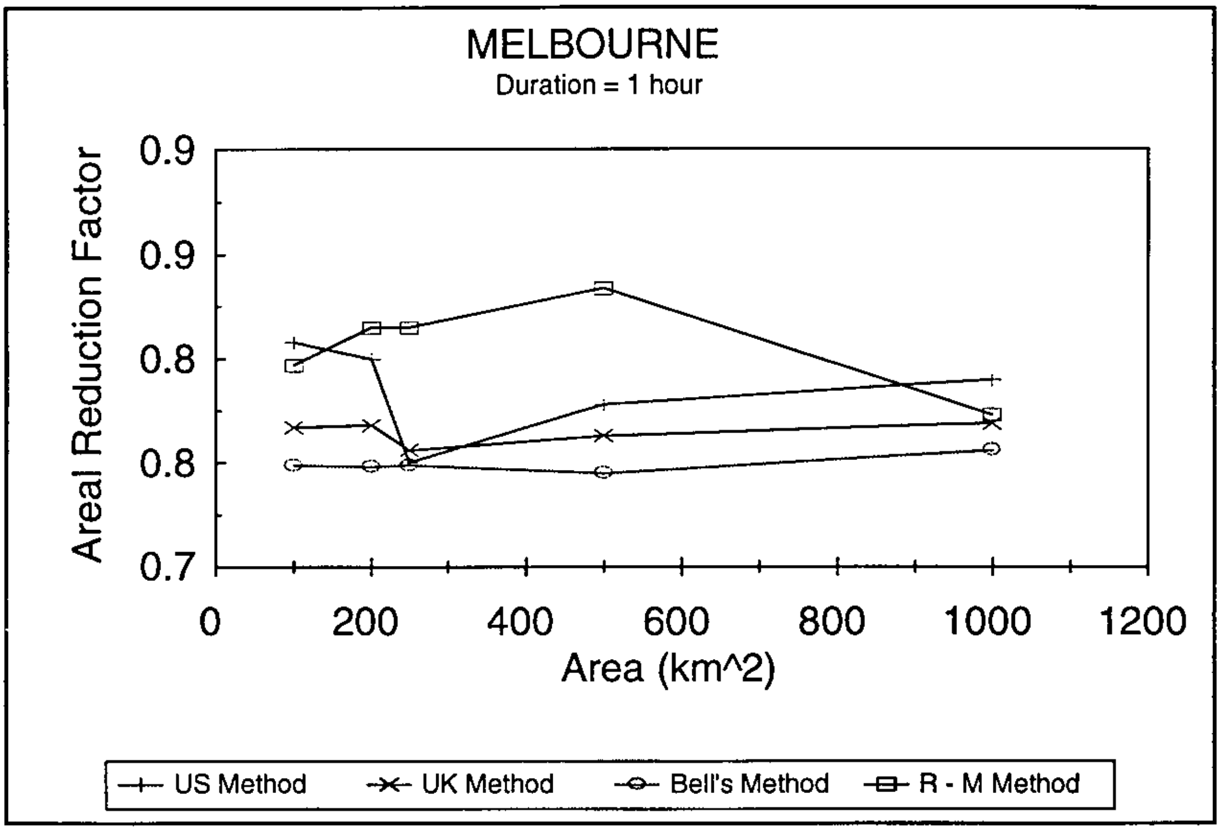

Figure 29. Comparison of ARFs obtained by different method for Melbourne (duration 1 hour)

Masters and Irish (1994) expressed reservations about Omolayo's implementation of the method of Rodriguez-Iturbe and Mejia for the derivation of ARFs. In their opinion, Omolayo's results should not be used without verification of his procedures.

Masters (1993) estimated areal reduction factors using the Myers and Zehr and Rodriguez-Iturbe and Mejia methods using data from a pluviograph network in Sydney. The decay parameter, α [in Eq (18)], was obtained from

$$\alpha = \frac{-E[\ln r(v)]}{E[v]} \tag{36}$$

where the symbol E denotes expected value. It should be noted that this is a variation of the method proposed by Rodriguez-Iturbe and Mejia (1974b) for the estimation of α.

The results from both procedures were lower than those from the U S Weather Bureau curves. It is reported that the areal reduction factors obtained by using the Rodriguez-Iturbe and Mejia method were found to be substantially lower than those from the Myers and Zehr method. The ARFs derived by the Myers and Zehr method were not presented in Masters (1993).

The areal reduction factors obtained for an area 2200 km² in Sydney by the Rodriguez-Iturbe and Mejia method are given in Table 14.

