ESTIMATION OF EXTREME RAINFALLS FOR VICTORIA USING THE CRC-FORGE METHOD

(For Rainfall Durations 24 to 72 Hours)

N. Nandakumar
P. E. Weinmann
R. G. Mein
R. J. Nathan

Report 97/4
May 1997

COOPERATIVE RESEARCH CENTRE FOR
CATCHMENT HYDROLOGY

ESTIMATION OF EXTREME RAINFALLS FOR VICTORIA USING THE CRC-FORGE METHOD

(For Rainfall Durations 24-72 Hours)

N. Nandakumar
P. E. Weinmann
R. G. Mein
R. J. Nathan.

Report 97/4

May 1997

COOPERATIVE RESEARCH CENTRE FOR
CATCHMENT HYDROLOGY

Nandakumar, Nadarajah, 1961 -

Estimation of extreme rainfalls for Victoria using the CRC-FORGE method (for rainfall durations 24-72 hours)

Bibliography.
ISBN 1 876006 20 X.

1. Hydrologic models. 2. Rain and rainfall - Victoria - Measurement. 3. Rain and rainfall - Victoria - Mathematical models. 4. Flood forecasting - Victoria - Mathematical models. I. Nandakumar, Nadarajah, 1961 -. II. Cooperative Research Centre for Catchment Hydrology. (Series: Report (Cooperative Research Centre for Catchment Hydrology); 97/4).

551.5772945

Keywords

Precipitation (Atmospheric)
Dams
Spillway
Design Data
Frequency Analysis
Flood Forecasting
Floods and Flooding
Rainfall/Runoff Relationship
Modelling (Hydrological)
FORGE

ISSN 1039-7361

PREFACE

CRC Project D3 "Probability and Risk of Extreme Floods" was formulated to meet the needs of Australian dam owners with respect to assessment of spillway adequacy. The analyses associated with reservoir risk assessment involve the computation of a flood frequency curve covering the range of events from "frequent" to "probable maximum"; the existing method tended to be too conservative, due to the uncertainties of extreme event estimation.

The research described in this report focusses on rainfall, since the standard method of estimating extreme flood events uses design rainfall as input. To extend the frequency range of site data, the project has made use of the regional information contained in the extensive daily rainfall data available in Victoria. The result is a confident estimate of events of duration 12 to 72 hours down to an annual exceedence probability of 1 in 2000; the derived frequency curves show consistency with corresponding probable maximum precipitations.

The work in this project has already been used in Victoria to provide more accurate estimates of the risk of spillway overtopping. Plans are afoot to extend the work to provide similar design data for the other states of Australia.

Dr Nanda Nandakumar and Erwin Weinmann (Project Leader), with input from several others, have produced here research results which have important practical outcomes for the water industry. I commend their report to you.

Russell Mein
Program Leader, Flood Hydrology
Cooperative Research Centre for Catchment Hydrology

ABSTRACT

This report describes research work undertaken as part of Project D3 on "Probability and Risk of Extreme Floods" to develop and evaluate a methodology for estimating extreme design rainfalls. The specific range of interest is design rainfalls of one to several days duration and annual exceedance probabilities (AEPs) of 1 in 50 to 1 in 2000. The main field of application of this research relates to the design and safety assessment of dam spillways.

The CRC-FORGE method resulting from this research is a further development of the Focussed Rainfall Growth Estimation (FORGE) concept originating from the UK Institute of Hydrology (IH) [Reed and Stewart, 1989]. Modifications to the IH-FORGE methodology were necesssary to allow its application to the estimation of design rainfalls in the extreme range of AEPs.

The development and testing of the CRC-FORGE methodology was based on the analysis of daily rainfall data from more than 1400 medium to long record rainfall stations in Victoria and the bordering regions, made available by the Bureau of Meteorology.

The important questions of regional homogeneity of extreme rainfall, appropriate probability distributions, and effects of inter-site dependence of rainfall data were addressed in detail, using appropriate statistical tests and the results of rainfall data generation experiments. A method for estimating confidence limits on the estimated rainfall frequency curve was also developed.

The report concludes that the CRC-FORGE method yields consistent extreme design rainfall estimates to an AEP of 1 in 2000, and thus establishes an improved basis for the estimation of design floods in the extreme range. The method has been applied to produce design rainfall estimates for any site in Victoria for rainfall durations from 12 to 72 hours and for AEPs in the range from 1 in 50 to 1 in 2000. The CRC-FORGE method is considered suitable for application to extreme design rainfall estimation in other regions of Australia.

ACKNOWLEDGMENTS

The work presented in this report has benefited directly and indirectly from the contributions of many people. The authors would like to specifically express their appreciation to the following people and organisations:

- The Bureau of Meteorology for the provision of the basic rainfall data set and for assistance with quality control of this data;
- Jim Irish for valuable inputs to the project in its initial stages and for reviews in subsequent stages;
- Duncan Reed and Elisabeth Stewart (UK Institute of Hydrology) for helpful discussions and making available relevant reports;
- John Hosking for providing programs for estimation of frequency distribution parameters using L-moments (public domain);
- Steve Burges, Vijay Gupta and Vit Klemes for stimulating discussions on their visits to the CRC for Catchment Hydrology;
- Fiona McConachy and Lionel Siriwardena for their help as team members of Project D3 and for valuable inputs from parallel sub-projects;
- The members of the Project Reference Panel - Eric Laurenson (Monash University), Tom McMahon and QJ Wang (University of Melbourne), Mike Cawood, Helen Pearce, Sri Srikanthan and Brian Taylor (Bureau of Meteorology) - for assisting the project in many ways and keeping it on track; and
- Scott Frazer for assistance with the derivation of design rainfall values for Victoria;
- The CRC Industry Partners and Sinclair Knight Merz for provision of data and assistance with benchmarking studies;
- Trevor Daniell for undertaking a commissioned review of the draft report which provided many valuable suggestions for corrections and further work.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

1. INTRODUCTION

This report presents the evaluation and further development of the IH-FORGE (Institute of Hydrology *FO*cussed *R*ainfall *G*rowth *E*stimation) method by Reed and Stewart (1989), and the application of the developed method (CRC-FORGE) to estimate extreme rainfalls for Victoria. It summarises the research undertaken as part of Project D3 "Probability and Risk of Extreme Floods" of the CRC for Catchment Hydrology. The aim was to allow estimation of point design rainfalls up to an annual exceedance probability of 1 in 2000 for any location in Victoria.

In recent years, regional rainfall frequency estimation methods have been favoured over conventional methods based on single site data. In the regional methods, uncertainty in rainfall quantile estimation is reduced by using information from several sites. The FORGE concept involves pooling of data from regions of increasing size to estimate point rainfalls of decreasing frequency.

The next chapter provides the background of this study. The data used for the work are described in Chapter 3. Chapter 4 covers important aspects of regional frequency estimation in the context of the FORGE concept; Chapter 5, the distribution choice and homogeneity issues of annual rainfall maxima for the Victorian region. In Chapter 6, the development of spatial dependence models is given, leading to modifications to the IH-FORGE method (Chapter 7) Chapter 8 deals with the estimation of uncertainty in the CRC-FORGE estimates. Chapter 9 describes a method to derive the design values for ungauged sites. The final chapter presents the summary and conclusions.

2. BACKGROUND: THE NEED FOR IMPROVED ESTIMATES OF EXTREME RAINFALLS

The adoption of nonlinear flood estimation methods, and increased probable maximum precipitation (PMP) estimates, have led to significantly higher values of probable maximum floods (PMF) used in the design of spillways. As a consequence, dam owners in most states of Australia are reviewing the spillway adequacy of their dams, and planning remedial action if necessary. Some are considering design floods in the extreme range, but less than PMF.

The decision on the type and extent of rehabilitation needed for a particular dam can be based on the risk of failure associated with the current spillway capacity compared to the level of risk that the community considers acceptable. The Australian National Committee on Large Dams (ANCOLD) Guidelines on Design Floods for Dams (ANCOLD, 1986) provide the basis for classifying existing dams in Australia into an incremental flood hazard category which determines the acceptable level of risk of failure of the dam from flooding. These guidelines are currently undergoing a process of revision.

Risk assessment techniques require a continuous flood frequency curve up to the annual exceedance probability (AEP) of the probable maximum flood (PMF). The flood frequency curves can be obtained from rainfall frequency curves using a rainfall-runoff model.

Errors in frequency curve estimates have considerable economic and social implications. An under-estimate of the design flood will lead to an under-designed spillway which has the potential to result in increased flood damage costs and social hardship. On the other hand, an overestimate will lead to extra costs, possibly up to tens of millions of dollars, from spillway over-design (Nathan and Weinmann, 1995).

The currently recommended estimation method of rare events between an AEP of 1 in 100 and the AEP of the probable maximum event (PME), as described in Chapter 13 of Australian Rainfall and Runoff (1987) [ARR87], is based on the criteria of providing reasonable and consistent answers, and some degree of conservatism, rather than being based on rigorous analysis of data. In this method, the shape of the frequency curve for rare events depends on the slope of the frequency curve between the AEPs of 1 in 50 and 1 in 100, and on the magnitude and assigned probability of the PME. The interpolation method can be applied to both rainfall and flood events.

Since the publication of ARR87, frequency estimation methods have been considerably improved. This has been achieved by the use of regional data and better parameter estimation methods [eg. L-moments (Hosking, 1990)]. In addition, the availability of more data also contributes to improved frequency estimates. As such, there is a need to revise the currently recommended method in the light of improved methodologies.

The CRC for Catchment Hydrology have undertaken several projects related to the estimation of extreme floods to fill gaps in current knowledge on extreme floods. The objectives of this project (Project D3) are:

(i) to better estimate the flood frequency curve up to an AEP of 1 in 2000, through improvements in the associated extreme precipitation frequency curve, and

(ii) to define the error band associated with extreme flood estimates for various AEPs.

To meet these objectives, Irish, in a commissioned review (Irish, 1994) recommended evaluation and adaptation of two regional rainfall frequency estimation methods: (i) Schaefer's (1990) method and (ii) the IH-FORGE method (Reed and Stewart, 1989). The methods were to be applied to daily rainfall data for the Victorian region. This report describes the evaluation and further development of the IH-FORGE method and application of the further developed IH-FORGE method which is called the CRC-FORGE method. The evaluation of Schaefer's method is described in McConachy (1996).

3. REGIONAL FREQUENCY ESTIMATION

3.1 INTRODUCTION

This chapter describes the important aspects of regional frequency estimation. The aim is to introduce important concepts of regional frequency estimation in the context of the FORGE approach.

Regional frequency methods facilitate derivation of quantile estimates for any site in that region using data from several sites, including data from the site of interest. When data are available for a particular site, the regional methods utilise the information from other compatible sites to increase the accuracy of quantile estimates. On the other hand, for an ungauged site, these methods estimate the quantiles using regional relationships between frequency distribution parameters and site characteristics.

This chapter firstly describes the important factors that affect regionalisation. Then approaches used for regional frequency estimation at gauged and ungauged sites are presented. This is followed by a description of the regional frequency estimation concept used in this study (the FORGE concept).

3.2 FACTORS AFFECTING REGIONAL FREQUENCY ESTIMATION

The regional frequency estimation methods attempt to obtain maximum possible information from data at all the gauged sites in the region. But in the process, they check for (i) compatibility or relevance of data from different sites *(homogeneity)* and (ii) usefulness of additional information (effects of *inter-site dependence of data).* These are briefly described in the following section.

3.2.1 Regional Homogeneity

The regional frequency estimation methods utilise compatible information from several sites to derive the at-site frequency curve. The compatibility of data in relation to a specific objective is known as *regional homogeneity.*

In the ideal situation, data from a homogeneous region are considered to be from a single population, and assumed to be identically distributed. In reality, this assumption is rarely satisfied, and data from different sites will exhibit some degree of heterogeneity. The definition of regional homogeneity is dependent on which aspect of frequency behaviour is considered. It is normally assumed that the differences in at-site frequency curves in a homogeneous region are due to sampling variability.

If regional data exhibits a significant degree of heterogeneity, either an adjustment or selection procedure needs to be applied to achieve the necessary degree of homogeneity to make regional analysis meaningful. With the first approach, regional homogeneity can be assumed after adjusting data by a known relationship that explains at least a significant part of the variation over the region. As an example, the *index methods* of regional frequency analysis assume regional homogeneity after *standardising* at-site data by the at-site mean (or another statistical index).

A homogeneous group can also be obtained by selecting a sub-sample of sites. The selection can be based on geographical contiguity [eg. IH-FORGE method (Reed and Stewart, 1989), region of influence method (Burn, 1990)]. In contrast to this, the Schaefer's method (1990) forms homogeneous data sets by choosing sites with similar L-CV and L-Skewness values, ie. parameters which can characterise a 3-parameter distribution of standardised data.

Statistical tests for homogeneity

A large number of statistical tests for regional homogeneity can be found in the recent literature. These tests check whether the between-site variability of a frequency curve or distribution parameter (eg. coefficient of variation, 1 in 10 year estimate) can be explained by sampling variability. The usefulness of different homogeneity tests for the purposes of this study depends on the specific test statistic used, and how relevant it is to the estimation of extreme events. The homogeneity of Victorian rainfall data is further discussed in Section 5.2.

3.2.2 Inter-site dependence

The annual maximum data in a region will show some spatial dependence because of both large and small scale meteorological influences. Large scale meteorological phenomena, such as El Nino conditions, influence wet or dry years which may affect annual maxima in a larger region alike; small scale factors are more responsible for the high correlation between data from close stations. However, other factors such as topography and location also affect inter-site dependence.

Inter-site dependence can be viewed as disadvantageous for gauged sites, as it reduces the value of additional information for regional analysis, ie. inter-site dependence limits the increase in information from an increase in the number of stations in a region. On the other hand, it is beneficial to the derivation of quantiles for ungauged sites, as it allows transfer of information from gauged to ungauged sites. The effects of inter-site dependence on Victorian rainfall frequency estimation are discussed in Chapter 6.

3.3 REGIONAL FREQUENCY ESTIMATION FOR GAUGED SITES

Regional frequency estimation approaches for gauged sites can be divided into two groups: (i) methods that use regional average values (eg. Schaefer, 1990, Hosking, 1990) and (ii) methods that pool recorded data from several sites (eg. station-year method). The first uses some form of averaging of at-site parameters to derive the regional frequency curve, whereas the second assumes the space-time equivalence of data and jointly analyses data from all sites.

In either of the above groups, the derivation of a frequency curve can be based on two approaches: (i) parametric and (ii) non-parametric,.

3.3.1 Parametric approach

The parametric approaches are methods that

(i) assume a particular theoretical probability distribution of the data and

(ii) estimate the parameters of that distribution using a statistical estimation method.

Probability distributions

A number of probability distributions with a range of number of parameters can be found in the literature. Some of the most popular distributions for floods are Log-Pearson 3 (LP3), Log-Normal, Generalised Extreme Value (GEV) (Cunnane, 1989). Of these, LP3 has been widely used for flood frequency analysis in Australia. For rainfalls, the GEV distribution appears to be the current popular choice (eg. Dales and Reed (1989), Schaefer, 1990).

The number of parameters of a distribution characterises its flexibility. Distributions with a small number of parameters tend to have small standard errors in parameter estimates but large biases in quantile estimates. However, standard errors in parameter estimates decrease with an increase in length of record. The use of a distribution with a larger number of parameters is justifiable if a long record of data is available.

The choice of a distribution is very important when the extrapolated portion of the frequency curve is used to estimate quantiles. The ability of a distribution to describe a given data set is usually tested using statistics derived from the whole data set (see Section 5.1), even if only one tail of the distribution is of direct interest. An inappropriate distribution has the potential to result in a significant bias in extreme quantile estimates, which is the range of interest in this study.

Parameter estimation

Some of the available methods for estimating parameters are: (i) probability weighted moments, (ii) L-moments, (iii) method of moments (product moments) and (iv) maximum likelihood method. The first two are used in this study and are briefly described below.

Probability weighted moments (PWMs) and L-moments are based on order statistics. PWMs of a random variable X are computed as the expectation of the product of X and a power function of the cumulative probability F(X) (Greenwood et al, 1979).

L-moments developed by Hosking (1990) are linear functions of PWMs. L-moment ratios are standardised L-moments and are analogous to product moment ratios (eg. coefficient of variation, skewness and kurtosis). Hosking (1990) presented L-moment ratios for some common distributions; these can be used to identify an appropriate distribution for observed data. In addition, he developed relationships to estimate parameters of a number of distributions using sample L-moment estimators. Recently Wang developed direct sample estimators of L-moments (Wang, 1996b), thus eliminating the need for computation of PWMs as an intermediate step.

Since sample estimators of L-moments are linear combinations of the ranked observations, they are virtually unbiased and have relatively small sampling variance. In addition, they are relatively insensitive to outliers. Nevertheless, L-moment estimators can be too robust, because large sample values reflecting important information on the tail of the parent distribution are given too little weight in the estimation (Bobee and Rasmussen, 1995).

In the parametric approaches to regional frequency analysis (eg. Hosking and Wallis, 1990), the parameters of the regional distribution are determined as *weighted averages* of the parameter values at individual sites. This averaging procedure further reduces the influence of the largest regional observations in determining the shape of the upper distribution tail. While this may be an advantage in the regional estimation of more frequent events (reduced sensitivity to 'outliers'), it limits the usefulness of the parametric approach for the estimation of extreme rainfalls.

A recent extension of L-moments are LH-moments, which are linear combinations of higher PWMs. The LH-moments are more weighted towards larger observations than the L-moments, and hence lead to improved high quantile estimates (Wang, 1996a). LH-moments therefore appear to be better suited to regional estimation of extreme events, but there is still a danger that the parameter averaging process reduces the influence of the largest regional observations in determining the shape of the tail of the regional distribution.

3.3.2 Non-parametric approach

Non-parametric approaches first assign a plotting position to the observed data, and fit an empirical curve locally to points on a probability plot, assuming both continuity and differentiability to some order in the neighbourhood (Lull, 1995). This is in contrast to the *a priori* assumption of the global form of the entire distribution function used in the parametric approaches.

The frequency curves obtained from a non-parametric approach cannot be readily extrapolated, due to the local fitting involved. However, in the case of regional methods, the pooling of data from many sites in the region allows the definition of an empirical frequency curve that extends to the range of extreme events, thus avoiding the need for extrapolation. The shape of the frequency distribution in the region of the extreme upper tail is determined by the plotting position of the largest events in the region, rather than being inferred from data in the body of the distribution, as is the case in the parametric approaches.

For these reasons, the non-parametric approach is preferred for regional frequency estimation of extreme events. An example of this approach is the IH-FORGE method, described in Section 3.5, which was evaluated in detail and further developed during the course of this study.

A possible drawback of the non-parametric approach is the greater dependence of quantile estimates on individual observations which makes the estimates more susceptible to data errors, particularly errors in the largest observations. Special attention to the data checking of these most influential observations is therefore required (see Section 4.4).

3.4 ESTIMATION OF FREQUENCY CURVES FOR UNGAUGED SITES

Estimating the frequency curves for ungauged sites involves the transfer of 'appropriate' information from gauged sites to ungauged sites. Homogeneity in the relevant characteristics (eg. flood behaviour) is the important factor in determining the 'appropriateness'. Inter-site dependence then determines the value of the information to the estimation at the site of interest.

The transfer of information from gauged sites to an ungauged site involves two steps:

(i) identification of a homogeneous region to which the ungauged-site belongs, and

(ii) utilisation of information from the homogeneous region, with or without adjustments, to give an estimate at the ungauged site.

The identification of a homogeneous region can be based on geographical contiguity and/or similarity of other available characteristics. For example, Burn (1990) selected a homogeneous region based on geographical contiguity in his 'region of influence approach', whereas Acreman and Sinclair (1986) and Schaefer (1990) formed the homogeneous group of stations using catchment characteristics and mean annual rainfall respectively.

The information from an homogeneous region can be transferred to an ungauged site in three ways: (i) direct use of the regional frequency curve determined for the homogeneous region to which the ungauged site belongs (eg. Acreman and Sinclair, 1986), (ii) estimation of the quantiles at gauged sites and interpolation for ungauged sites using a geostatistical method or other spatial interpolation method (eg. Stewart et al., 1995), or (iii) estimation of statistical parameters such as L-CV and L-Skewness for the ungauged site, using relationships with site characteristics obtained from gauged sites, and calculation of quantiles (eg. Schaefer, 1990).

3.5 THE INSTITUTE OF HYDROLOGY FOCUSSED RAINFALL GROWTH CURVE ESTIMATION (IH-FORGE) METHOD

A large number of regional frequency estimation methods are described in the literature; they cover different combinations of parameter estimation methods, homogeneity assumptions and parameter regionalisation methods. This section first presents the FORGE concept, as it is the basis for the IH-FORGE and the CRC-FORGE methods. Then the IH-FORGE method. is described.

3.5.1 FORGE Concept

The FORGE concept developed by Reed and Stewart (1989) enables a 'growth curve' (ratios of quantile estimates to the mean) to be drawn for a location of interest, the 'focal point', using regional data. The FORGE concept is identical to the basic concept of station-year methods; observed data from a homogeneous region is pooled and a non-parametric frequency curve is fitted on a probability plot. The homogeneity assumption for the FORGE concept is the same as that used in the index-flood approaches, ie. an acceptable degree of homogeneity within the region is achieved by standardising the at-site data by at-site mean or median values.

It is well known that any station-year method suffers from problems associated with inter-site dependence. These problems have been minimised in the FORGE concept by using an *effective number of independent stations* concept, described in Chapter 6.

3.5.2 IH-FORGE Method

The IH-FORGE method developed by Reed and Stewart (1989) uses the FORGE concept to derive at site frequency curves from regional data.

Steps of the IH-FORGE method

The following steps are used to obtain a growth curve for a given focal point using annual maxima from an homogeneous region.

Step 1:
Standardise annual maximum data for all stations in the region by their respective at-site mean

Step 2:
Consider the three gauges nearest to the focal point

Step 3:
Select the six highest independent standardised values from the pooled data and plot them against their AEP estimated from a plotting position formula

Step 4:
Double the number of stations nearest to the focal point and repeat Steps 3 to 4 until all the stations in the region are included in the pooled data set.

Step 5:
Fit a frequency curve by eye as a line of best fit to the plotted values.

In Step 3, a *spatial dependence function* is used to calculate the effective record length (L_e) of the pooled data, calculated using a modified station-year method. In this method, the effective number of independent stations, N_e, is calculated using a spatial dependence function for each year; when these are summed over the number of years, the result is the effective record length (L_e). The L_e value is used in the procedure to calculate the plotting positions assigned to the six highest standardised annual maxima. The spatial dependence model used by Reed and Stewart (1989) and improved models for Victorian data are described in Chapter 6.

3.6 SUMMARY

This chapter has described important concepts of regional frequency estimation, with the aim to place the FORGE concept in context. Key factors affecting the regionalisation procedure were shown to be homogeneity and inter-site dependence. Regional estimation approaches for gauged and ungauged sites require decisions on probability distribution, parameter estimation and curve extrapolation. The FORGE concept appears to be well suited to form the basis of the regional estimation procedure needed for this study.

4. DATA

This chapter describes the collation of a data base of annual maximum rainfalls from daily rainfall records provided by the Bureau of Meteorology.

4.1 DAILY RAINFALL DATA FROM BUREAU OF METEOROLOGY

The Bureau of Meteorology's daily rainfall data base for Victoria, a total of 2144 daily rainfall stations, were available for this study. In addition, daily data for rainfall districts adjacent to the Victorian border from NSW and SA were also obtained to avoid "edge effects". Figure 4.1 shows the location of a total of 3030 daily-rainfall stations considered for this study. The rainfall records cover the period from 1835 to 1993, although most of the stations have data for only part of the period.

4.2 EXTRACTION OF ANNUAL MAXIMA

One to five day annual maxima were extracted using computer programs especially written for this project. These programs are described in detail in Nandakumar, Siriwardena and Weinmann (1996).

One of the programs, *DayMaxExtract*, graphically displays the daily rainfall data for a given station with data from the 10 nearest stations within a 50 km radius. An example display is given in Figure 4.2. This figure shows an aggregated amount of 280 mm for station 090001 on 28/03/1932, together with nearby station data; the distances between the main station and other nearby stations are also displayed on the left side of the plot. This plot enables the user to visually check the quality of the rainfall record and to disaggregate accumulated data using an appropriate temporal pattern chosen from one of the nearby stations. Such a procedure allows maximum use to be made of the available records.

Years with periods of missing data were analysed, by comparison with neighbouring gauges, to check if the annual maximum event was likely to have occurred during the missing period(s). If this was not the case, the observed annual maximum value for the year with missing records could be included in the data set.

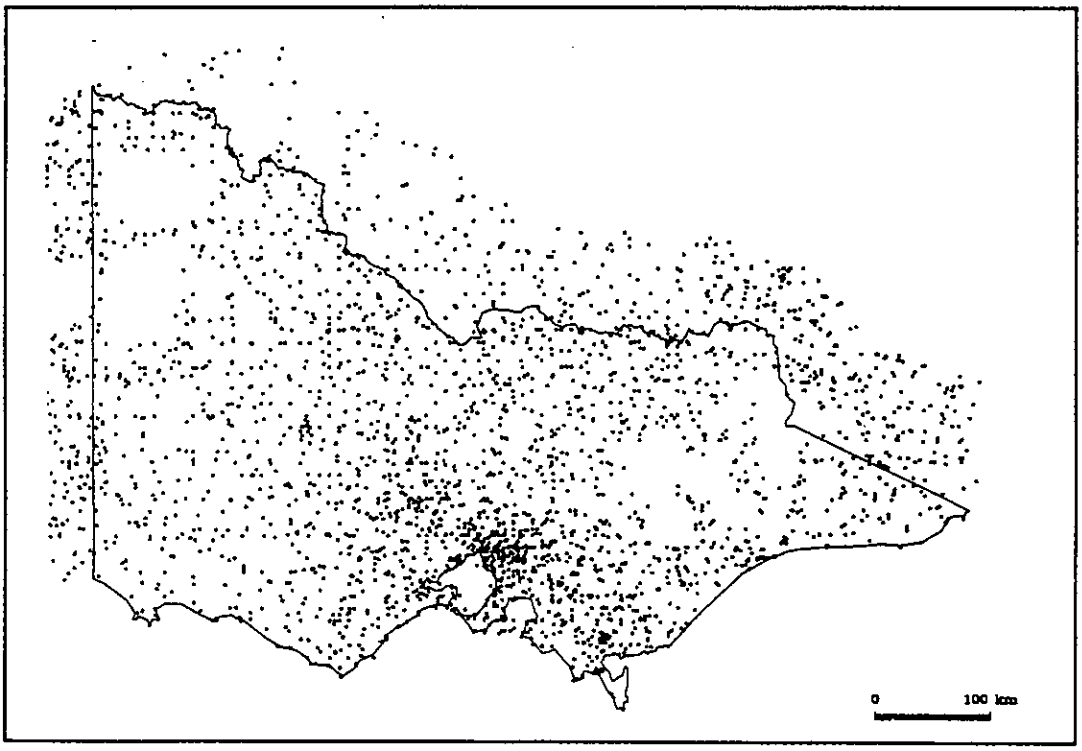


Figure 4.1 Daily rainfall stations available for Victoria and neighbouring regions

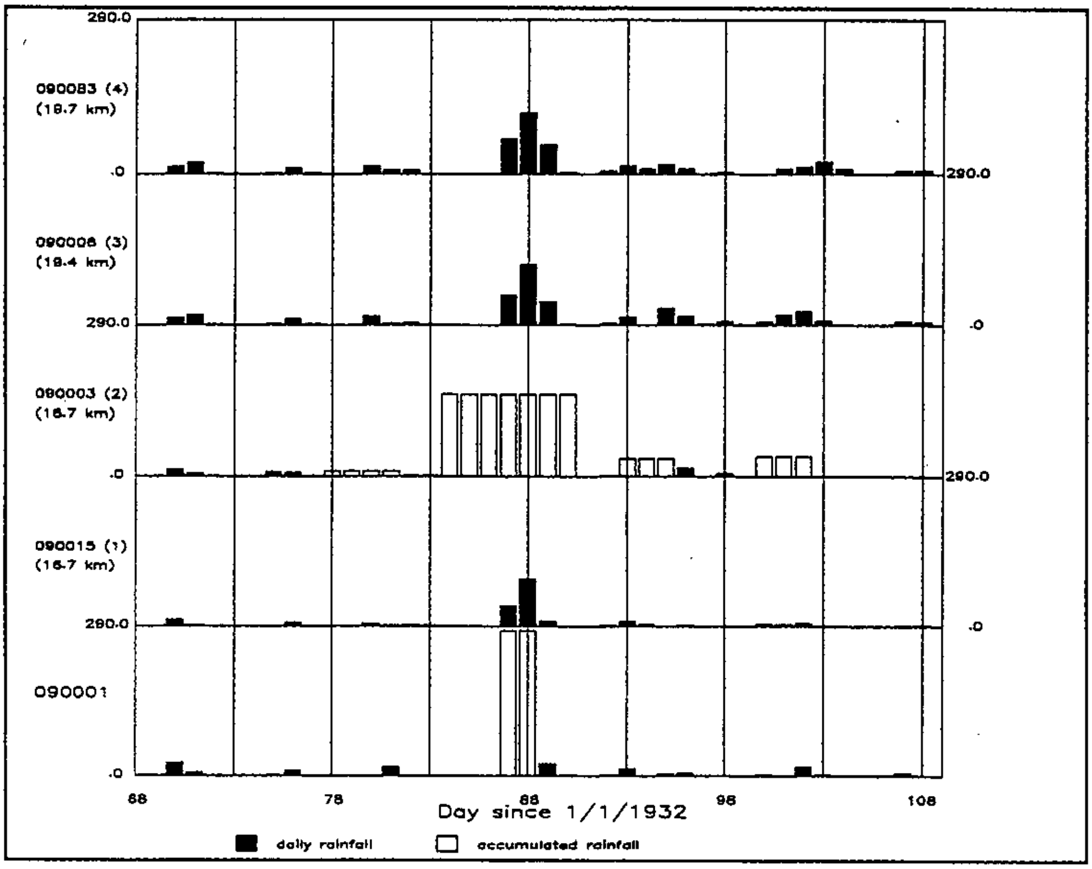


Figure 4. 2 An example plot for aggregated rainfall

4.3 ANNUAL MAXIMUM DATA SETS USED FOR THE FORGE APPLICATIONS

As mentioned in Section 4.1, about 3000 daily rainfall stations are available for Victoria and the surrounding region; these have record lengths varying from 1 to 120 years.

It is reasonable to expect that the useful information for regional frequency analysis increases with the increasing number of stations in the region. However, the net information does not increase proportionally with the increasing number of stations within a given area, due to spatial dependence between stations. In addition, shorter record lengths introduce more uncertainty in parameter estimates. Thus there is a trade-off between the number of stations used and the useful information gained.

Three data sets were selected for this study after a subjective trade-off between the minimum record length and the number of stations; the former determines the errors in estimated parameters, whereas the latter affects the accuracy of spatially interpolated values. The selected data sets are:

Data Set I	data from stations in Victoria which have 100 or more years of record (Figure 4.3)
Data Set II	data from stations in Victoria and in Victorian border rainfall districts in SA and NSW which have 60 or more years of record (Figure 4.4)
Data Set III	data from stations in Victoria and in Victorian border rainfall districts in SA and NSW which have 25 or more years of record (Figure 4.5)

Data Set I was used in the development of models for the effective number of stations, because long records of concurrent annual maxima are needed to reliably assess the effects of inter-site dependence (see Chapter 6). Data Set I does not include the long record stations from Victorian border rainfall districts, as this part of the analysis was undertaken before the additional data became available.

For the application of the IH and CRC FORGE methods, a significant amount of extra information ca be gained by reducing the cutoff length (ie. minimum record length) from 100 to 60 years, without losing much accuracy. Accordingly, Data Set II was used to obtain the design growth factors (see Chapter 7). It will be shown in Chapter 9 that the spatial variability of the index variable (mean annual maximum rainfall) is much higher than that of the growth factors. To reduce the errors in spatially interpolated values, Data Set III was used to map the index variable (24-hour annual rainfall maxima).

Table 4.1 summarises the properties of the raingauge networks in the above data sets. It is evident from Figures 4.3 and 4.4 that the stations in both data sets give a reasonable cover of the region, except in alpine and mallee areas.

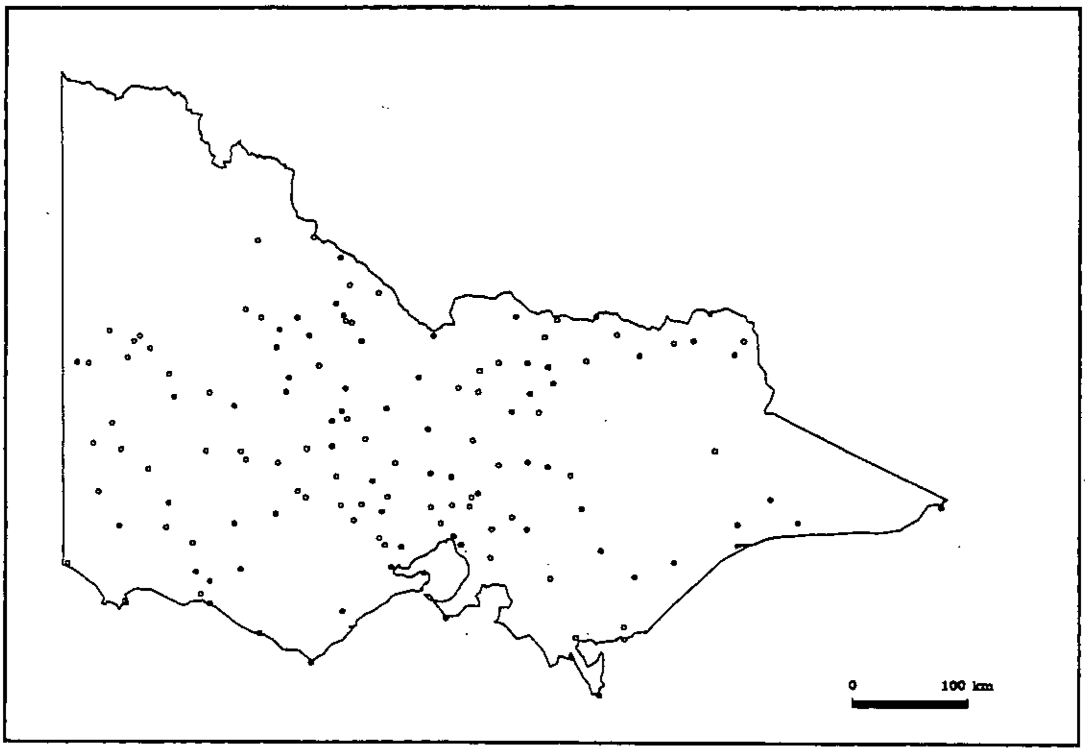


Figure 4.3 Stations with 100 or more years of 1-day annual maximum rainfall

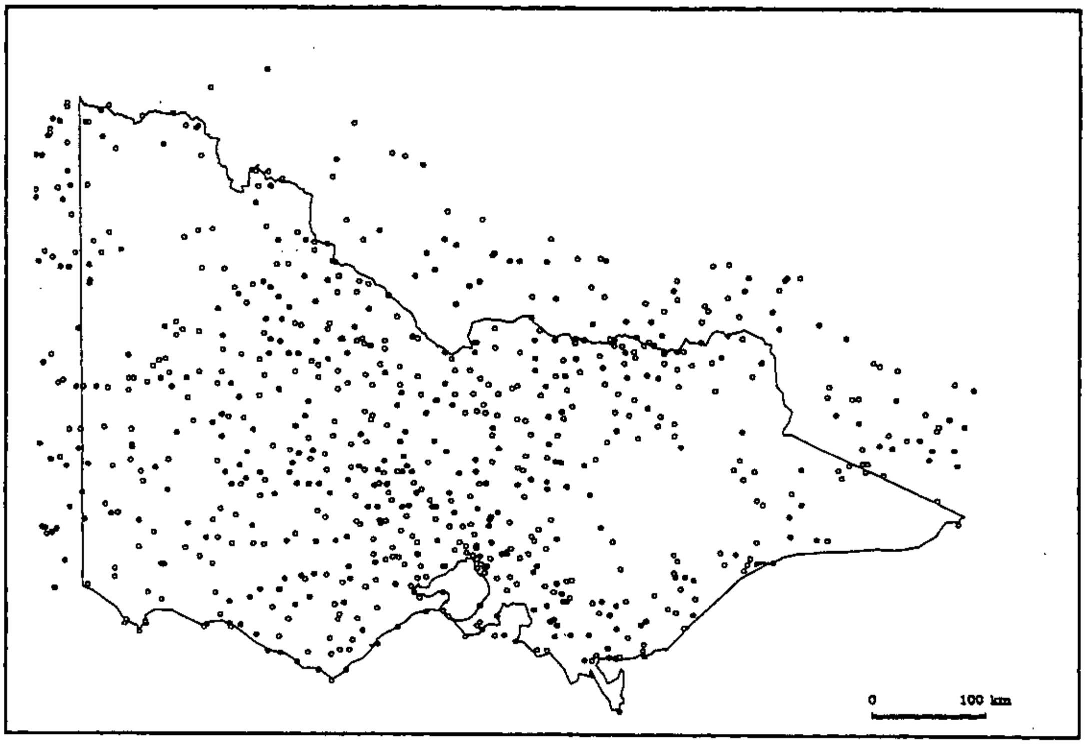


Figure 4.4 Stations with 60 or more years of 1-day annual maximum rainfall

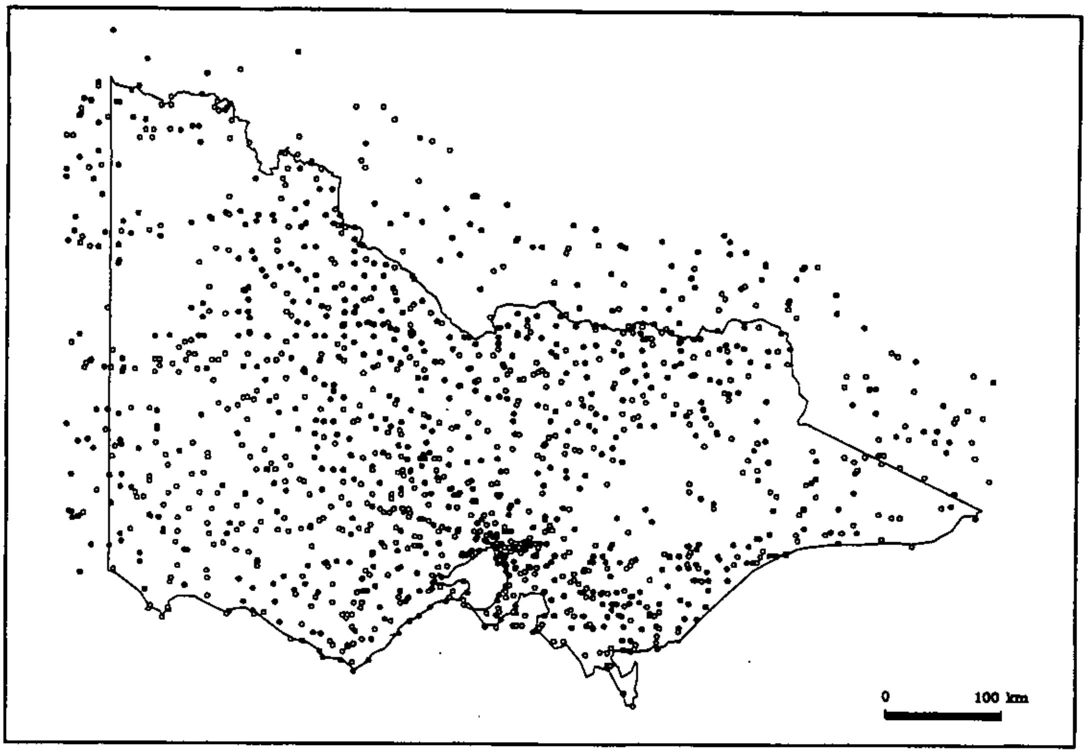


Figure 4.5 Stations with 25 or more years of 1-day annual maximum rainfall

Table 4.1 Properties of raingauge network in Data Sets I, II and III

	Duration (day)		
	1	2	3
Data Set I			
Number of stations	136	137	137
Average record length (years)	107	107	107
Average area covered by each station (km^2)	1770	1770	1770
Data Set II			
Number of stations	756	756	760
Average record length (years)	86	86	86
Average area covered by each station (km^2)	390	390	390
Data Set III			
Number of stations	1396	1398	1399
Average record length (years)	65	65	65
Average area covered by each station (km^2)	210	210	210

4.4 QUALITY CHECK OF LARGEST ANNUAL MAXIMA

Because of a large volume of data processing involved, there is a possibility of failing to observe anomalies in the extracted annual maxima, or to find other errors in the original data provided by the Bureau of Meteorology.

As described in Section 3.5, the FORGE method uses a relatively small number of the highest observed data values in the region. Hence any errors in these observations have the potential to introduce significant error into the FORGE estimates. Accordingly, it was decided to closely examine the 50 largest 1-day annual maxima used in the FORGE applications.

The check on the largest maxima was done by comparing them with concurrent rainfall at nearby stations. This process was facilitated by specially developed software. Rainfalls that appeared to be very high compared to adjacent observations were considered suspicious; 17 suspicious values were identified. It is possible that these nonconforming data points were due to very localised rainfalls (ie. thunderstorms), or due to recording or processing errors.

Figures 4.6 and 4.7 show two suspicious readings which are much higher than rainfalls at nearby stations. By revisiting the original data sheets, the Bureau of Meteorology found processing errors in two of the 1-day maxima. One of the daily rainfall readings with processing errors is shown in Figure 4.6; a value of 663 mm in this figure was amended to 167 mm. Figure 4.7 is an example of a localised thunderstorm which was noted in the original recording sheet. The data for this event was therefore accepted.

4.5 ASSESSMENT OF STATIONARITY OF VICTORIAN RAINFALL DATA

The data to be used in a regional frequency analysis have to satisfy the condition of homogeneity, both with respect to their variation in space (regional homogeneity, see Section 5.2), and in time (stationarity). McConachy (1996) assessed the stationarity of Victorian rainfall data, based on studies reported in the literature and additional stationarity tests on data sets similar to the ones used in this study. She concluded from the literature review that there was no evidence to suggest any significant trend in the annual maximum rainfall series.

McConachy applied the Mann-Kendall rank correlation test (Srikanthan and Stewart, 1991) and the distribution-free CUSUM test (McGilchrist and Woodeyer, 1975) to a data set of 2144 Victorian rainfall stations and a subset including 694 stations with record lengths exceeding 50 years. She concluded from these tests that the Victorian annual maximum rainfall series data was sufficiently stationary to be used in a regional rainfall analysis.

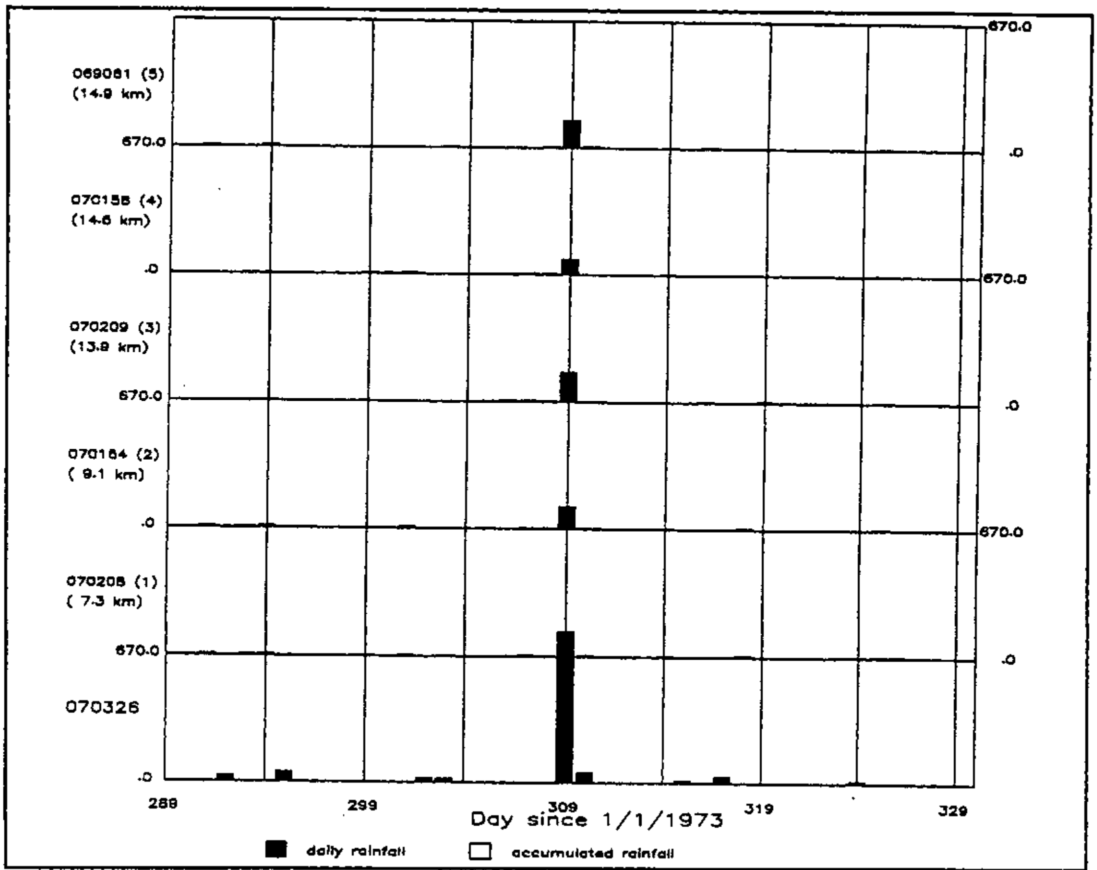


Figure 4.6 A suspicious rainfall reading at Station 070326

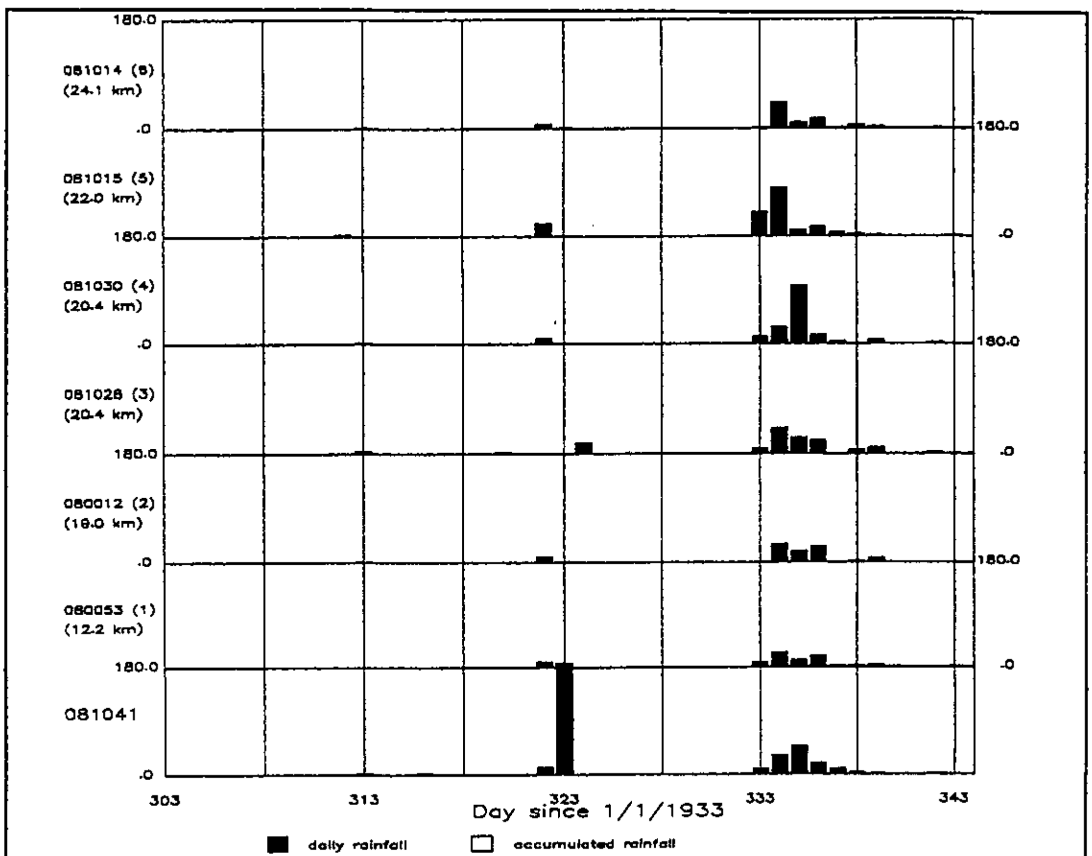


Figure 4.7 A suspicious rainfall reading at Station 081041

4.6 STATIONS SELECTED FOR EVALUATION OF THE CRC-FORGE METHOD ESTIMATES

In Chapter 7, the potential of the IH-FORGE and CRC-FORGE method for application in Victoria is evaluated at the eight selected sites shown in Figure 4.8. These sites from Data Set I are located close to storages where PMP estimates from the Generalised South-east Australia Method (GSAM) are available for spillway adequacy studies. Accordingly, these sites were initially chosen as focal points for the application of the IH-FORGE and the CRC-FORGE methods.