Table 14. Areal reduction factors for an area 2200 km^2 in Sydney using the Rodriguez-Iturbe and Mejia method (Masters, 1993)

Duration (h)	Areal reduction factor
1	0.31
2	0.44
3	0.52
6	0.60
12	0.70
24	0.75

Masters and Irish (1994) report consistent variation of ARF with area and duration using the Rodriguez-Iturbe and Mejia method and results which agree with those of Avery and Nittim. They also found that the decorrelation distance (which is the reciprocal of α) varies smoothly with duration and indicated the possibility that ARF values from short durations can be derived from long duration data.

5 SUMMARY

A review of the literature on the estimation of ARFs revealed basically six methods to estimate fixed area ARF. These methods are broadly classified into three categories: empirical, analytical and analytical-empirical methods. The appropriateness of these methods for the derivation of ARF for Australia is briefly discussed below.

5.1 Empirical Methods

Three methods (US Weather Bureau, UK and Bell's methods) are considered in this category. A direct comparison of the ARF values was not possible because of the slight difference in the way Bell's method was applied by Nittim (1989), Avery (1991), Porter and Ladson (1993) and Omolayo (1993). The first three studies above only used variations of Bell's method to derive ARFs and compared them with the values obtained from ARR87. However, from the extensive comparison carried out by Omolayo (1993), it appears that the UK and Bell's methods tend to give reasonable and consistent values for the ARFs. The basic differences between these two methods are that the former does not use areal rainfall explicitly in the derivation of ARF and it does not take into account the probability of exceedance of the rainfall events. Bell's method seems appropriate to derive ARFs empirically.

5.2 Analytical Methods

Four methods (Roche, Rodriguez-Iturbe and Mejia, Meynink and Brady and Omolayo) are considered in this category. All the methods assume that the rainfall process is stationary and isotropic. This is unlikely to be true in many cases. Of these four methods, Roche's method is complicated and a lot of effort is needed to obtain an estimate for ARF. The Rodriguez-Iturbe and Mejia method appears to be easier to use but the estimates obtained by Omolayo (1993) seem to be highly variable and do not follow the expected behaviour of ARF with area.

However, Masters (1993) and Masters and Irish (1994) report consistent variation of ARF with area and duration using Rodriguez-Iturbe and Mejia methodology and expressed concern about Omolayo's implementation of this method. The ARF is derived in Rodriguez-Iturbe and Mejia method by assuming a zero mean process and it is not stated anywhere as to how one would use it in practice with observed or design rainfall with non-zero mean.

The expression for ARF derived by Meynink and Brady (1993) equates ARF to the arithmetic mean of the interstation correlation. This contradicts with the expression derived by Rodriguez-Iturbe and Mejia which equates ARF to the square root of the expected value of the spatial correlation coefficient. This apparent contradiction is due the fact that the variance of the mean is considered in one study (Meynink and Brady, 1993) and the standard deviation is considered in the other study (Rodriguez-Iturbe and Mejia, 1974b). It is more appropriate to match the standard deviation than the variance. The ARF values obtained by Meynink and Brady appear to be low compared with the estimates from other methods and it appears that this method is not suitable to estimate ARF.

The statistical derivation of ARF given in Omolayo (1989) assumes a log-normal distribution for the rainfall amounts and does not make the assumption of zero-mean process. There are no comparisons available in the literature to evaluate this method. However, when a normal distribution is used for the rainfall events, it reduces to the same expression for ARF as that of Rodriguez-Iturbe and Mejia method. The difficulty with this method is the estimation of the standard deviation for a region. This means that one has to first look for a homogeneous region to perform the analysis.

As all the analytical methods use spatial correlation for an area, the issue of estimating correlation at the characteristic distance needs to be resolved or an average value obtained as in Masters (1993).

5.3 Analytical-Empirical Method

The Myers and Zehr method requires a large amount of data and the process of deriving various statistical surfaces is an extremely time consuming process (Masters, 1993). In addition, their method assumes that the data fits an extreme value type I distribution and this may not be true for the Australian data.

6. RECOMMENDATIONS FOR DERIVING ARF

Based on the above review of the methods for deriving ARFs, the following recommendations are made for deriving ARF:

- Bell's method should be used to derive the ARFs for regions and range of durations for which sufficient data is available.

- Regionalise the ARFs if different patterns of variation in the ARF values are observed at different locations.

Even though the Rodriguez-Iturbe and Mejia method is elegant and attractive in the sense that the ARFs for shorter durations can be derived from the relationship between decorrelation distance and duration, its usage in practice is not clear. Also, rainfall data relevant to design are, in general, highly skewed and a normal distribution for them is completely inappropriate and, consequently, this method would require further reasearch before it could be recommended for application in areas with less data.

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