In addition to the sites shown in Figure 4.8, five storages were selected to compare 1 in 10^6 AEP *areal* rainfalls estimated using the CRC-FORGE method and the GSAM method in Chapter 9. The areal GSAM PMP estimates given in Table 4.2 were provided by the Bureau of Meteorology.

Table 4.2 GSAM method estimates of areal PMP in mm (Source: the Bureau of Meteorology)

Storage	Area (km^2)	Duration (hours)		
		24	48	72
Lake Buffalo	1145	760	910	960
Rosslynne Reservoir	90	880	1040	1090
Thomson Reservoir	487	850	1010	1090
Lake Bellfield	100	611	737	772
Dartmouth Dam	3564	520	620	640

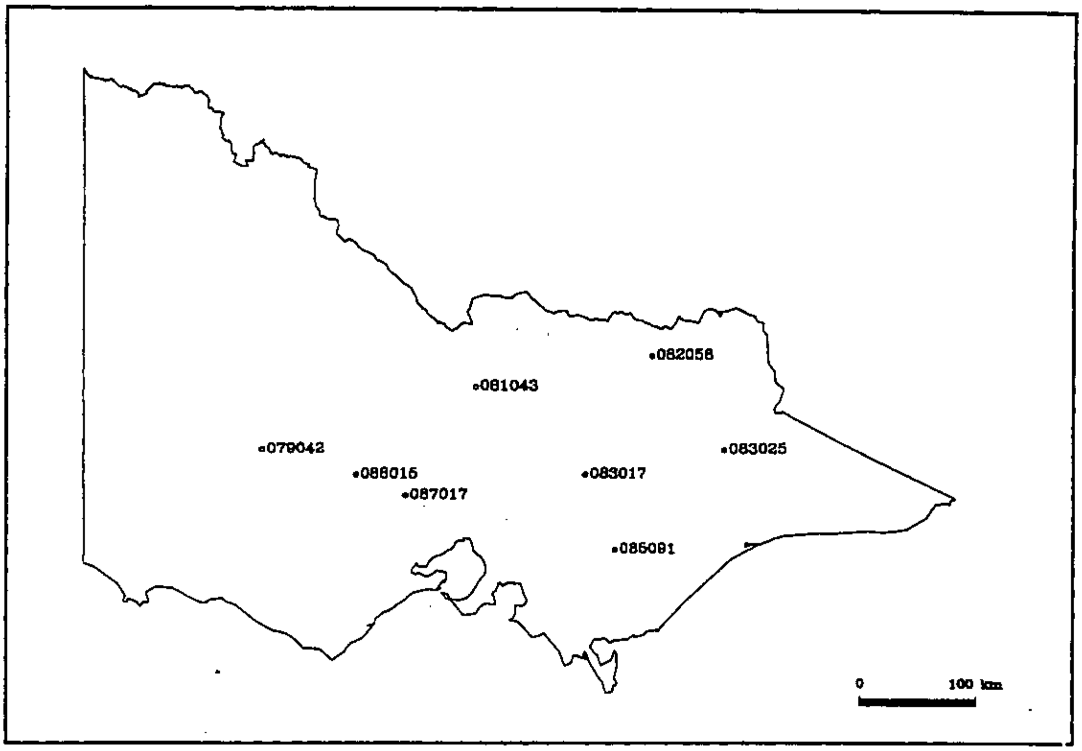


Figure 4.8 Selected stations for the evaluation of the FORGE concept

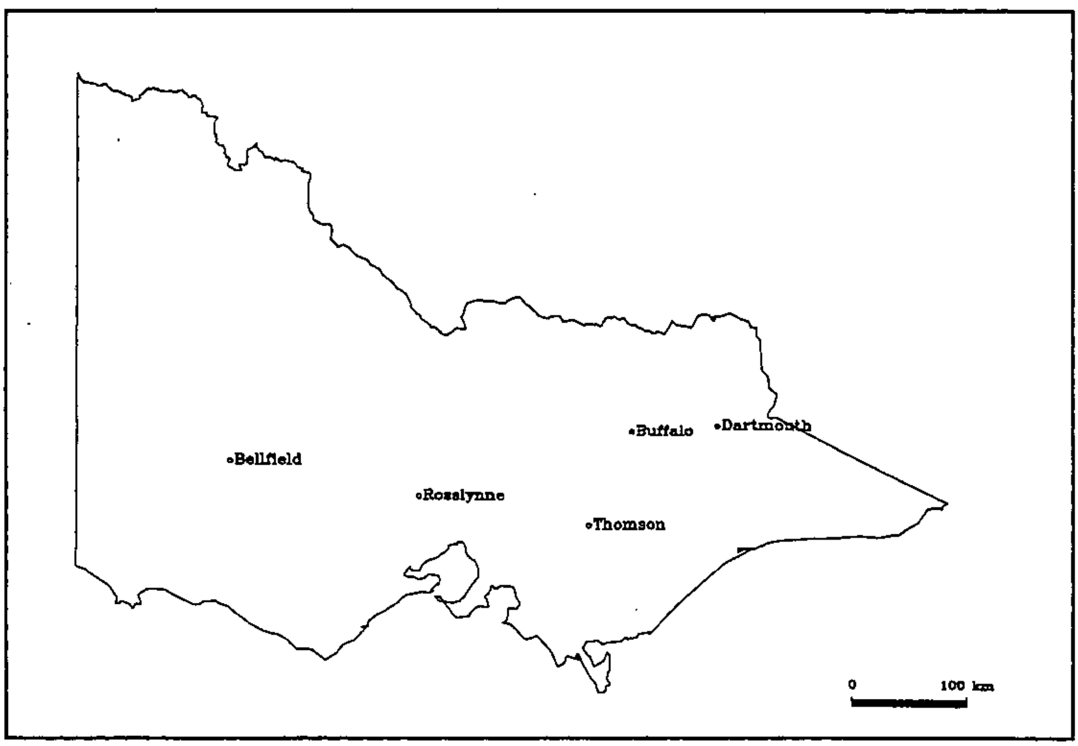


Figure 4.9 Locations of selected storages for comparison of PMP estimates

5. IDENTIFICATION OF APPROPRIATE PROBABILITY DISTRIBUTION AND EXAMINATION OF HOMOGENEITY OF ANNUAL MAXIMA FOR VICTORIAN REGION

In this chapter, the most appropriate probability distribution and the homogeneity of annual maxima for the Victorian region are examined in the context of the application of the FORGE method.

5.1 PROBABILITY DISTRIBUTION

As stated in Section 3.5, the FORGE concept is non-parametric and therefore an assumption of a particular distribution is not needed. However, it will be shown in Chapter 6 that a parametric frequency curve is fitted to annual maxima in order to derive a generic relationship for the effective number of independent stations (used in the calculation of plotting positions for the FORGE points). In addition, one of the modifications suggested to the FORGE method in Chapter 7 is a parametric distribution fit to the FORGE points instead of an eye-ball fit.

It is evident from the literature that the Generalised Extreme Value (GEV) distribution has been widely used to describe rainfall frequency curves (eg. Dales and Reed, 1989; Schaefer, 1990,). A GEV distribution fitted using the regional PWM method has been shown to be relatively insensitive to violations of the distributional assumptions and to modest regional heterogeneity, and to have low variability and bias (Lettenmaier et al., 1987).

A number of techniques to evaluate distributional assumptions are available in the literature (eg. Chowdhury et al., 1991; Hosking, 1990). Two of these methods are used to show that the GEV is an appropriate distribution to describe annual rainfall maxima for the Victorian region: (i) the L-moment diagram and (ii) the probability plot correlation coefficient. For this analysis, data from stations with more than 60 years of data were used (Data Set II, Chapter 4)

5.1.1 L-Moment Ratio Diagram

Hosking (1990) introduced the L-moment ratio diagram technique for the purpose of selecting a suitable distribution. An L-moment ratio diagram compares sample estimates of L-skewness and L-kurtosis with their population counterparts, for a range of assumed distributions.

Figure 5.1 shows L-kurtosis vs L-skewness plots for 1, 2 and 3 -day maxima for the Victorian region, along with theoretical curves for some popular 3-parameter distributions (GEV, Pearson 3, Logistic and Pareto). It is evident from this figure that annual maxima for the Victorian region cannot be fully described by any of the single distributions considered. However, the GEV seems to be the best single distribution to describe data for the Victorian region, as the spread of the points is approximately centred around the L-kurtosis-L-skewness curve for the GEV distribution.

Some of the scatter in Figure 5.1 can be attributed to sampling variability. The significance of sampling variability can be quantified using the probability plot correlation coefficient described in the following section.

5.1.2 Probability Plot Correlation Coefficient (PPCC) Test

The PPCC test introduced by Filliben (1975) measures the linearity of a probability plot and is given by:

$$PPCC = \frac{\sum_{i=1}^{n}(x_{(i)}-\bar{x})(M_i-\overline{M})}{\left[\sum_{i=1}^{n}(x_{(i)}-\bar{x})^2\sum_{i=1}^{n}(M_i-\overline{M})^2\right]^{1/2}} \qquad (5.1)$$

where $x_{(i)}$ is an ordered observation, M_i is the expected value from the selected distribution for an observation of order i, n is the number of observations, and $\bar{x}$ and $\overline{M}$ are the means of x and M respectively.

PPCC is near to unity when the sample is drawn from the selected distribution; the plot of ordered observations versus corresponding expected values M_i for the selected distribution is expected to be nearly linear.

The average values of PPCC for fitting GEV distributions to 1, 2 and 3 day maxima are 0.990, 0.990 and 0.989 respectively. This illustrates that the GEV distribution generally fits annual maximum rainfall data for the Victorian region.

The critical values of PPCC at the 10% significance level were used to test whether the GEV distribution describes at-site data. Chowdhury et al. (1991) provided the critical values up to a record length equal to 100. For record lengths greater than 100, the critical values were estimated using Eq. 5.2. This relationship was obtained using a regression analysis to the critical value data given by Chowdhury et al. (1991):

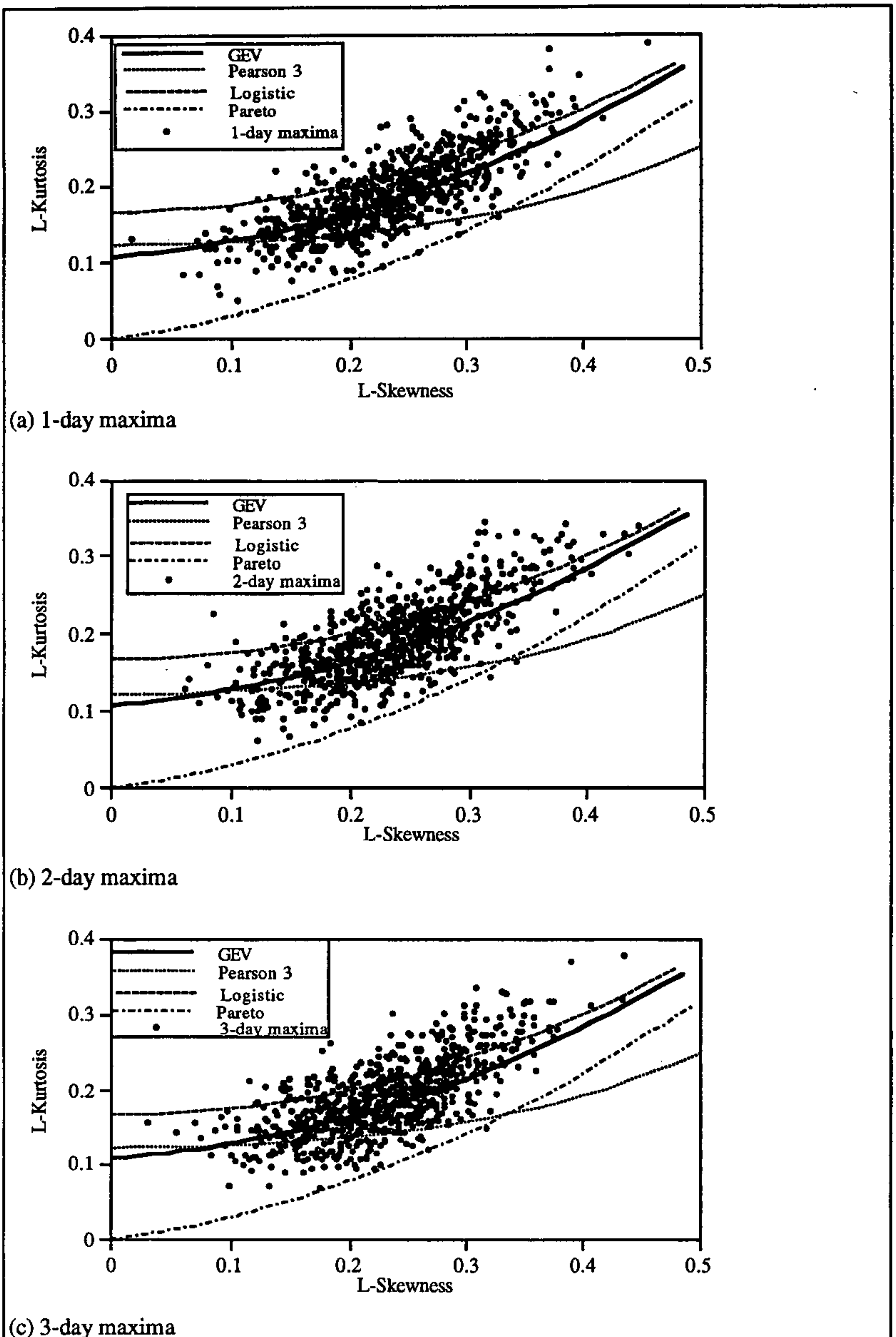


Figure 5.1 L-moment ratio diagrams of mean annual maximum rainfall for the Victorian region

$$PPCC_{crit} = 1 - (246.3 + 265.5)\kappa n^{-(0.534+0.947\kappa)} \qquad (r^2 = 0.961) \qquad (5.2)$$

where n is the record length and κ is the shape parameter of the GEV distribution.

Figure 5.2 shows the locations of stations whose data can be described by the GEV distribution at the 10% significance level. From this figure, it is evident that distributions of annual maxima from a relatively small number of stations are not GEV, but there is no clear spatial pattern in the occurrence of these stations, despite some consistency in the results for different durations. For this reason, and from the evidence from the L-moment diagrams in the previous section, it is reasonable to assume that the GEV distribution describes at-site annual maximum rainfall data for the Victorian region.

5.2 HOMOGENEITY

Statistical tests for regional homogeneity are based either on sample statistics (eg. CV, L-CV) or on quantile estimates from an assumed parent distribution (eg. AEP 1 in 10 quantile estimates). The following sections present the results of these tests for Victorian data, using stations with more than 100 years of data (Data Set I).

5.2.1 Homogeneity Based On Sample Statistics

Nandakumar (1995), using a CV-based test (Wiltshire, 1986), showed that the Victorian region is homogeneous at the 1% significance level. However, the region was found not to be homogeneous using Hosking and Wallis' (1993) heterogeneity measure based on the sampling variability of L-CV. This test has a limited range of applicability, as L-CV can only explain the between-site variation of rainfall quantiles for exceedance probabilities greater than 0.002 (Hosking and Wallis, 1993). McConachy (1996) found no evidence of any distinct spatial pattern in the sites that did not fit into a homogeneous region with respect to L-CV.

5.2.2 Homogeneity Based on Parent Distributions

Another Wiltshire test, based on the distribution of annual maxima, shows the Victorian region as *not* being homogeneous at the same significance level. Using the same data set, McConachy (1996) showed that the region *is* homogeneous with the Lu and Stedinger's (1992) test which is based on the AEP 1 in 10 rainfall quantile from a GEV distribution.

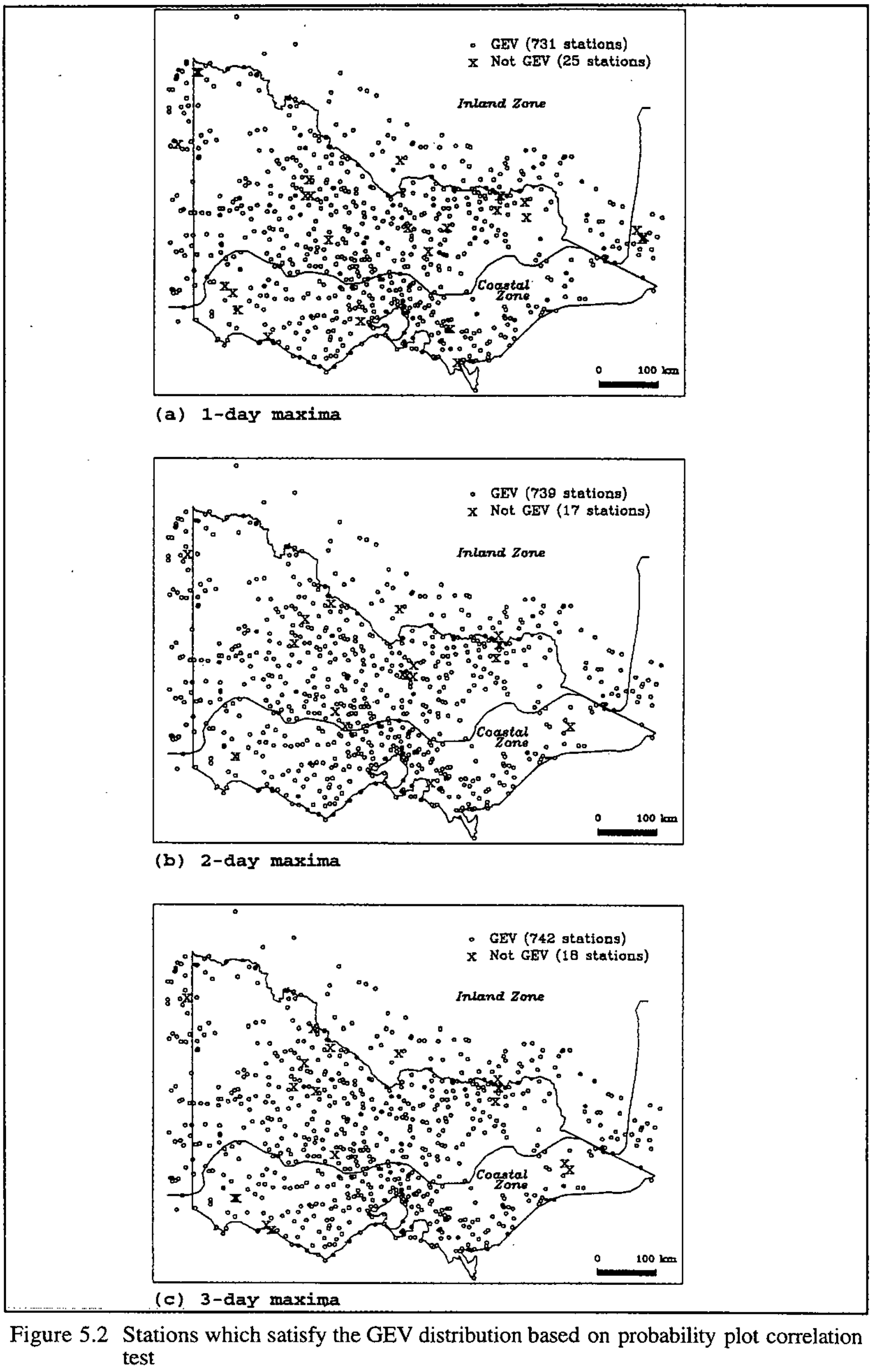


Figure 5.2 Stations which satisfy the GEV distribution based on probability plot correlation test

5.2.3 Interpretation of Test Results and Implications for Application of FORGE Concept

Unfortunately, the four homogeneity tests referred to above are of limited relevance to extreme rainfall estimation. The tests are based either on the overall fit of a distribution at different sites (sample statistics tests) or on the fit in a specified quantile range (quantile tests). In contrast to other index-flood methods which use all the annual maxima available, the methods based on the FORGE concept use only the highest standardised values to derive the regional growth curve. The tests give little direct information on the homogeneity of these data points used to fit the extreme tail of a regional distribution.

Out of the four tests, the Lu and Stedinger (1992) test, which relates to the AEP 1 in 10 rainfall, is probably the most relevant. It indicated homogeneity of the Victorian rainfall data with respect to the quantiles for an AEP of 1 in 10 (McConachy, 1996). McConachy also concluded that standardised maximum annual rainfall data from the Victorian region was homogeneous with respect to L-Skewness and higher L-moments. As these play a primary role in determining the upper tail of a distribution, it is reasonable to assume that the *highest standardised annual rainfall maxima* for the Victoria form a homogeneous data set.

It is worth noting that the generalised methods of PMP estimation used in Australia (BOM 1994, 1996) also assume homogeneity of standardised extreme rainfall values over relatively large areas. In the case of Victoria, the Bureau of Meteorology used a "working hypothesis" (BOM 1996) to divide the overall GSAM region into two sub-regions: the Coastal Zone (south of the Great Dividing Range) and the Inland Zone (North of the Great Dividing Range). No evidence was given to justify the adopted dividing line between the two sub-regions.

It is shown in Section 7.5, that the applications of the CRC-FORGE method to the whole Victorian region and the GSAM sub-regions produced similar growth curves. These results confirm that the Victorian region can be assumed homogeneous for standardised extreme rainfalls.

5.3 SUMMARY

This section has examined the distribution and homogeneity of annual maxima for the Victorian region. Using the L-moment diagram and the probability plot correlation coefficient techniques, it was shown that the general extreme value (GEV) distribution satisfactorily describes the annual maximum data for the Victorian region. Based on results from other studies, the extremes for Victoria and the neighbouring region can be assumed to be a homogeneous region in terms of the rainfall extremes of interest in the FORGE methodology.

6. EFFECTS OF INTER-SITE DEPENDENCE ON REGIONAL FREQUENCY CURVE

6.1 INTRODUCTION

This chapter presents the effects of inter-site dependence on regional frequency analysis. The aim is to develop a spatial dependence model which will be used in application of the methods using the FORGE concept.

As stated in Section 3.2.2, spatial dependence in annual maximum data reduces the net information available in regional frequency analysis of site data. Accordingly, the presence of spatial dependence reduces the accuracy of quantile estimates, because of the reduced number of independent stations.

This chapter begins with an introduction of the effective number of independent stations concept. The estimation methods of the effective number of independent stations (N_e) are then described. Finally, models for the estimation of N_e are developed and tested using Victorian annual rainfall maxima.

6.2 EFFECTIVE NUMBER OF INDEPENDENT STATIONS

The effective number of independent stations concept has been introduced to quantify the effects of inter-site dependence (or spatial correlation) on regional estimates of frequency distribution parameters. The value of N_e depends on which specific parameter is being estimated.

The following sections describe measures of N_e for two basic regional approaches: (i) methods that use some form of regional average parameters and (ii) methods that pool annual maximum data.

6.2.1 N_e for Methods Using Regional Averages

In regional averaging approaches, distribution parameters are obtained using some form of averaging of at-site parameters (following standardisation); this has the effect of reducing the sampling error of the parameter estimates. In these methods, the intersite dependence does not introduce any bias, but increases uncertainty in quantile estimates (Stedinger, 1983), thus negating some of the benefit of averaged parameter estimates.

The effective number of stations for these approaches is defined with respect to a particular statistic, such that the variance of the mean of the chosen statistic calculated from N_e independent stations is equal to the variance of the value calculated from N spatially dependent stations.

N_e defined with respect to regional mean

For a region with N sites with concurrent data of length of n, the variance of the pooled mean is given by (Alexander, 1954):

$$\sigma_m{}^2 = \frac{\sigma^2}{Nn} [1+(N-1)\bar{\rho}] \qquad (6.1)$$

where σ is the standard deviation of the combined data and $\bar{\rho}$ is the arithmetic mean of the N(N-1)/2 correlations between every pair of the N terms.

From Eq. 6.1, Alexander (1954) obtained N_e^1 to give the same standard deviation of regional mean as in an uncorrelated data set as:

$$N_e^1 = \frac{N}{1+(N-1)\bar{\rho}} \qquad (6.2)$$

For large N values, $N_e^1 \to 1/\bar{\rho}$

N_e defined with respect to regional variance and skewness

Hosking and Wallis (1988) argued that Eq. 6.2 is not applicable for regional flood frequency analysis, because the regional data are not used to estimate the mean but rather the second and higher order moments required for parameterisation of a regional distribution.

From Stedinger's (1983) analyses, the equivalent number of independent stations with respect to regional averages of the second and third moments can be given by Eqs. 6.3 and 6.4 respectively:

$$N_e^2 = \frac{N}{1+(N-1)\overline{\rho^2}} \qquad (6.3)$$

$$N_e^3 = \frac{N}{1+(N-1)\overline{\rho^3}} \qquad (6.4)$$

where $\overline{\rho^2}$ and $\overline{\rho^3}$ are the average of the squares and cubes of the correlations respectively.

For large N values, $N_e^2 \to 1/\overline{\rho^2}$ and $N_e^3 \to 1/\overline{\rho^3}$. This illustrates that for given N, N_e increases with increasing order of moments since $-1.0 \le \rho \le 1.0$.

For annual maximum data which can be described by a 3-parameter distribution, the quantile estimates are function of moments up to the third order. The correct value of N_e should therefore be a combination of Eqs. 6.2 to 6.4. As the three moments have a varying degree of influence on different quantile estimates, the effective number of independent stations should vary with the exceedance probability of the quantile to be estimated. The third-order moment has more influence at lower exceedance probabilities. Consequently N_e will be larger for high exceedance probability quantiles.

6.2.2 N_e for Methods Pooling Recorded Data

In the regional frequency estimation approaches that pool the standardised annual maximum data from several sites, time sampling is substituted by space sampling. If the spatial data were independent, each maximum value in the pooled data could be assigned a plotting position computed from the aggregated period of the record (the total record length: L = Nn). This is often referred to as the "station-year method". However, the effective record length (L_e) is invariably less than the total number of annual maxima in the pooled data because of presence of inter-site correlation.

The effective record length of pooled data determines the positions of observed annual maxima on a probability plot. Thus, the effective number of stations for this approach can be defined such that N_e independent stations will provide the same record length as N spatially independent stations. Thus, N_e is defined as the ratio of effective record length (L_e) and the average record length at each station ($\bar{n}$).

$$N_e = \frac{L_e}{\bar{n}} \tag{6.5}$$

As N_e determines the positions of the points on a probability plot, any errors in this measure of spatial dependence in annual maxima will introduce bias in the quantile estimates.

Measure of N_e

The effective number of independent stations can be calculated from the relative displacement of two rainfall frequency curves, the *regional maximum curve* and the *regional average curve (or typical curve)* (Dales and Reed, 1989). For this, a regional maximum series is formed by choosing the highest value of the standardised annual maxima from all the stations in the raingauge network (or region) considered each year. The regional average curve is the average curve for the raingauge network considered (ie. the average of the standardised at-site curves).

Consider a region with N stations, each station having a record length of n. If the regional average curve and the regional maximum curve are drawn as shown in Figure 6.1, the regional maximum curve is fitted to the points that would formerly have been considered as outliers in the frequency curves of individual stations. However, in the regional context they are not outliers, the question is only at what probability they should be plotted.

If the regional average curve is assumed to represent the best estimate of the frequency curve for the region, it indicates by how much the annual exceedance probability of a particular rainfall depth on the regional maximum curve needs to be adjusted to produce an *equivalent probability estimate* for that given rainfall amount. This provides another method for estimating N_e.

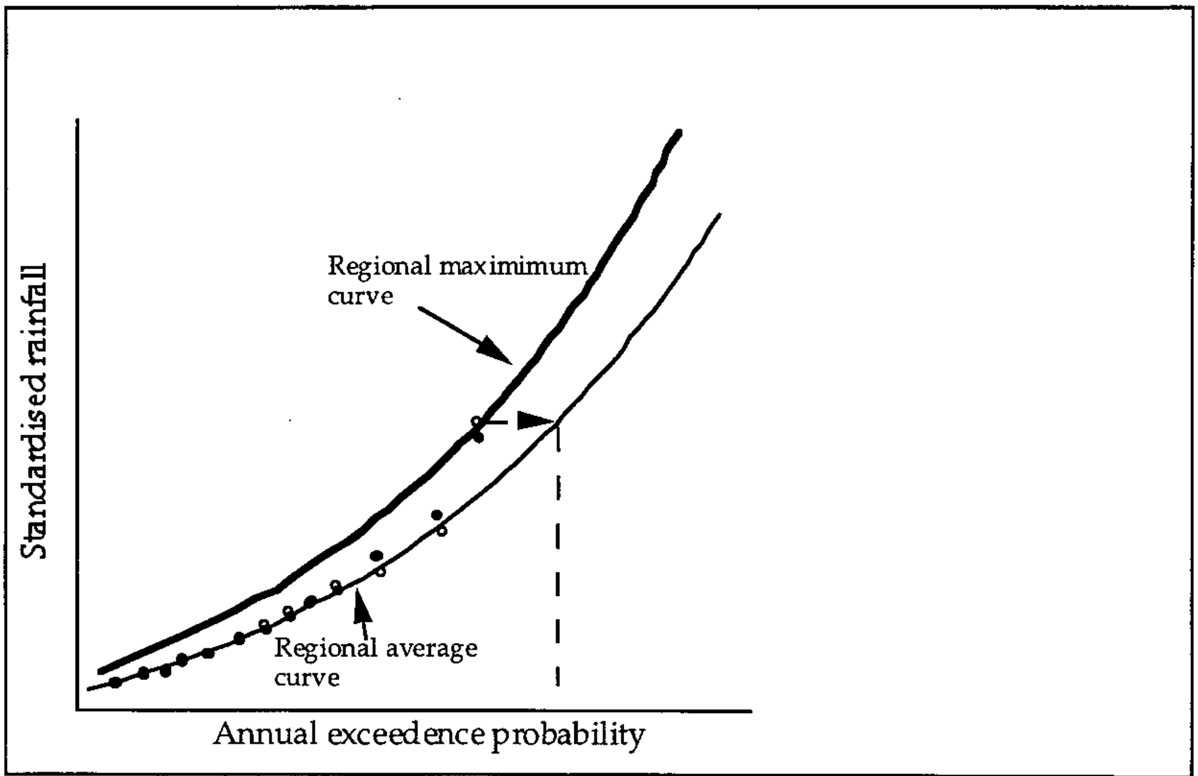


Figure 6.1 Illustration of regional maximum and regional average curves

Let the *regional average* cumulative frequency function be $F_t(x)$ and the cumulative frequency function for the *regional maxima* be $F_r(x)$. If we use a simple plotting position formula, the exceedance probability of the largest rainfall amount from n regional maxima is

$$P_r = \frac{1}{n} \tag{6.6}$$

This treats the regional maxima as if they came from a single station (i.e. (N_e=1) and will require an adjustment. The exceedance probability of the rank one event in the region would be estimated from the regional average curve for the region as

$$P_t = \frac{1}{L_e} \tag{6.7}$$

Since $\bar{n} = n$, using Eqs. 6.5, 6.6 and 6.7 it can be shown that

$$N_e = \frac{P_r}{P_t} \tag{6.8}$$

For a given quantile value x, N_e can thus be interpreted as the ratio between the exceedance probabilities indicated by the regional maximum curve and the regional average curve. Dales and Reed (1989) came to the same result following a different derivation. They defined an effective number of independent stations ($\hat{N}_e$) based on the cumulative frequency functions for the regional maximum curve and the regional average curve as

$$F_r = F_t^{\hat{N}_e} \tag{6.9}$$

But, $F_t = 1 - P_t$ and $F_r = 1 - P_r$

Thus

$$1 - P_r = (1 - P_t)^{\hat{N}_e}$$

$$= 1 - \hat{N}_e P_t + \frac{1}{2}\hat{N}_e(\hat{N}_e - 1)P_t^{\,2} + \ldots\ldots \tag{6.10}$$

At low exceedance probabilities, we may neglect higher order expansions and hence

$$\hat{N}_e = \frac{P_r}{P_t} \tag{6.11}$$

This is equivalent to Eq. 6.8 and hence,

$$\hat{N}_e = N_e \tag{6.12}$$

It can be shown that Eq. 6.8 or 6.11 could also have been derived using other plotting position equations, such as the Cunnane plotting position equation for lower exceedance probabilities.

Using Eq. 6.9,

$$N_e(x) = \frac{\ln F_r(x)}{\ln F_t(x)} \cdot \qquad (6.13)$$

Dales and Reed (1989) showed that $\ln(N_e)$ represents the horizontal separation between the regional maximum curve and a regional average growth curve on a Gumbel plot, as shown in Figure 6.2.

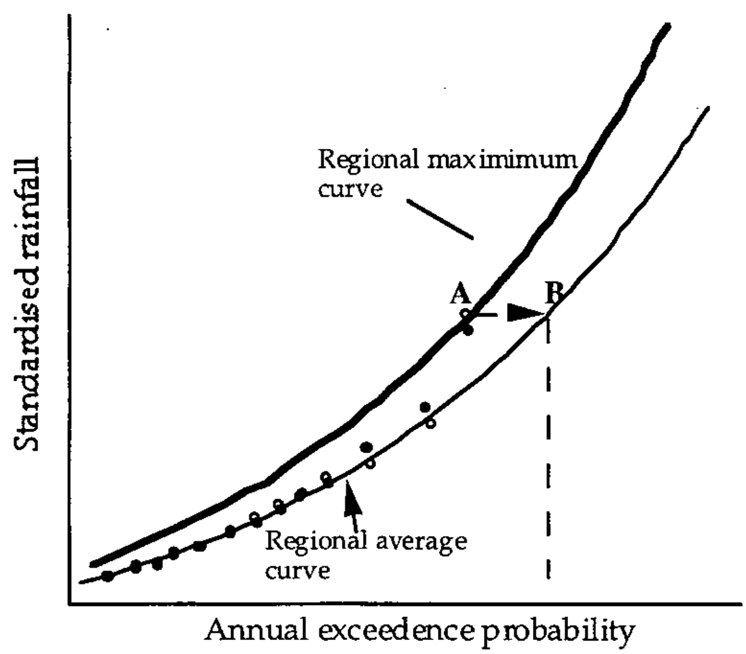


Figure 6.2 The effective number of stations, based on a Gumbel plot

Dales and Reed (1989) estimated N_e assuming that (i) annual maxima are GEV distributed and (ii) the value of N_e is constant irrespective of rainfall quantile. This means that the distribution of regional maxima was assumed to be GEV with a shape parameter equal to that for the regional average curve. The constant N_e is hereafter referred to as N_e^c.

If the shape parameters of fitted regional maximum and regional average curves are not equal, then N_e varies with quantiles and hence exceedance probabilities. The variable N_e is referred to as N_e^v.

6.3 ESTIMATION OF THE CONSTANT EFFECTIVE NUMBER OF INDEPENDENT STATIONS

The following section describes the development of generic estimation equations for an N_e^c assumed to be invariant with AEP using easily obtainable parameters. This enables fast estimation of N_e for a given sample network and reduces the effects of sampling variability in estimating N_e from a single network. Thus, N_e estimated with these generic equations would have smaller uncertainty than that from a single network estimate, provided the equation gives a satisfactory fit to the data.

6.3.1 Dales and Reed's Spatial Dependence Model

Dales and Reed (1989) obtained an empirical equation that relates N_e to the effective area (A) spanned by the selected stations and the number of stations (N) in the form

$$\frac{\ln N_e^C}{\ln N} = c + d \ln A + e \ln N \qquad (6.14)$$

where A = 2.5*(mean distance between stations)^2 and c, d and e are regional coefficients. The data points used in the derivation of this equation were the values of, N and A for each sample network of rainfall stations in the region. ***This equation is hereafter called the Dales and Reed Constant N_e Model.*** It should be noted that Reed and Stewart used this model in the IH-FORGE method.

In the present study, Eq. 6.14 was calibrated using annual rainfall maxima from stations with 100 or more years of data (Data Set I). For this, the network sizes were: 2, 4, 8, 16 and 32 stations. A value in the regional maximum series was only chosen if the annual maximum for the corresponding year was available at all stations in that network. Data Set I has more years with complete records at all stations than Data Set II (60 or more years of data). Therefore, Data Set I has the potential to produce a longer regional maximum series than Data Set II.

Table 6.1 shows the calibrated coefficients of Eq. 6.14. A goodness of fit measure, the coefficient of efficiency, given in this table indicates the degree of departure of points from the 1: 1 line. The coefficient of efficiency is given by Aitken (1973) as:

$$E = \frac{\sum (N_e^c - \overline{N_e^c})^2 - \sum (N_e^c - \hat{N_e^c})^2}{\sum (N_e^c - \overline{N_e^c})^2} \qquad (6.15)$$

where $\hat{N}_e^c$ is the estimate of N_e^c by the spatial dependence model and $\overline{N_e^c}$ is the mean of the N_e^c values computed directly from the data. A value of E close to 1 indicates only a small systematic deviation from the 1:1 line, which is the case for the models in Table 6.1. (It should be noted that the directly computed values of N_e used as the basis for this comparison are also *assumed to be invariant with AEP*.)

The comparison of N_e^c computed using regional maximum and regional average curves, and that estimated by the spatial dependence model (Eq. 6.14) is shown in Figures 6.3(a), 6.4(a) and 6.5(a). It should be noted that log scales in these figures were chosen because the errors in plotting position estimates are indicated by the errors in $\ln(N_e^c)$. These figures illustrate that the scatter in the spatial dependence model estimates increases with increasing N. This introduces higher uncertainties in N_e estimates for larger N values which has serious implications for high quantile estimates from the FORGE approach. Thus this model appears not to be suitable for Victorian data.

Table 6.1 Coefficients for Dales and Reed Constant N_e Model (Eq. 6.14) and CRC Constant N_e model (Eq. 6.16), and coefficient of efficiency (E) for each one.

Duration	D. & R. Constant N_e model				CRC Constant N_e model		
(days)	c	d	e	E	a	b	E
1	0.177	0.061	-0.029	0.949	0.991	-0.699	0.985
2	0.053	0.072	-0.042	0.955	0.981	-0.713	0.983
3	0.006	0.075	-0.045	0.960	0.982	-0.724	0.981

6.3.2 CRC Constant N_e model - An Empirical Relationship for N_e based on Average Correlation Coefficient

In this section, a relationship between N_e^c and the average correlation coefficient is investigated. The aim is to find an alternative model for the Dales and Reed Constant N_e Model found to be unsuitable for Victorian data in the previous section.

The CRC Constant N_e model given by Eq. 6.16 was calibrated using Data Set I (as for the spatial dependence model). The identification of the equation form was based on an investigation of a generated data set with known correlation coefficients and is described in detail in Nandakumar (1996).

$$\frac{\ln N_e^c}{\ln N} = a + b\overline{\rho} \tag{6.16}$$

The fitted parameters of the CRC Constant N_e model are given in Table 6.1. The closeness of the parameter 'a' to 1.0 suggests that Eq. 6.16 could be reduced to a one parameter model. The E values close to unity indicate that the use of the CRC Constant N_e model will result in improved N_e estimates. This is also reflected in Figures 6.3(b), 6.4(b) and 6.5(b) which show that the scatter at higher N values is significantly reduced. The CRC Constant N_e model is therefore recommended for use in the FORGE procedure to reduce the uncertainty in rainfall quantile estimates.

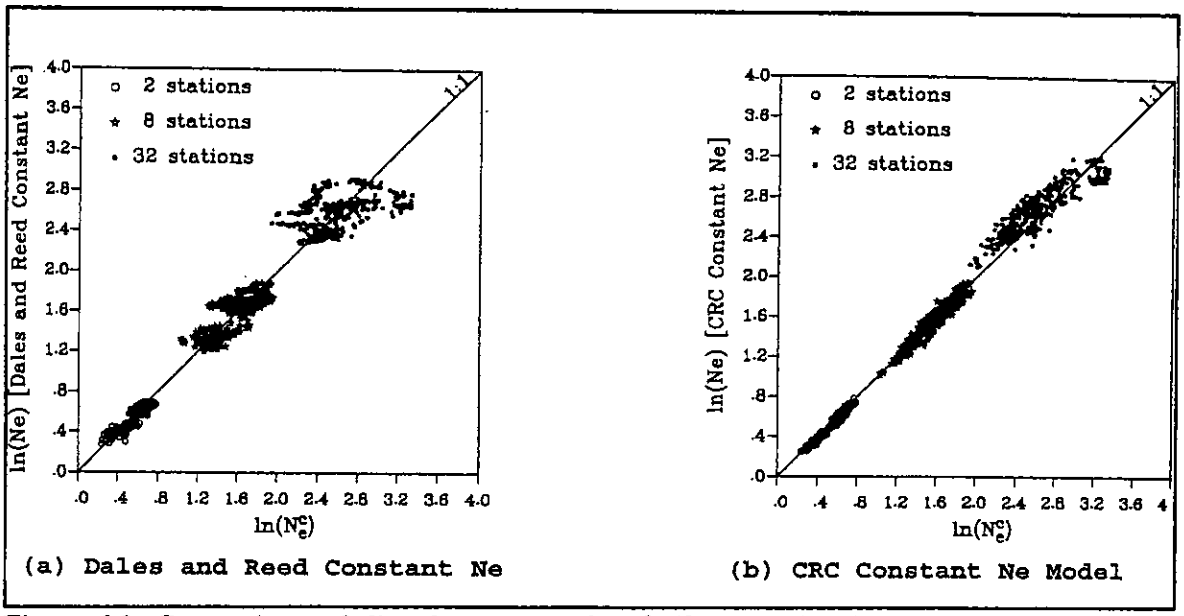


Figure 6.3 Comparison of directly computed N_e^c (from regional average and maximum distributions) and N_e^c estimated using: (a) Dales and Reed Constant N_e model and (b) CRC Constant N_e for 1-day maximum rainfall

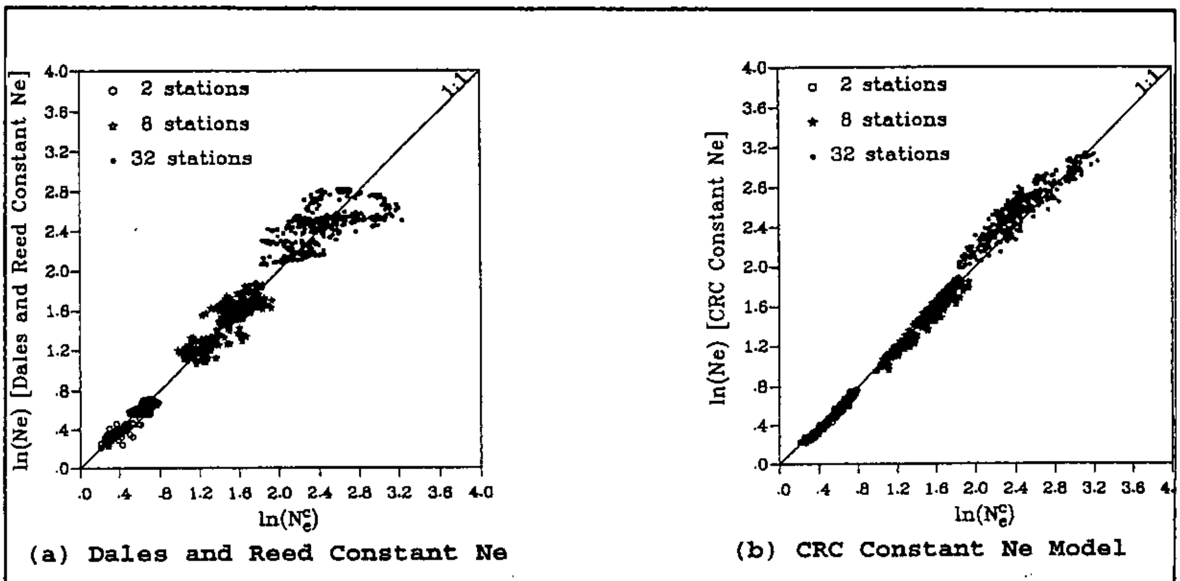


Figure 6.4 Comparison of directly computed N_e^c (from regional average and maximum distributions) and N_e^c estimated using: (a) Dales and Reed Constant N_e model and (b) CRC Constant N_e for 2-day maximum rainfall

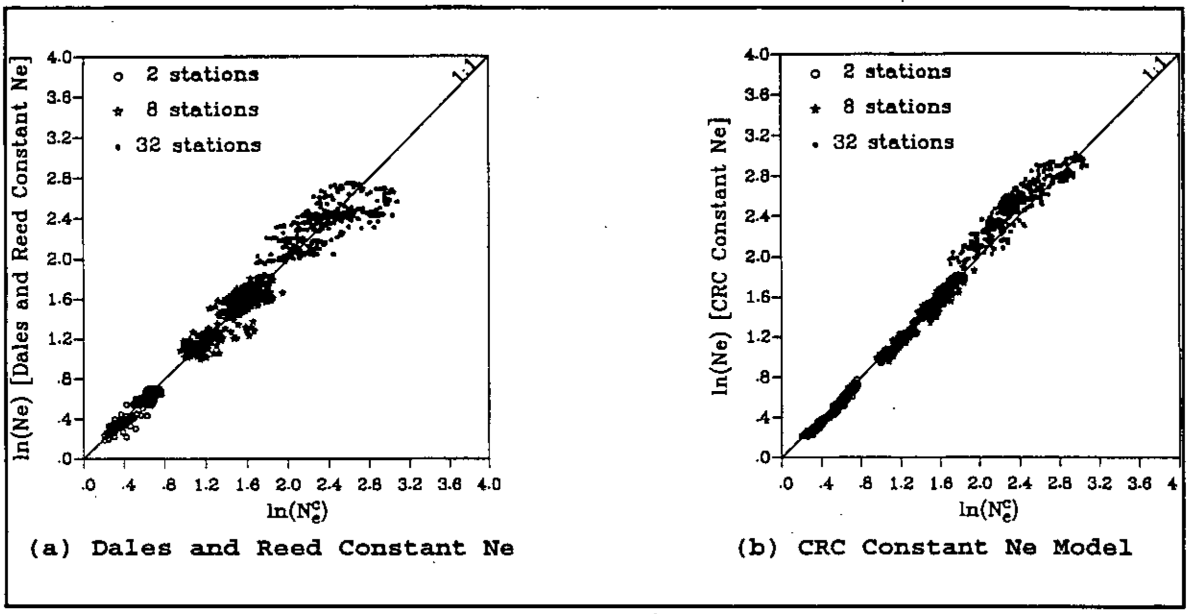


Figure 6.5 Comparison of directly computed N_e^c (from regional average and maximum distributions) and N_e^c estimated using: (a) Dales and Reed Constant N_e model and (b) CRC Constant N_e for 3-day maximum rainfall

6.3.3 Discussion

The coefficients of Eq. 6.16 given in Table 6.1 suggest that $N_e^c \approx N$ for independent stations $(\bar{\rho}=0)$ as should be the case. However, for totally dependent rainfall amounts $(\bar{\rho}=1)$, $N_e^c \neq 1$, in contrast to the theoretical expectation. This could be an artefact of the simple equation for the relationship between N_e^c, N and $\bar{\rho}$ used. Thus, for the case of $\bar{\rho}=1$, the errors in N_e^c are high for large N values. However, this has little effect on the estimates from the methods using the FORGE concept, as the average correlation coefficient between sites is normally much less than one.

It is generally accepted that the correlation of rainfalls recorded at two stations is inversely related to the distance between the two stations. Figure 6.6 shows that for Victorian rainfall data this relationship can be expressed by a logarithmic equation. Using this form of relationship and the substitution ln A = a+ b ln(distance), Eq. 6.16 can be rearranged in the following form:

$$\frac{\ln N_e}{\ln N} = c' + d' \ln A \qquad (6.17)$$

where c' and d' are coefficients.

This is equivalent to the Dales and Reed model (Eq. 6.14) without the last term. As the values of coefficient 'e' in that model are small, the influence of this term on calculated N_e^c would be minimal; in this event, Eq. 6.14 reduces to Eq. 6.17. Thus it can be concluded that the large scatter in estimates of N_e from the Dales and Reed Constant N_e Model is due to the scatter in the relationship between the correlation coefficient and distance shown in Figure 6.6.

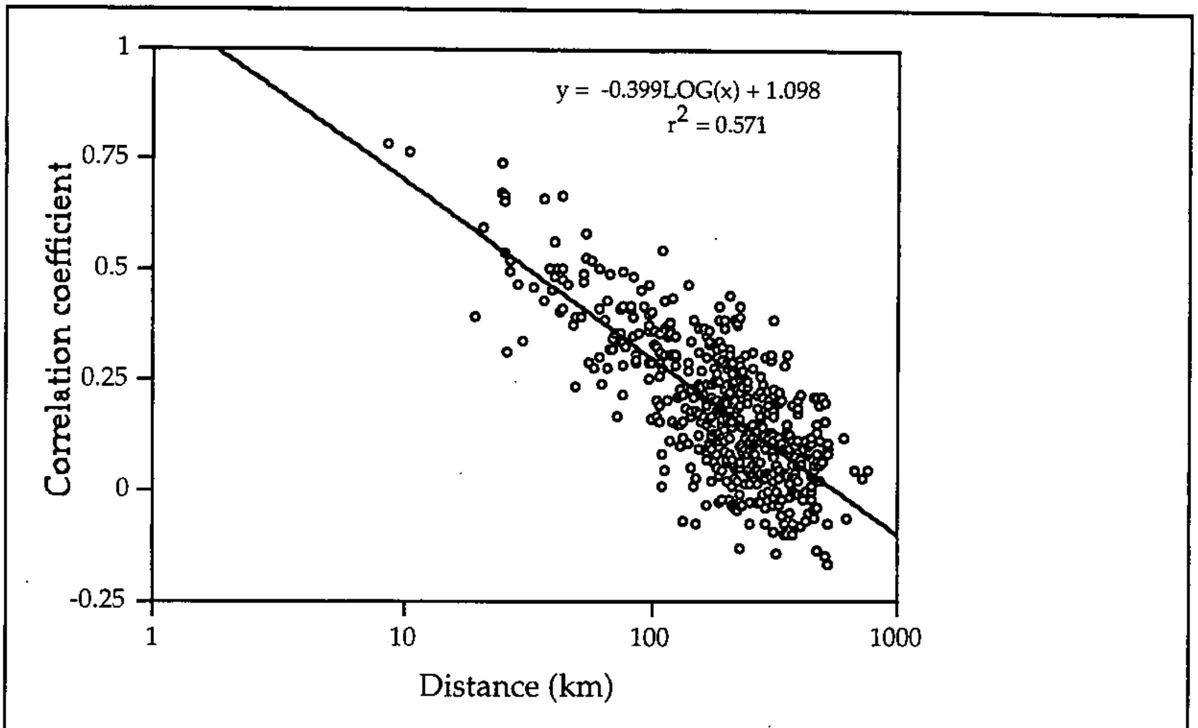


Figure 6.6 Relationship between correlation coefficient and distance for Victorian 1-day annual maxima (500 points randomly selected from 9180 values are shown)

The use of the CRC Constant N_e model in the applications of methods using the FORGE concept has an advantage over a constant N_e model based on distance or area (eg. Dales and Reed Constant N_e model), as it significantly reduces the uncertainty in N_e^c, especially for large N. The only difficulty is in calculating the correlation coefficient for pairs of stations where only limited concurrent rainfall record is available. In such a case the fall back position would be to firstly compute the correlation coefficient from a regional relationship with distance and then to apply Eq. 6.16. This is equivalent to disaggregating the Dales and Reed Constant N_e Model into two stages. Thus a spatial dependence model of the Dales and Reed type can be replaced with the CRC Constant N_e model to improve the rainfall quantile estimates if sufficient data (more than 20 years concurrent record) exists to compute average correlation coefficients.

6.4 ESTIMATION OF THE EFFECTIVE NUMBER OF INDEPENDENT STATIONS FROM A VARIABLE N_e MODEL

In Section 6.3, a constant N_e was estimated by assuming that the shape parameters of regional and regional average frequency curves were equal, i.e. that the two curves were parallel. In reality, this may not be true; Pescod (1991) and Buishand (1984) have found that N_e increases as the exceedance probability decreases. The considerations in Section 6.2.1 also support the proposition of a variable N_e. Hence, the following sections describe the development of a Variable N_e model.

6.4.1 Estimation of a Variable N_e for Victoria

For a given rainfall quantile, N_e is expressed by the horizontal displacement between the regional maximum and the regional average curves on a Gumbel plot (Section 6.2.1). The variable effective number of independent stations (N_e^v) for a given rainfall quantile was computed using Eq. 6.13; the rainfall quantiles were derived from the regional average GEV distribution for a range of Gumbel reduced variate (y) values, where y = -ln -ln(1-AEP).

It was shown in Section 6.3.2 that N_e^c is a function of the average correlation coefficient ($\bar{\rho}$). Therefore, N_e^v is also expected to be dependent on $\bar{\rho}$. Accordingly, the estimates of N_e^v for different $\bar{\rho}$ values were treated separately.

For a selected focal point, the network size was varied, and N_e^v calculated for a range of y and $\bar{\rho}$ values. To ensure a wide range of $\bar{\rho}$, three different networks of N stations with different inter-station distances were selected for each focal point. The distance groups are: (i) the stations closest to the focal point, (ii) the stations falling in the mid-distance range and (iii) the stations furthest from the focal point. The N values used were 2, 4, 8, 16 and 32.

The variation of N_e^v with y was investigated down to the smallest exceedance probability justified by the effective record length of the pooled data. To obtain this initial estimate of the effective record length, data was assumed to be independent. Accordingly, the total record length of pooled data was used to calculate the limiting value of the reduced variate for the regional average curve.

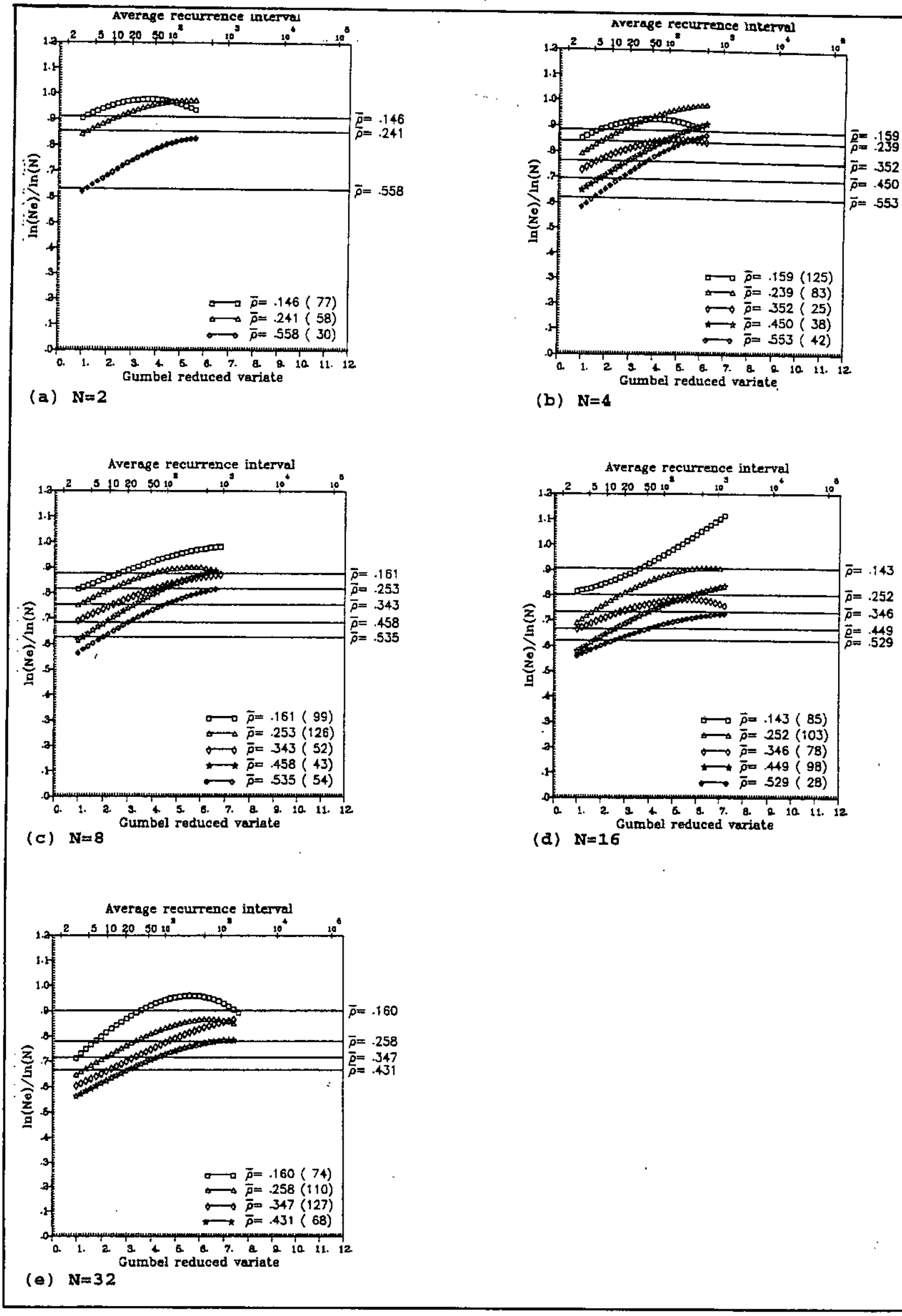


Figure 6.7 Variation of $\ln N_e / \ln N$ calculated from GEV distribution fit for Victorian 1-day rainfall maxima (horizontal lines indicate constant N_e values for shown $\bar{\rho}$)

The station networks obtained for the N-station groups were divided into 10 classes based on their $\bar{\rho}$. These average correlation coefficient classes had equal width and ranged between 0 and 1. Figures 6.7 (a) to (e) show the variation of the average of $\ln N_e^v / \ln N$ with y for the 1-day rainfall duration. In these figures, only $\bar{\rho}$ classes which have more than 20 cases are shown. The average correlation coefficient is also shown along with the number of cases. The horizontal lines indicate the average N_e^c estimated using the CRC Constant N_e model. Corresponding plots for 1-day and 3-day rainfall durations show similar variations of $\ln N_e^v / \ln N$ (see Appendix A).

From Figure 6.7, it is evident that N_e^v increases with increasing y values (or reducing exceedance probability). Typically, N_e^v is higher than N_e^c. As the FORGE-concept mainly deals with six highest standardised annual maxima of pooled data, the use of N_e^c for N_e would underestimate the effective record length and, as a result, quantile estimates would be overestimated. This is demonstrated in Chapter 7 using generated data with known population parameters.

Although Figure 6.7 indicates a general tendency for N_e^v to increase with increasing y values, it does not show explicitly any regular variation with N or $\bar{\rho}$. This is possibly due to inherent heterogeneity in the data and sampling variability. Accordingly, to identify a suitable model function for N_e^v, a synthetic data set with known population parameters was used; this is described in the following section.

6.4.2 Identification of Model Function Using Synthetic Data

The relationship between N_e^v, y, N and $\bar{\rho}$ was investigated using generated data with known population parameters. This allows control over the issues of homogeneity and sampling variability in this investigation.

Data generation

For the generation of annual maximum data, it was assumed that: (i) generated maxima come from the same population, (ii) data from different years are spatially independent (ie. a particular year data for a given site is not correlated with other year data for other sites) and (iii) data from the same year are dependent with a given degree of correlation. To represent the Victorian data, the regional average Victorian GEV distribution parameters of 1-day maxima were used for data generation. The multi-site maxima data were generated according to the following steps.

(i) For a given correlation coefficient, a vector of random multivariate normal deviates with zero mean and a covariance matrix whose elements are the constant cross correlations is generated using the Matalas (1967) method.

(ii) The normal variate vector is transformed into a GEV distribution with the regional average parameters for Victoria.

To reduce the sampling variability due to a limited record length, sequences of annual maximum data for a region with 48 stations, each having a record length of 1000 years was generated. The (constant) correlation coefficient between the annual maxima from different stations was varied from 0.0 to 0.5 in steps of 0.1. In all, 99 replicates of regional data (each replicate consists of data for 48 stations) were generated for each constant correlation coefficient.

Table 6.2 gives the GEV parameters for the parent distribution used in the data generation and the mean parameters for the generated data. The parent distribution parameters used to generate the data seem to be reasonably well preserved by the generated model. The correlation coefficients (ρ) are not quite as well preserved, as ρ was not directly introduced in the GEV data generation (correlated standard normal deviates were first generated and then transformed to a GEV distribution). However, strict preservation of a target ρ is not essential for this exercise; the important requirement is to know what the average ρ value is. This is then assumed to represent the population correlation coefficient.

Table 6.2 Comparison of the parameters of the parent distribution and the distribution for generated data (distribution: $F(x)=\exp[-\{1-\kappa(x-\xi)/\alpha\}^{1/\kappa}]$) and correlation coefficient, ρ.

Parameters							
ρ		ξ		α		κ	
Parent	Gen.*	Parent	Gen.*	Parent	Gen.*	Parent	Gen.
0.000	0.002 (0.032)	0.811	0.810 (0.011)	0.280	0.280 (0.007)	-0.092	-0.092 (0.021)
0.100	0.084 (0.033)	0.811	0.810 (0.011)	0.280	0.279 (0.007)	-0.092	-0.091 (0.023)
0.200	0.172 (0.033)	0.811	0.812 (0.011)	0.280	0.279 (0.007)	-0.092	-0.089 (0.025)
0.300	0.263 (0.032)	0.811	0.810 (0.011)	0.280	0.278 (0.007)	-0.092	-0.088 (0.026)
0.400	0.356 (0.031)	0.811	0.811 (0.011)	0.280	0.276 (0.007)	-0.092	-0.087 (0.026)
0.500	0.452 (0.028)	0.811	0.811 (0.010)	0.280	0.275 (0.007)	-0.092	-0.087 (0.026)

* the values given in parentheses indicate the standard deviation of estimates

Variable N_e for generated data

The variation of N_e^v with Gumbel reduced variate y is examined for networks of 4, 8, 16 and 32 stations in Figure 6.8. For a network of a given size, the correlation coefficient was varied from 0.0 to 0.5 in a step of 0.1. Each point in Figure 6.8 was obtained by averaging ln(N_e^v) values from 10 random combinations of stations (for a given number of stations and fixed correlation coefficient) to reduce the sampling variability. The ln(N_e^v) values were divided by ln(N) to standardise the value for the variation of N.

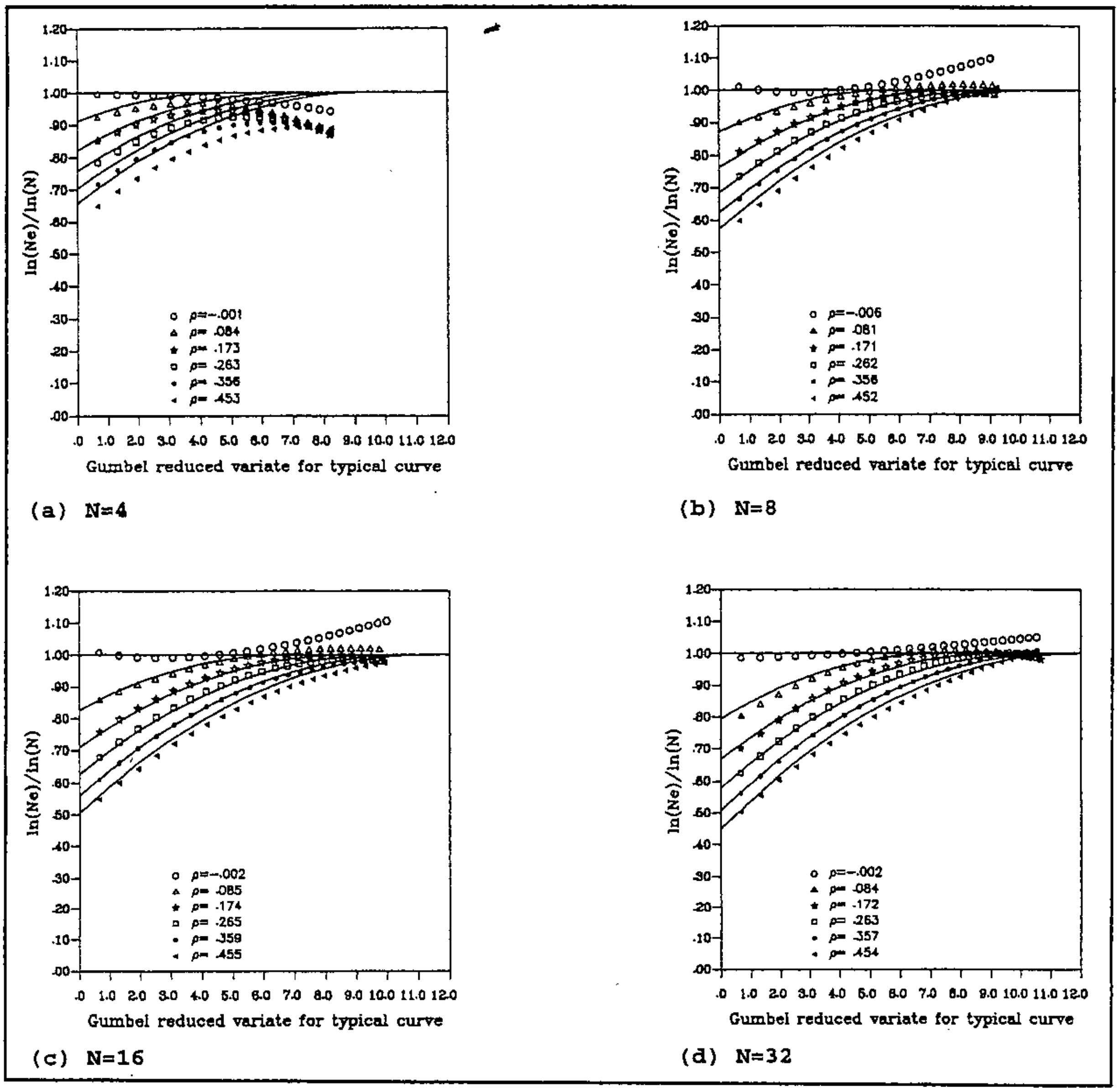


Figure 6.8 Variation of $\ln N_e / \ln N$ with Gumbel reduced variate (y) for given N values for generated data

It is evident from Figure 6.8 that N_e^v increases with increasing Gumbel reduced variate (y) value and, for a given y value, N_e^v decreases with increasing correlation coefficient. The ratio ln(N_e^v)/ln(N) approaches unity for large y values. The following form of a relationship seems to satisfy the general behaviour of these points for $\bar{\rho} \geq 0.1$.

$$\frac{\ln N_e^v}{\ln N} = 1 - \alpha\{y - \beta \ln(\bar{\rho} \ln N) - \gamma\}^2 \quad \text{for} \quad y \leq \beta\bar{\rho} \ln N - \gamma \qquad (6.18)$$

For the data points with $\bar{\rho} \geq 0.1$, the least squares estimates of α, β and γ are 0.0039, 2.74 and 10.63.

For $\bar{\rho} \leq 0.1$, the coefficient α was linearly weighted such that it will be zero when $\bar{\rho}=0$ (ie. N_e^v=N, for $\bar{\rho}=0$).

Figure 6.8 also shows the curves for Eq. 6.18 with the fitted parameters given above. It appears from this figure that the uncertainty in the ln(N_e^v)/ln(N) estimates is large for low values of N. However, this has only limited impact on the estimated ln(N_e^v) values. Figure 6.9 shows the overall satisfactory performance of Eq. 6.18.

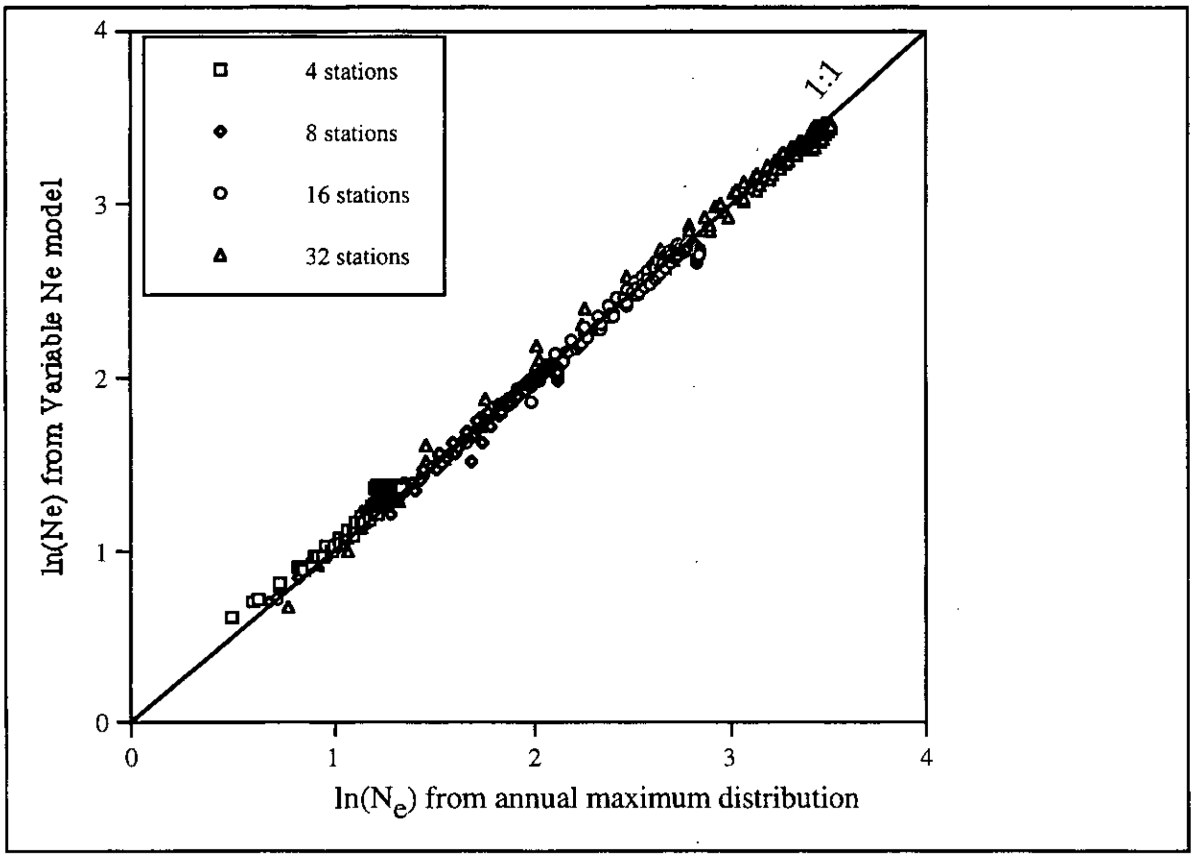


Figure 6.9 Comparison of N_e^v estimated directly from regional average and regional maximum growth curves and N_e^v estimated by Eq. 6.18 for generated data

6.4.3 Estimation of Variable N_e Model Parameters for Victoria

The Variable N_e model given by Eq. 6.18 was calibrated for points in Figure 6.7 and corresponding values for 2-day and 3-day durations. In the calibration, the model response was log-transformed to achieve constant variance of residuals. The residuals are the differences between model responses and corresponding values calculated directly from the regional average and regional maximum growth curves. The variance of residuals needs to be constant in order to estimate uncertainty in quantile estimates (Chapter 8).

Figure 6.10 shows comparisons of $\ln(N_e)$ estimated using the CRC Constant N_e model (N_e^c) and the Variable N_e model (N_e^v) with that calculated directly from the regional average and the regional maximum curves for a range of Gumbel reduced variates. Although the scatter about the 1:1 line appears to be similar for both models, the systematic bias in the Variable N_e model estimates is lower than that with estimates using a fixed N_e.

Table 6.3 gives calibrated coefficients of Eq. 6.18 using Victorian data (Data Set I) along with the coefficients of efficiency (E), a goodness of fit indicator. For comparison, E values for fitting the CRC Constant N_e model are also given. Higher E values for the Variable N_e model than those for the CRC Constant N_e model also suggest that there is less bias in the Variable N_e model estimates than is present in the CRC Constant N_e model estimates.

6.4.4 "Asymptotic Independence" and Implications for CRC-FORGE Estimates

The dependence of $\ln N_e^V/\ln N$ on $\ln N$ may be somewhat surprising. It is thought that the property of asymptotic independence indicated by the analysis of observed and generated rainfall data is associated with sampling effects related to the highest ranking events in the regional maximum distribution. For large data sets (ie. large values of N) the asymptotic independence occurs at larger values of y (or lower AEPs) than for smaller data sets (or smaller regions). As the FORGE points to be plotted are also affected by these sampling effects, it is appropriate to apply an N_e model that accounts for the effect of sample size, such as the model given by Eq. 6.18.

Although the effects of record length on N_e^V were not investigated in detail, the above considerations on sample size suggest that asymptotic independence is also affected by the average record length at the N stations. The fitted coefficients in Eq. 6.18 are based on 1000 years of data (generated data set) and approximately 100 years of data (Victorian Data Set I) respectively. For stations with shorter records from Data Set II (minimum 60 years) the computed N_e values may thus be slightly underestimated.

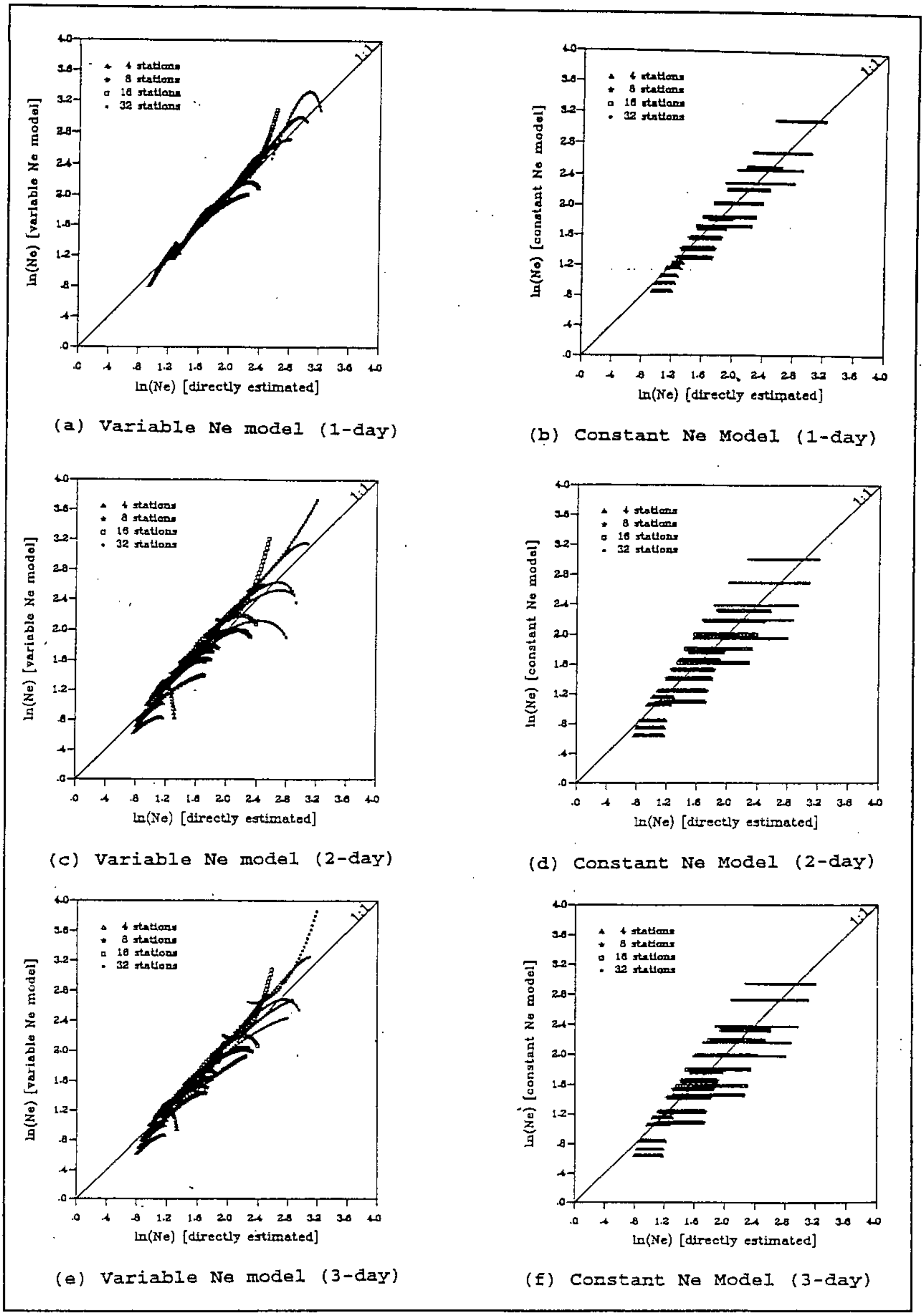


Figure 6.10 Comparison of (i) N_e estimated by the Variable N_e model and N_e directly calculated from typical and regional maximum curves and (ii) N_e estimated by the CRC constant N_e model and N_e directly calculated from typical and regional maximum curves for shown durations

Table 6.3 Parameters of Variable N_e model (Eq 6.18)

Duration (days)	α	β	γ	E^*
1	0.0013	4.41	17.9	0.963 (0.877)*
2	0.0022	2.74	15.1	0.895 (0.781)
3	0.0021	3.03	15.2	0.927 (0.756)

* E values in parentheses are for CRC Constant N_e model
(It should be noted that the E values listed in this table are based on a comparison with directly computed N_e values which allow for variation of N_e with AEP. The E values for the CRC Constant N_e model are therefore different from the ones listed in Table 6.1)

6.5 COMPARISON OF THE EFFECTIVE RECORD LENGTH ESTIMATES USING CRC CONSTANT AND VARIABLE N_e MODELS

The effective record lengths were estimated using CRC Constant N_e model and Variable N_e models for station networks with varying sizes. For this, stations with more than 60 years of data were used (Data Set II). For a selected focal station, the station network was selected as for the IH-FORGE method (Section 3.5) ie. number of stations (N) were 3, 6, 12, 24, 48, 96, 192, 384 and 756.

Figure 6.11 shows the typical variation of total record length and the effective record lengths from the CRC Constant N_e and Variable N_e models, when the number of stations around a focal station (079042) gradually increases. As expected, the effective record length estimates from the Variable N_e model are higher than those from the CRC Constant N_e model. The deviation of effective record length curve from total record length curve first increases with increasing N, and then decreases, although the average correlation consistently decreases with increasing number of stations. This is due to the fact that extreme observations from a network tend to be more independent, regardless of the high degree of correlation which more frequent rainfalls may exhibit (see Section 6.4.4).

The effective record length and the effective number of independent stations for Data Set II are given in Table 6.4. The effective record length for the Variable N_e model is significantly higher than for the CRC Constant N_e model. Table 6.4 also shows that the N_e estimates from the CRC Constant N_e model decrease with duration, as the average correlation coefficient increases with duration. However, this was not observed for the Variable N_e model, which could indicate that the extremes for longer durations are more independent than those for shorter durations.

Table 6.4 Total record length (L), effective record length (L_e) for Data Set II

Duration	N	L	CRC Const. N_e Model		CRC Var. N_e Model	
(days)		(years)	L_e (years)	N_e*	L_e (years)	N_e*
1	756	64971	27088	315 (42%)	41560	483 (64%)
2	756	65005	21757	253 (33%)	48191	560 (74%)
3	760	65230	20661	241 (32%)	46947	547 (72%)

* N_e values in parentheses are percentages of N

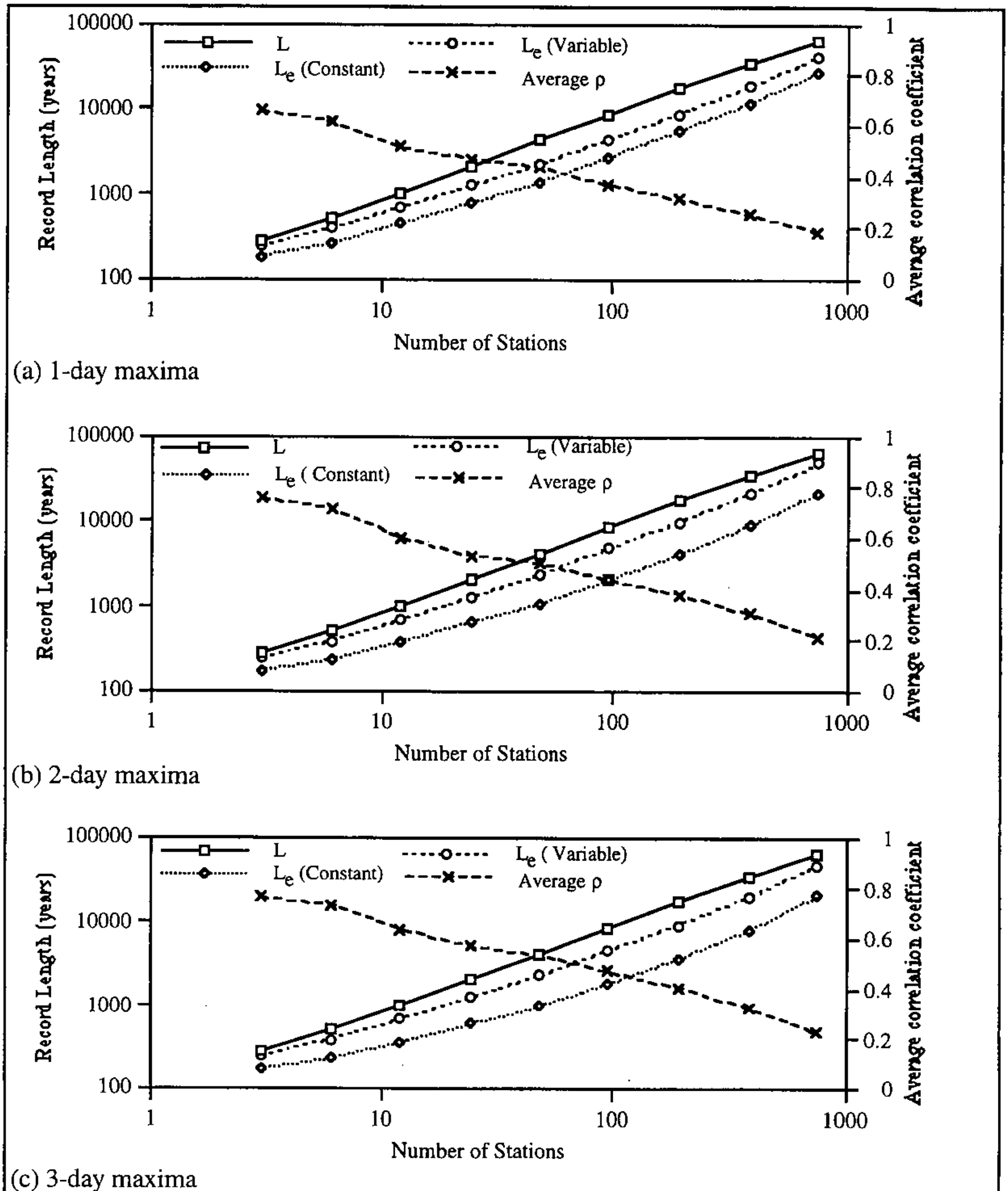


Figure 6.11 Variation with number of stations of: total record length (L), effective record lengths estimated using CRC Constant N_e model [L_e (Constant)] and Variable N_e model [L_e (Variable)], and average correlation coefficient (ρ)

6.6 EFFECTS OF DISTRIBUTIONAL ASSUMPTION ON ESTIMATES OF THE EFFECTIVE NUMBER OF INDEPENDENT STATIONS

The methods using the FORGE concept require estimates of the plotting positions of the highest regional rainfalls; the basic principles are explained in Section 6.2.2. In applying these principles, the effective number of independent stations is calculated by extrapolating the regional average curve. The degree of extrapolation increases with increasing size of the raingauge network, as the regional maximum curve shifts upward on a Gumbel plot. As illustrated in Figure 6.12, different distributions fitted to the same data result in different values for N_e especially in the extrapolation zone.

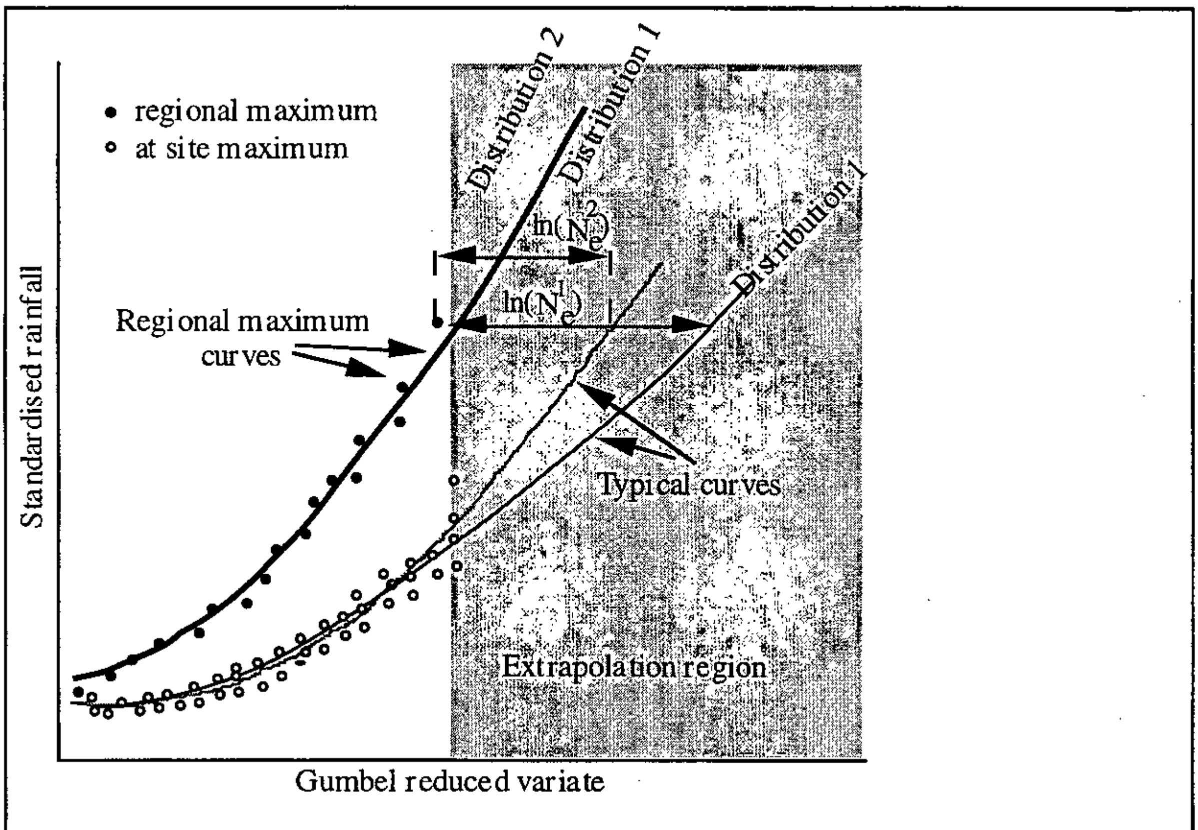


Figure 6.12 Illustration of the effects of distribution assumption on N_e estimates for higher exceedance probabilities

To illustrate the effects of the distribution assumption on N_e estimates, four 3-parameter distributions were fitted for a typical raingauge network with 32 stations. They were: (i) generalised extreme value (GEV) (ii) generalised logistic (GLO) (iii) Pearson 3 (PE3) and generalised Pareto (GPA). Figure 6.13 shows the fitted regional average and regional maximum curves for the typical raingauge network. Also shown in this figure is the variation of $\ln(N_e)/\ln(N)$ with y. [Note: $\ln(N_e)/\ln(N)$ values greater than one were restricted to unity.]

In Figure 6.13, the $\ln(N_e)/\ln(N)$ ratio increases with y for all distributions and exceeds unity, except for the GLO distribution. This is most pronounced for the GPA distribution, where the ratio tends towards infinity, as the distribution has an upper bound. As illustrated in the figure, the GPA distribution is clearly not appropriate for this application. The PE3 distribution shows a similar behaviour as the GEV, but the $\ln(N_e)/\ln(N)$ ratio reaches unity at a lower value of the Gumbel reduced variate.

With the GLO distribution, the $\ln(N_e)/\ln(N)$ ratio peaks to a value lower than unity and decreases with increasing y. This means that the upper tails of the regional maximum and the regional average distributions tend to converge. A similar variation was found when this distribution was applied to the whole Victorian data (Data Set I), as shown in Figure 6.14. The $\ln(N_e)/\ln(N)$ ratios for this figure were computed as described in Section 6.4.1 fitting the GLO distribution to annual maxima instead of the GEV distribution.

The variation of the $\ln(N_e)/\ln(N)$ ratio with y derived using the GLO distribution contradicts the expectation that extreme rainfalls tend to be spatially independent. As the $\ln(N_e)/\ln(N)$ ratio decreases with y, the CRC-FORGE estimates based on that distribution would be higher than assuming the other distributions.

The inconsistent behaviour of the $\ln(N_e)/\ln(N)$ ratio for the GLO distribution may be attributed to the fact that the distribution does not describe data for the Victorian region satisfactorily (see Figure 5.1 in Chapter 5). Nevertheless, the GLO distribution was considered to be a possible alternative distribution, and it was therefore used to further examine the implications on estimates of N_e. This was investigated using synthetic data as summarised in the following section.

6.6.1 Estimation of N_e from GLO distribution using synthetic data

To reduce the effects of sampling variability and to have a known population distribution, the variation of the $\ln(N_e)/\ln(N)$ ratio was examined for a long synthetic data set with a GLO distribution. Synthetic GLO data were generated using a method similar to that described in Section 6.4.2. For this, the average Victorian GLO parameters were used, as determined from Data Set (I).

The variation of the average $\ln(N_e)/\ln(N)$ ratio with y for each correlation group is illustrated in Figure 6.15. The value greater than one for the $\ln(N_e)/\ln(N)$ ratio in this figure reflects the effects of extrapolation of two curves. This figure shows that extreme data tend to be independent, in contrast to the results obtained from the Victorian data, using the GLO distribution. Thus it appears that the hypothesis of extreme data tending to be independent will be accepted, if an appropriate distribution for the given data set is selected.

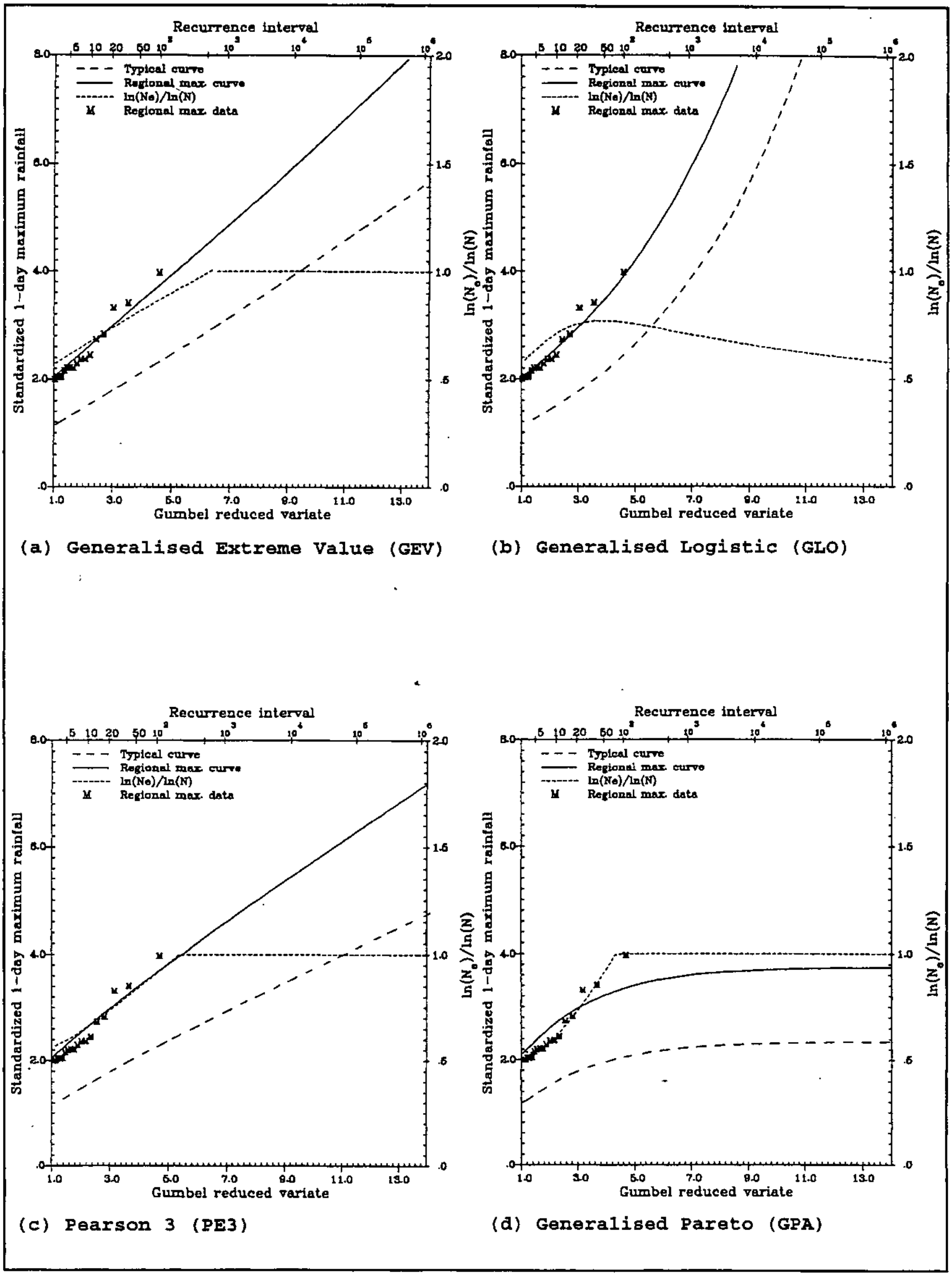


Figure 6.13 Illustration of dependence of N_e on shown distributions assumed for regional maximum and typical curves

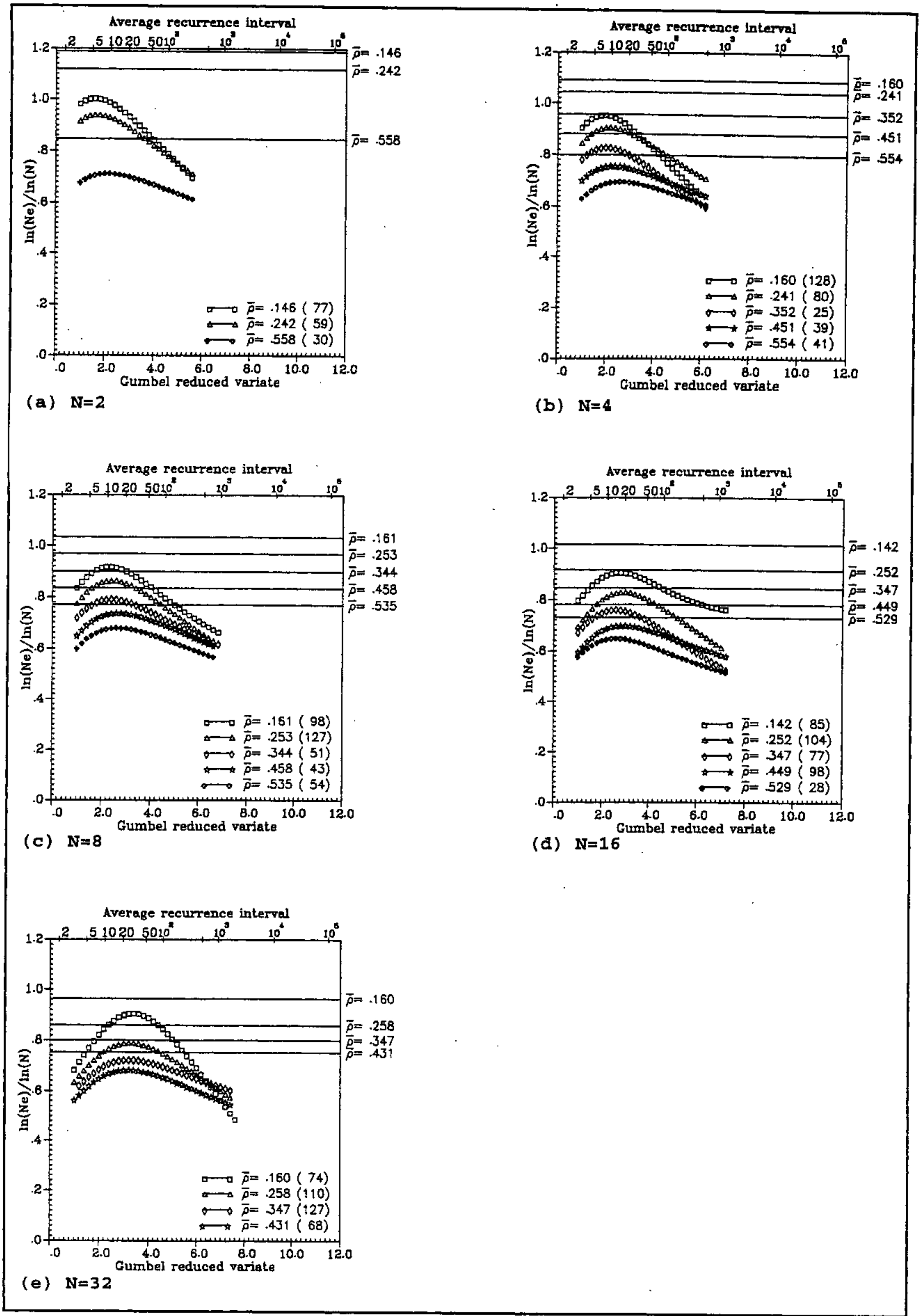


Figure 6.14 Variation of $\ln N_e / \ln N$ calculated from GLO distribution for Victorian 1-day rainfall maximum data

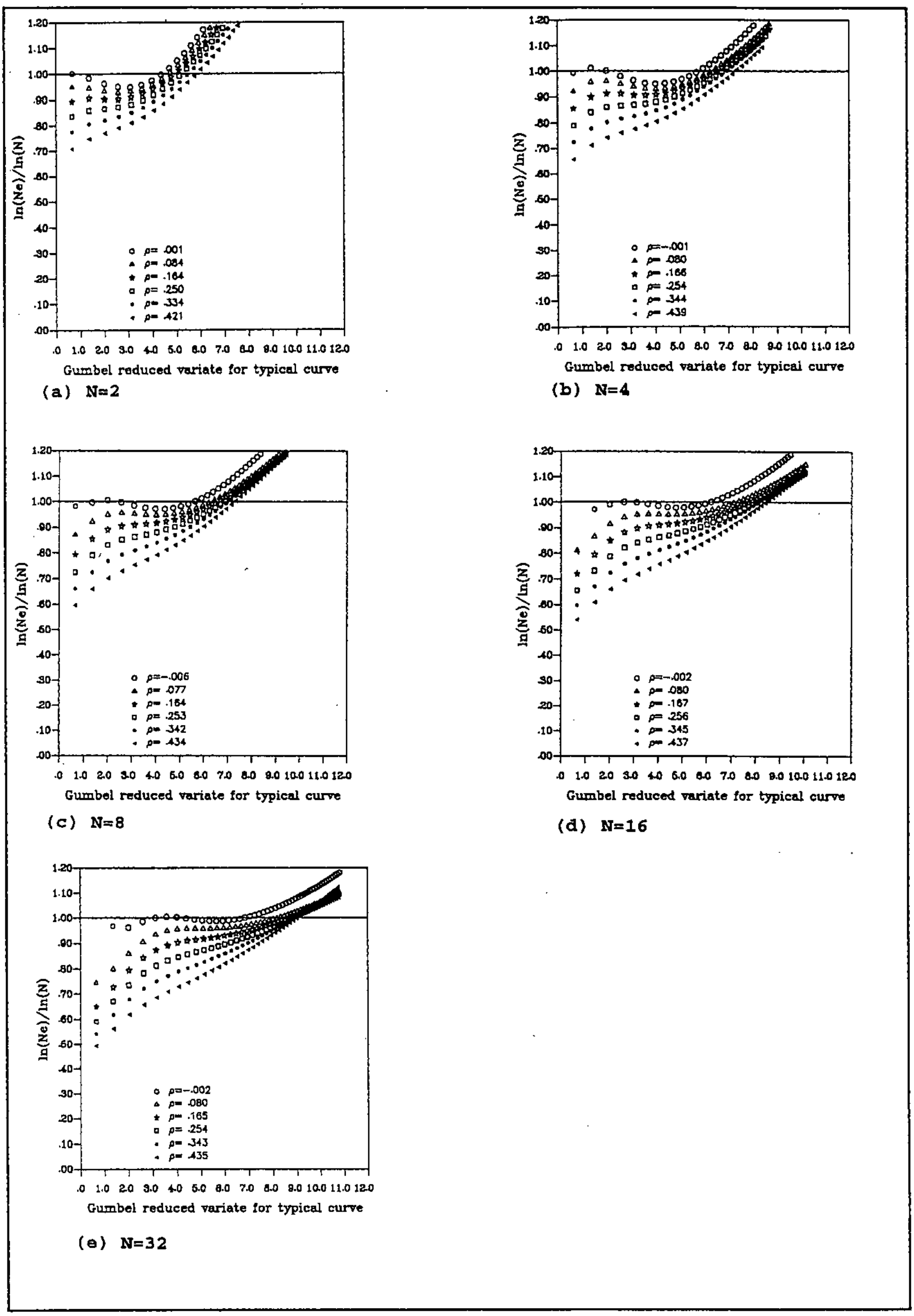


Figure 6.15 Variation of $\ln N_e / \ln N$ for generated data with GLO distribution

6.6.2 Implication of distribution assumption for estimates from the Methods using the FORGE concept

The methods using the FORGE concept (eg. IH-FORGE and CRC-FORGE) obtain the regional growth curve by assigning appropriate plotting positions to highest rainfalls observed. The plotting positions are calculated from a spatial dependence model, using the effective number of independent stations estimated from the regional maximum and the regional average rainfall frequency curves. Thus, in the FORGE concept, the highest observations (normally considered as outliers) shown at Point A are shifted to Point B (Figure 6.1) by a distance AB ($=\ln N_e$) on a Gumbel plot. If the appropriate displacements are applied, the FORGE data points will fall on the fitted regional average curve. For a large homogeneous data set (ie. data from all sites are identically distributed), the FORGE growth curve should be identical to the regional average curve. However, an inappropriate distributional assumption could result in the displacement being too small or too large, thus leading respectively to overestimation or underestimation of the correct regional average curve.

It is evident from comparison of Figures 6.8 and 6.15 that the values of the $\ln(N_e)/\ln(N)$ ratio for the same N, ρ and y values, are similar. This indicates that the displacement between the regional maximum and the regional average curves is not very sensitive to the distributional assumption. In addition, the $\ln(N_e)/\ln(N)$ ratio has a limiting value of unity irrespective of assumed distribution. Thus the distribution of the regional average curve can be determined with some confidence from the distribution of the regional maximum curve, if the latter is known.

As no distribution for the highest standardised annual maxima is assumed in the FORGE concept, the FORGE points tend to be free from any distribution assumption, despite the model for the effective number of independent stations beings based on the GEV distribution.

6.7 CONCLUSION

This chapter described the effects of inter-site dependence on various regional frequency estimation approaches. The concept of effective number of independent stations was investigated for the approaches that pool regional data(eg. methods using the FORGE concept) In addition, the effective number of independent stations computed from regional maximum and regional average curves was shown to be appropriate to use in calculation of plotting positions in the methods using the FORGE concept.

A model for a constant effective number of independent stations based on the average correlation coefficient in a network of stations was developed. This model was found to give more precise estimates of N_e than the Dales and Reed Constant N_e model, used in the IH-FORGE method.

A model for a variable effective number of independent stations was also developed, based on an investigation of generated data. The effective record length of 756 stations with 60 or more years of 1-day rainfall maxima was calculated to be 27,100 and 41,500 using the CRC Constant N_e model and the Variable N_e models respectively; these values are equivalent to 42% and 64% of the total number of station years of data.

7. APPLICATION OF IH-FORGE AND CRC-FORGE METHODS TO VICTORIAN DATA

This chapter describes the application of the IH-FORGE and the CRC-FORGE methods to Victorian data; the aim is to investigate the performance of both of these models. When evaluating these two methods, it should be kept in mind that the IH-FORGE method was not specifically intented for estimation of *extreme rainfalls* in the AEP range aimed for in this study.

In the sections below, the application of the IH-FORGE method to Victorian data is firstly described. This is followed by the validation of the IH-FORGE method using synthetic data, and a description of the resulting modifications. Then the modified FORGE method (CRC-FORGE) is applied to selected Victorian focal stations. The performance of both models is examined using independent rainfall quantile estimates.

7.1 APPLICATION OF THE IH-FORGE METHOD

The IH-FORGE method was applied to the eight selected focal stations described in Section 3.5. For this, at-site annual maxima were standardised using at-site mean annual maxima as in the IH-FORGE method. It might be noted that, in their recent IH-FORGE applications, Stewart et al. (1995) used the median annual maximum; this standardising variable has a fixed frequency of two years and is less sensitive to outliers. The influence of the outliers on the mean is minimal for the length of records used in this study (ie. record length of 60 or more years), thus the difference between two methods of standardisation is assumed negligible.

7.1.1 Fitting a Growth Curve

Reed and Stewart (1989) obtained the growth curves for South West England region by 'eyeball' fits to the data points. In drawing the growth curve, they assigned the theoretical value of the GEV type I distribution for the lower end ie. the curve passes through the point (0.577,1) in a Gumbel plot. The upper end of the curve was drawn in sympathy with growth curve plots for neighbouring points. In this fitting procedure, a fair degree of subjectivity is involved.

In their latest use of the IH-FORGE method, Stewart et al. (1995) first selected a number of segments on the reduced variate axis. Then, in each segment, straight lines were jointly fitted to pooled data and regional maximum data such that each pooled data segment joined up with the previous segment and the slopes of corresponding pooled and regional maximum segments were equal. The separation of the two parallel lines was determined from a spatial dependence model. As this method relies on the number of data points in the sub-samples to draw each segment line, it is expected that uncertainty in these estimates would be higher than fitting a single curve to the whole sample. However, the fitted line is little affected by parametric distribution assumptions. The curve is also forced through the point (0.577,1) on a Gumbel plot, as in the IH-FORGE method.

In the current study, to minimise subjectivity and reduce uncertainty, a GEV type I or II curve was fitted to the plotted FORGE points using a least-square method on logs of the standardised data. The use of a GEV curve is justifiable as it was found to fit Victorian annual maximum rainfall data (Chapter 5). In this procedure, the FORGE concepts aretherefore used as a parameter estimation method to fit the upper tail of a GEV distribution.

Figures 7.1 shows the FORGE growth curves on a log-normal probability plot for the selected focal stations for 1-day annual maxima; for durations 2 and 3 days, the growth curves are given in Appendix A. In these figures, the locations of focal stations are also illustrated. ARR87 1-day point rainfall estimates for annual exceedance probabilties (AEPs) of 1 in 50 and 1 in 100 are shown for comparison.

7.1.2 Removal of Bias in Fitting of Growth Curves

One of the drawbacks of the IH-FORGE method is that the growth curve is biased when the focal point is close to the rain gauge station where the highest rainfall in the region was observed. This is due to the fact that the highest observed values in the overall region are also included in the smaller regions used in the initial FORGE steps.

Because the same highest annual maximum is plotted at different exceedance probabilities (depending on the size of the region), the fitted growth curve is likely to shift to the left, as shown in Figure 7.2a. This highest annual maximum is really an outlier for lower recurrence intervals (smaller regions) and hence this point should be screened out before fitting the growth curve.

The identification of the outlier can be based on the spread of a group of points which have similar standardised rainfall values, in the direction of the exceedance probability axis. If the separation between a point and the extreme point on the right within this group is greater than a set limit, the point on the left can be assumed to be an outlier for the particular FORGE step, and be screened out for the curve fitting.

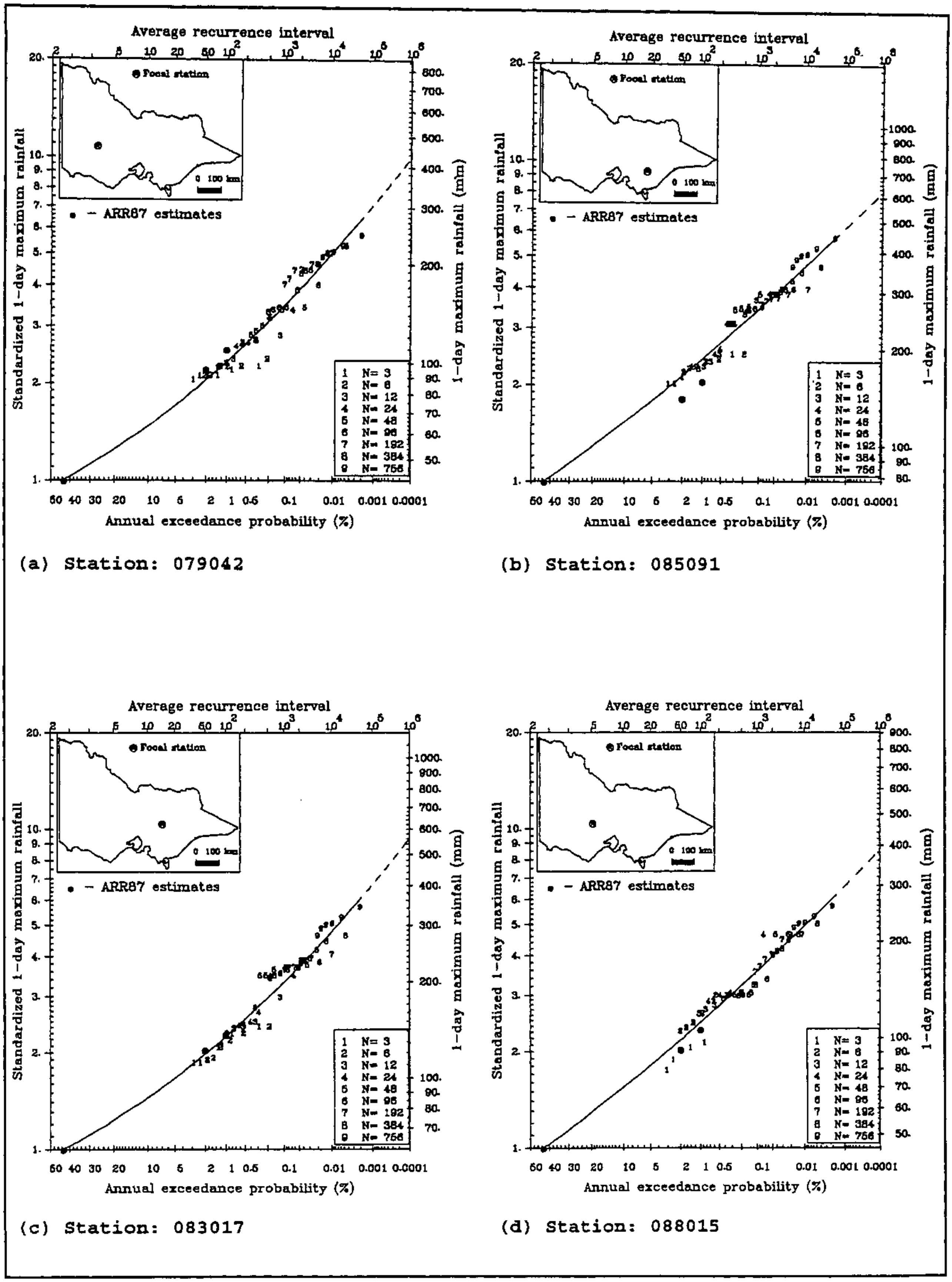
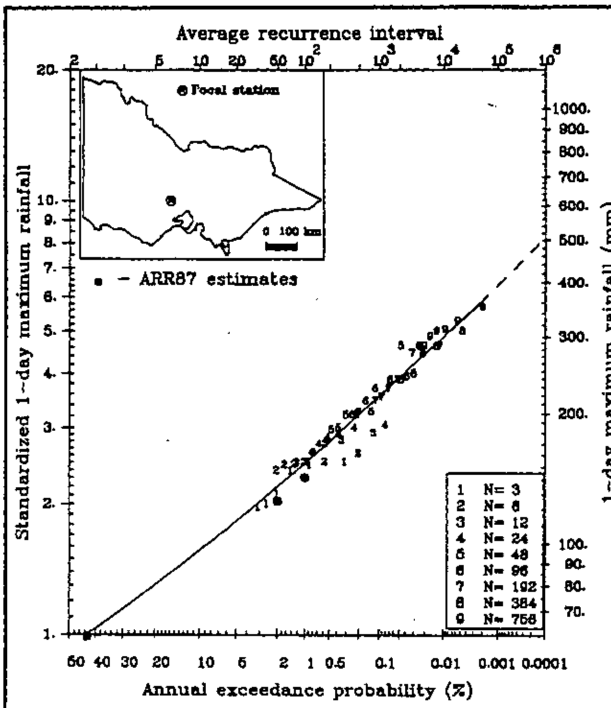
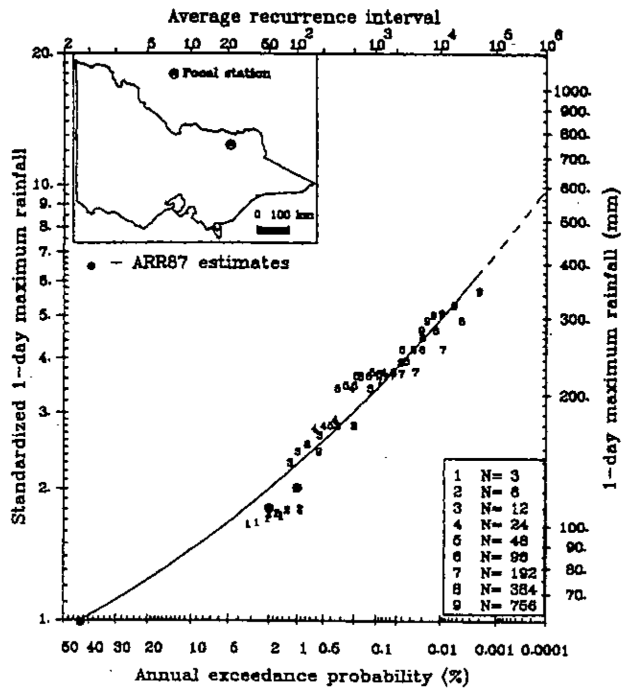


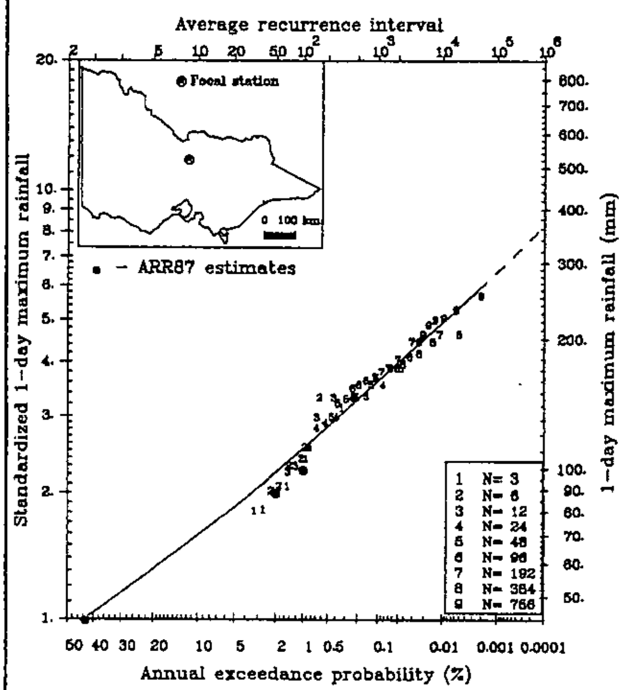
Figure 7.1 IH-FORGE growth curves for 1-day rainfall maxima at selected Victorian stations (continued overleaf)



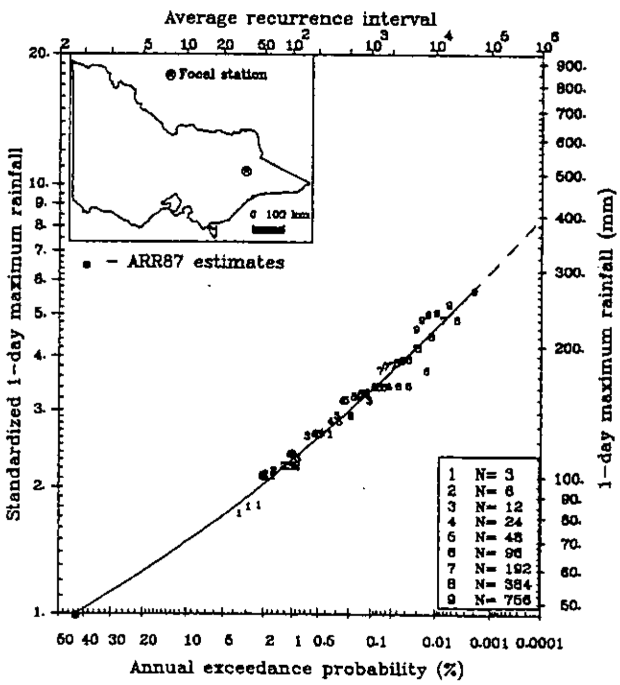
(e) Station: 087017



(f) Station: 082058



(g) Station: 081043



(h) Station: 083025

Figure 7.1 (continued) IH-FORGE growth curves for 1-day rainfall maxima at selected Victorian stations

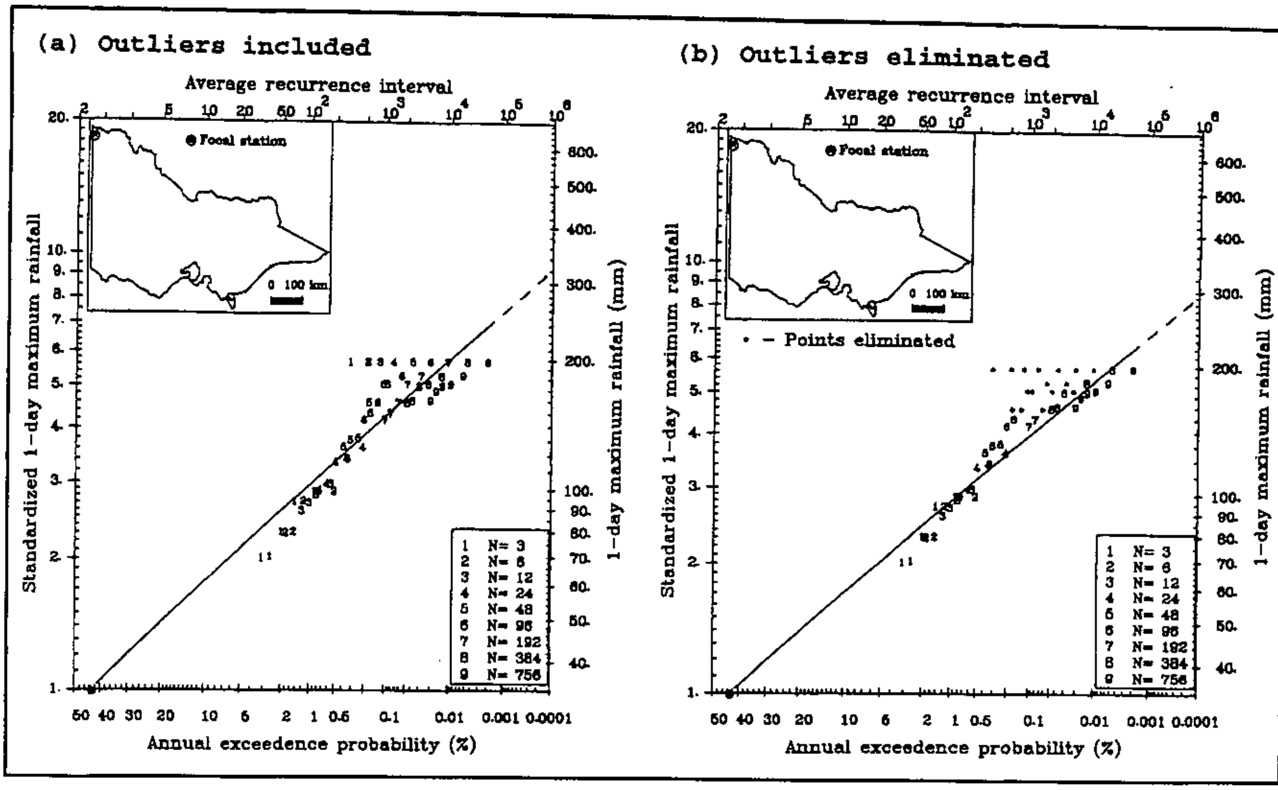


Figure 7.2 Effects of outliers on IH-FORGE growth curve at station 024015

Figure 7.2b shows the growth curve fitted to the FORGE points after outliers were screened out. For this, the set limit of separation was assumed to be a value of 1.16 on a Gumbel plot; this value was adopted as it represents the average of the standard deviations of residuals of FORGE points (along the Gumbel reduced variate axis) obtained for a number of focal points. Therefore, the points located further than 1.16 Gumbel reduced variate units to the left of the plotting position corresponding to the largest region were neglected in the fitting of the distribution. The omitted points are shown as stars on Figure 7.2b.

7.1.3 A Comment on Selecting Six Data Points for each FORGE Sub-region

In the IH-FORGE method, the adopted number of data points (the six largest values) to be selected in each FORGE sub-region was not justified by any theoretical argument. However, it is clear that the number should be (i) large enough to make use of the information from the highest rainfall events in the region and (ii) small enough that the effective number of stations (N_e) for the computation of plotting positions can be assumed to be the same for all the data points selected from the sub-region.

The selection of six values appears to be a reasonable compromise between these two competing objectives. Detailed sensitivity studies would be required to allow the selection of an optimum number of data points.

7.2 MODIFICATIONS TO THE IH-FORGE METHOD BASED ON THE RESULTS OF DATA GENERATION

The FORGE concept involves pooling of data from regions of increasingly larger size to estimate point rainfalls of smaller frequency. Although the FORGE concept appears to be justifiable for uncorrelated data, it has not been validated for correlated data. The following sections validate the IH-FORGE method using generated data with known population parameters and inter-site dependence.

7.2.1 Validation of the IH-FORGE Method

The use of generated data with known population parameters ensures that the assumption of homogeneity (ie. the regional data is identically distributed) is satisfied. It also allows to check whether the IH-FORGE method reproduces the population growth curve.

To identify any systematic anomaly in the IH-FORGE method, the generated regional data was assumed to have the same degree of spatial dependence, ie. the correlation coefficient between the annual maxima of all pairs of stations in the region was assumed to be a known constant. In reality, however, the correlation between rainfall maxima decreases with increasing distance; thus the average correlation coefficient decreases with increasing size of a region. It also depends on the distribution of the rain gauges within a 'region'. However, the assumption that the correlation between all annual maxima in a region is constant does not contradict the FORGE concept as it still obtains more information from all the extremes by pooling the data.

The data sets generated in Section 6.4.2 were used for the IH-FORGE validation. As the IH-FORGE estimates are very sensitive to the highest rainfalls of the pooled data, the data generated with different correlation coefficients were checked for any systematic differences and were found to be free from any bias.

The IH-FORGE method was applied to 99 replicates of generated data of 1000 years. For each replicate, the first of 48 stations was considered as the focal station and the region size was progressively increased to include more stations. In the IH-FORGE applications, N_e^c was calculated using the CRC Constant N_e model given by Eq. 7.1; the coefficients of this equations were calibrated using the generated data set. A GEV distribution was fitted to the FORGE points. The parameters of the distribution were determined from a least square fit on a probability plot, as explained in Section 7.1.1.

$$\frac{\ln N_e^c}{\ln N} = 1.015 - 0.821\bar{\rho} \qquad r^2 = 0.999 \qquad (7.1)$$

The performance of the IH-FORGE method is evaluated by comparing the rainfall quantiles calculated from the parameters of the growth curve for each replicate with the quantiles calculated from the parameters of the average growth curve. As the parameters of the parent distribution were preserved well in the generated data (see Table 6.2), the parent distribution and the average growth curve were coincident (illustrated later in Figure 7.5). The parameters of the average growth curve were estimated using the probability weighted moment approach given by Dales and Reed (1989).

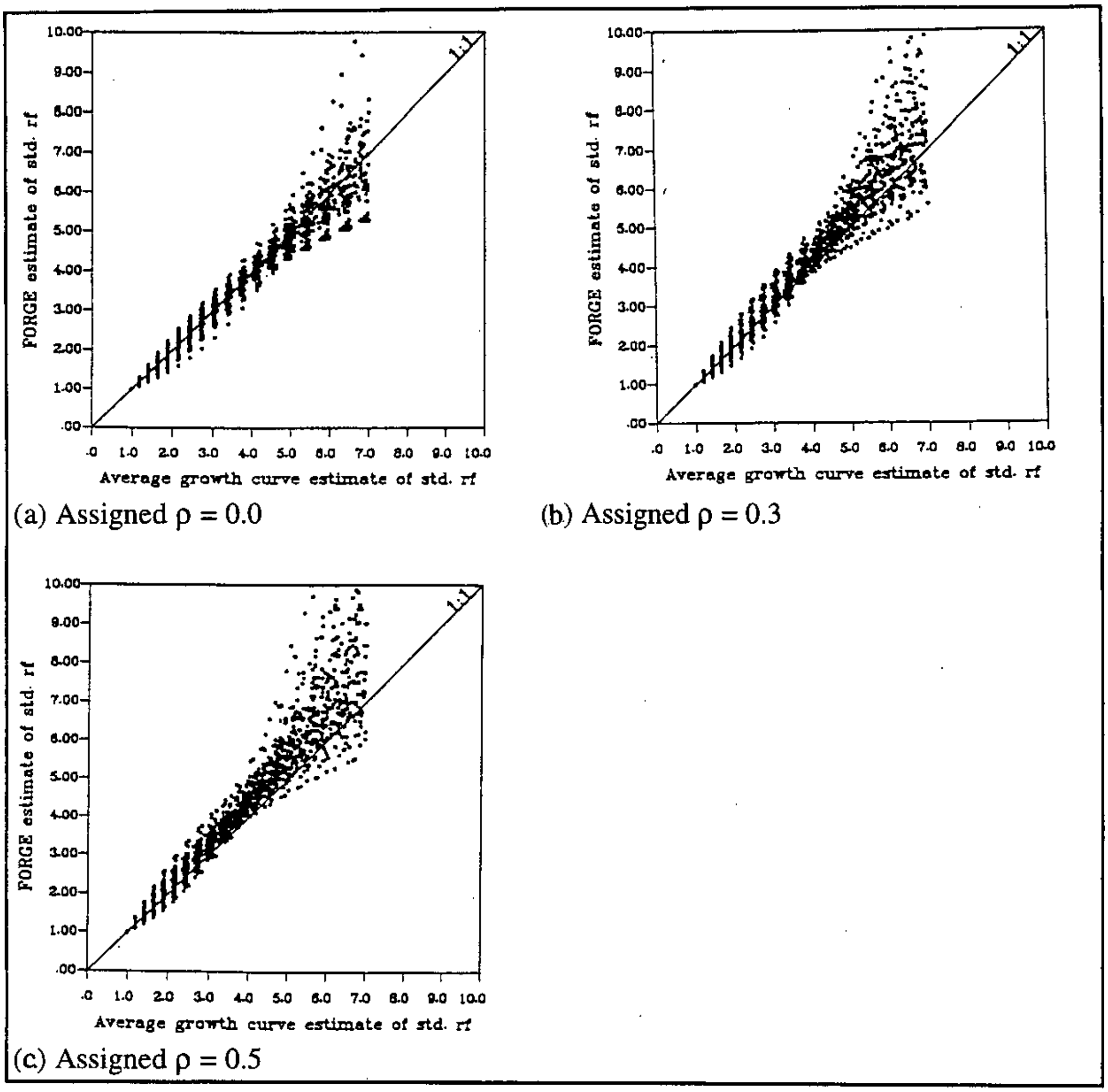
In Figure 7.3, standardised rainfalls calculated from the average growth curve for a range of annual exceedance probabilities (AEPs)are compared with those estimated by the IH-FORGE method for the corresponding AEP. The IH-FORGE growth curve and the average growth curve agree reasonably well and without bias for independent data (correlation coefficient = 0.0), although there is significant scatter for the large quantiles. However, on average, the IH-FORGE method significantly overestimates the extremes for spatially correlated annual maxima. The degree of overestimation increases with increasing correlation coefficient. The source of this overestimation appears to be an underestimated N_e using the CRC Constant N_e model (Eq. 7.1) for low AEPs, as the extreme events tend to be relatively independent (Buishand, 1984 and Pescod, 1990). The development of a Variable N_e model, which allows for reduced spatial dependence of extreme events, was described in Section 6.4. The performance of this Variable N_e model will now be evaluated.

7.2.2 Validation of the IH-FORGE Method with the Variable N_e Model

The IH-FORGE method with the Variable N_e model (Eq. 6.18) was applied to the generated data set. In the IH-FORGE method, the effective record length of pooled data is calculated using a modified station-year method (see Section 3.5). This needs to be modified, as the Variable N_e model requires the effective record length to calculate y from

$$y = -\ln -\ln[1-AEP(X_1)] \qquad (7.2)$$

where $AEP(X_1)$ is the plotting position of the rank 1 observation for the pooled data from the selected FORGE sub-region, as determined from the Cunnane plotting position formula. This calls for an iterative approach, as described below.



(a) Assigned ρ = 0.0

(b) Assigned ρ = 0.3

(c) Assigned ρ = 0.5

Figure 7.3 Comparison of IH-FORGE growth curve estimates and average growth curve estimates (from 99 generated sequences of 1000 years)

An iterative procedure to calculate the variable N_e

The initial effective record length L_e^0 is assumed to be the total number of station years of pooled data, and this is used to calculate an initial value of the Gumbel reduced variate y^0.. An initial value of N_e^0 can now be calculated from Eq. 6.18 for each year, and these values are added to obtain a new effective record length L_e^1. This revised effective record length is then used to calculate the a new Gumbel reduced variate value y^1. This procedure is repeated until there is no significant difference in successive values of the effective record length (or Gumbel reduced variate) estimates.

Comparisons

Figure 7.4 summarises the results of the IH-FORGE applications to the generated data for correlation coefficients of 0.0, 0.3 and 0.5. From the comparison of Figures 7.3 and 7.4, it is clear that, after the introduction of variable N_e, bias in IH-FORGE estimates is reduced for these three values of the correlation coefficient. This illustrates the need for use of the Variable N_e model in IH-FORGE applications with real data.

Use of focal point data

A close examination of Figure 7.4 reveals that the scatter near the lower end seems to be high, which indicates relatively large uncertainty in the IH-FORGE estimates for -high to medium AEPs, and could also have an effect on estimation for -low AEPs. This can be clearly seen in Figure 7.5a in which the fitted growth curve significantly deviates from the average growth curve. To eliminate this deviation, it was decided to introduce data from the focal point in the growth curve fitting procedure. If the data is not available for the focal point, at-site data from the nearest (or most similar) stations could be used instead.

Figure 7.5b shows that the IH-FORGE growth curve is close to the average growth curve, after introducing data from the focal point in the curve fitting. This modification thus makes the procedure less sensitive to data points in the extreme tail of the distribution. To give equal weights to both the regional and the at-site data, the number of data points introduced from the focal station was limited to the number of FORGE points (from the regional data). Only the highest ranking data points from the focal station were introduced to give weight to low frequency at-site data.

Figure 7.6 shows that the introduction of focal point data reduced the scatter at high AEPs. In addition, the introduction of at-site data reduced the tendency of the fitted curve to adjust to the largest regional observations. This produced more stable estimates of low frequency events - some of the largest events now seem to be treated as outliers in the fitting of the curve.

7.2.3 Summary of Proposed Modifications to the IH-FORGE Method

The modifications to the IH-FORGE method described in the previous sections appear to have the potential to improve its estimates significantly; the following modifications are therefore suggested.

Estimation of the plotting positions for FORGE data points:

(i) use of the Variable N_e model instead of a constant N_e model (Dales and Reed Constant N_e model or CRC Constant N_e model) to estimate the plotting positions of FORGE data points (Section 7.2.2).

Fitting of a distribution line:

(ii) inclusion of focal point data together with FORGE data points Section 7.2.2),

(iii) screening of outliers among the FORGE data points (Section 7.1.2),

(iv) fitting of a GEV distribution by a least squares method, rather than an eye-ball fit of a non-parametric distribution line (Section 7.1.1).

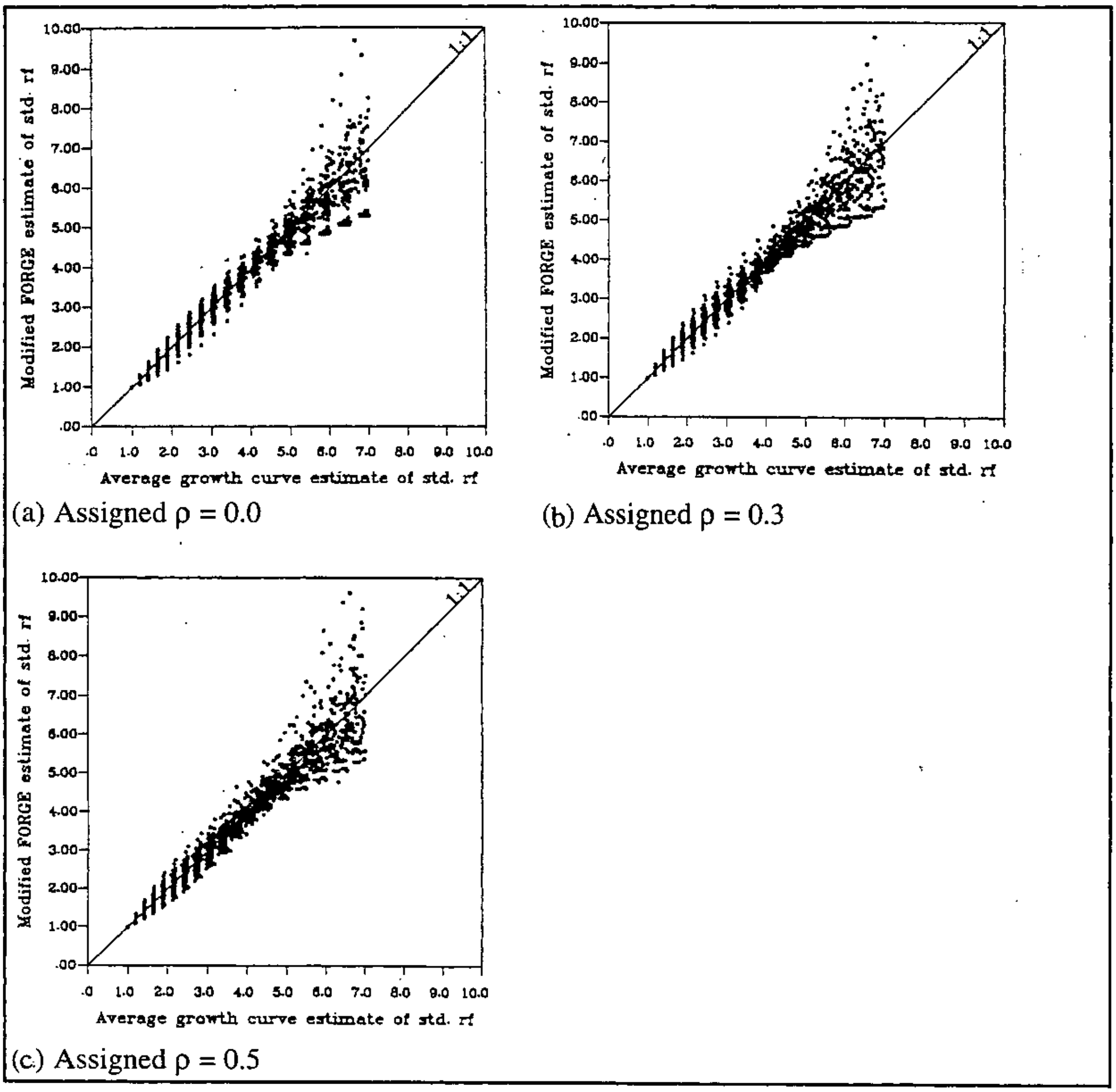
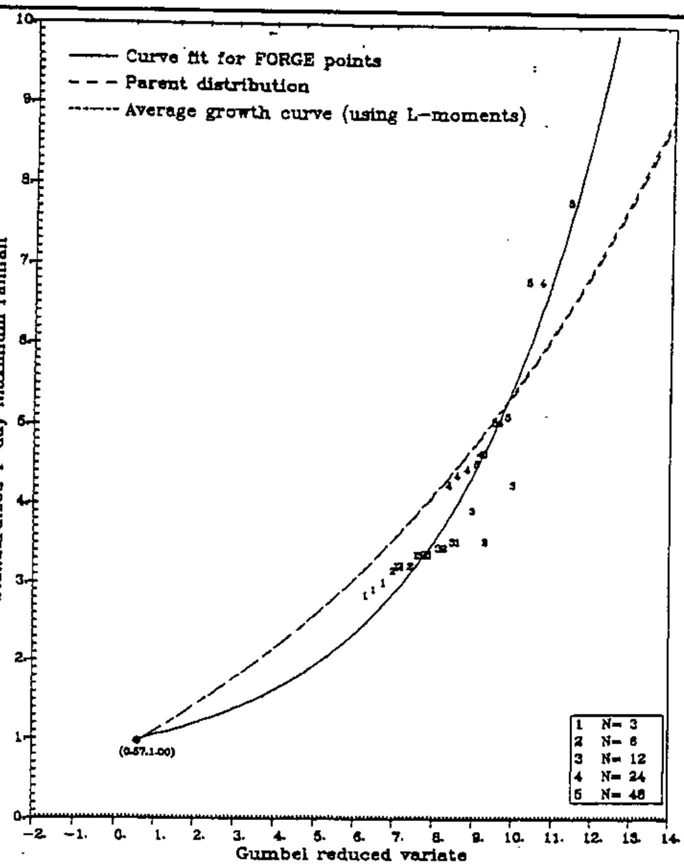
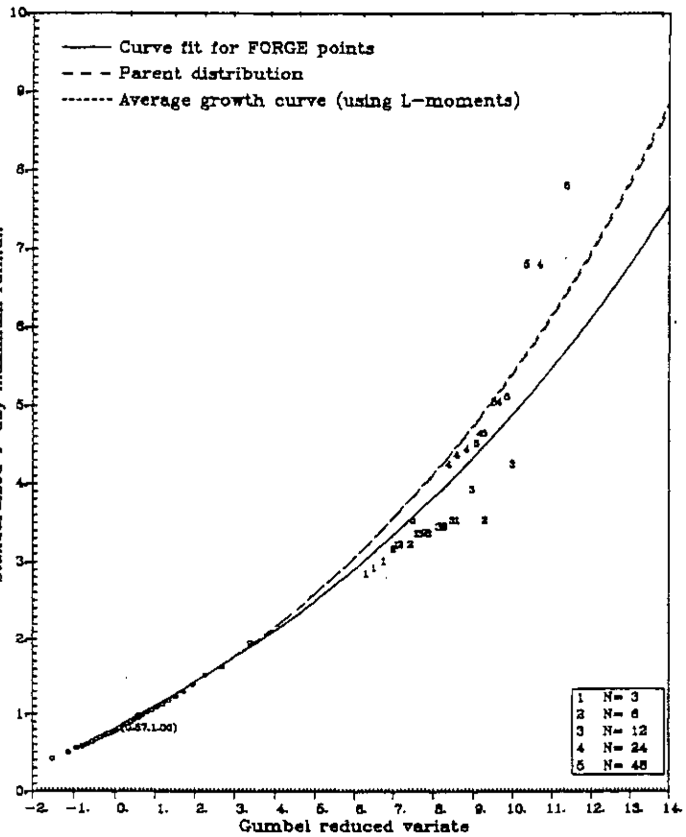


Figure 7.4 Comparison of modified IH-FORGE growth curve estimates and average growth curve estimates (from 99 generated sequences of 1000 years) [Note the bias at low probabilities been significantly reduced in compared with Figure 7.3]



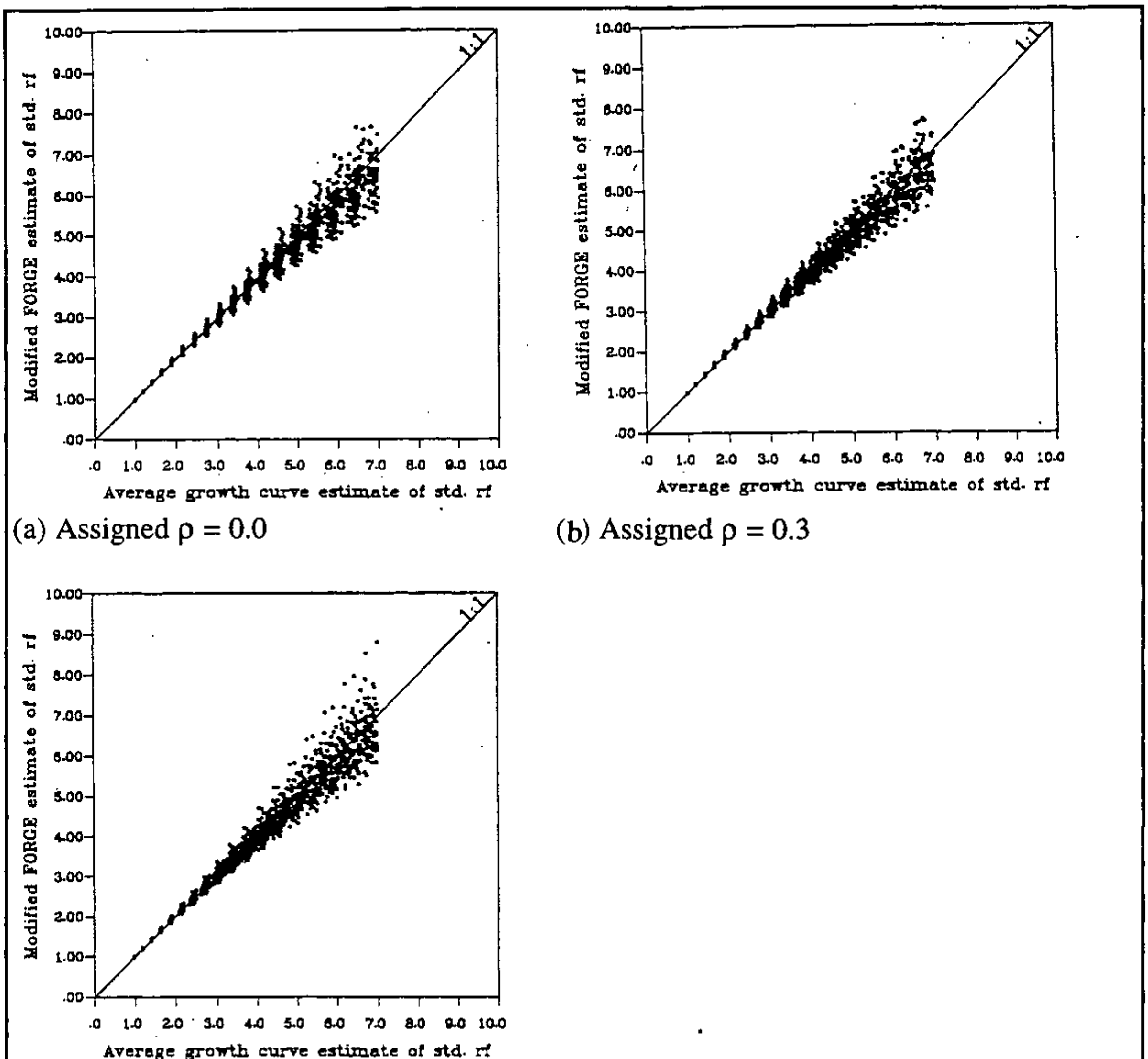
(a) Before introduction of focal point data



(b) After introduction of focal point data

Figure 7.5 Modified IH-FORGE growth curve for Replicate No. 10 (correlation coefficient 0.0)

The above modifications were adopted to improve the IH-FORGE method for the applications described in the following section. The improved FORGE method is hereafter called the *CRC-FORGE Method.* A list of all the steps in the CRC-FORGE design rainfall estimation procedure is given in Appendix D.



(a) Assigned ρ = 0.0

(b) Assigned ρ = 0.3

(c) Assigned ρ = 0.5

Figure 7.6 Comparison of modified IH-FORGE growth curve estimates and average growth curve estimates after introducing focal point data for growth curve fitting (from 99 generated sequences of 1000 years)

7.3 APPLICATIONS OF THE CRC-FORGE METHOD TO VICTORIAN DATA

The CRC-FORGE method was applied to Victorian data (Data Set II). Figures 7.7 (a) to (h) show the CRC-FORGE growth curves for 1-day annual maxima; for durations of 2 and 3 days, the growth curves are given in Appendix A. As for the application of the IH-FORGE method described in Section 7.1, the distribution of the FORGE data points was assumed to be GEV and the parameters of the curves were determined from a least square fit in log space giving equal weights to all points. Also shown in the figures are the I.E. Aust. (1987) [ARR87] point rainfall estimates for AEPs of 1 in 50 and 1 in 100.

7.3.1 Comparison of CRC-FORGE Estimates with Independent Design Rainfall Estimates

Comparison with ARR87 Estimates

For the selected stations, Figures 7.8, 7.9 and 7.10 compare the original and CRC-FORGE estimates with ARR87 estimates for AEPs of 1 in 50 and 1 in 100 for 1, 2 and 3 day maxima respectively. As expected, the CRC-FORGE estimates of 1-day rainfall for a given AEP are consistently lower than the IH-FORGE estimates for the same AEP, due to the higher N_e values used in the former. However, this is not always the case for very extreme rainfalls for the other two durations, as the effects of increased $N_e$values were offset by introduction of focal point data and by elimination of outliers (see Figures B.3 and B.4 in Appendix B).

On average, the CRC-FORGE estimates are closer to the ARR87 estimates than the IH-FORGE estimates. This improvement was achieved mainly by the introduction of focal point data. In most cases, the differences between the CRC-FORGE and the ARR87 estimates are marginal and can be explained by the regional smoothing applied to obtain the coefficients for estimation of the AEP 1 in 50 and 1 in 100 rainfalls in ARR87. For example, the differences appear to be higher for stations 085091 and 083017 which are located near alpine areas where the raingauge density is low. Two adjustments made to the at-site values in ARR87 are: (i) a statistical adjustment to allow for the influence of stations with short records and (ii) a meteorological adjustment when drawing contours to the data points (personal communication Canterford, 1995).

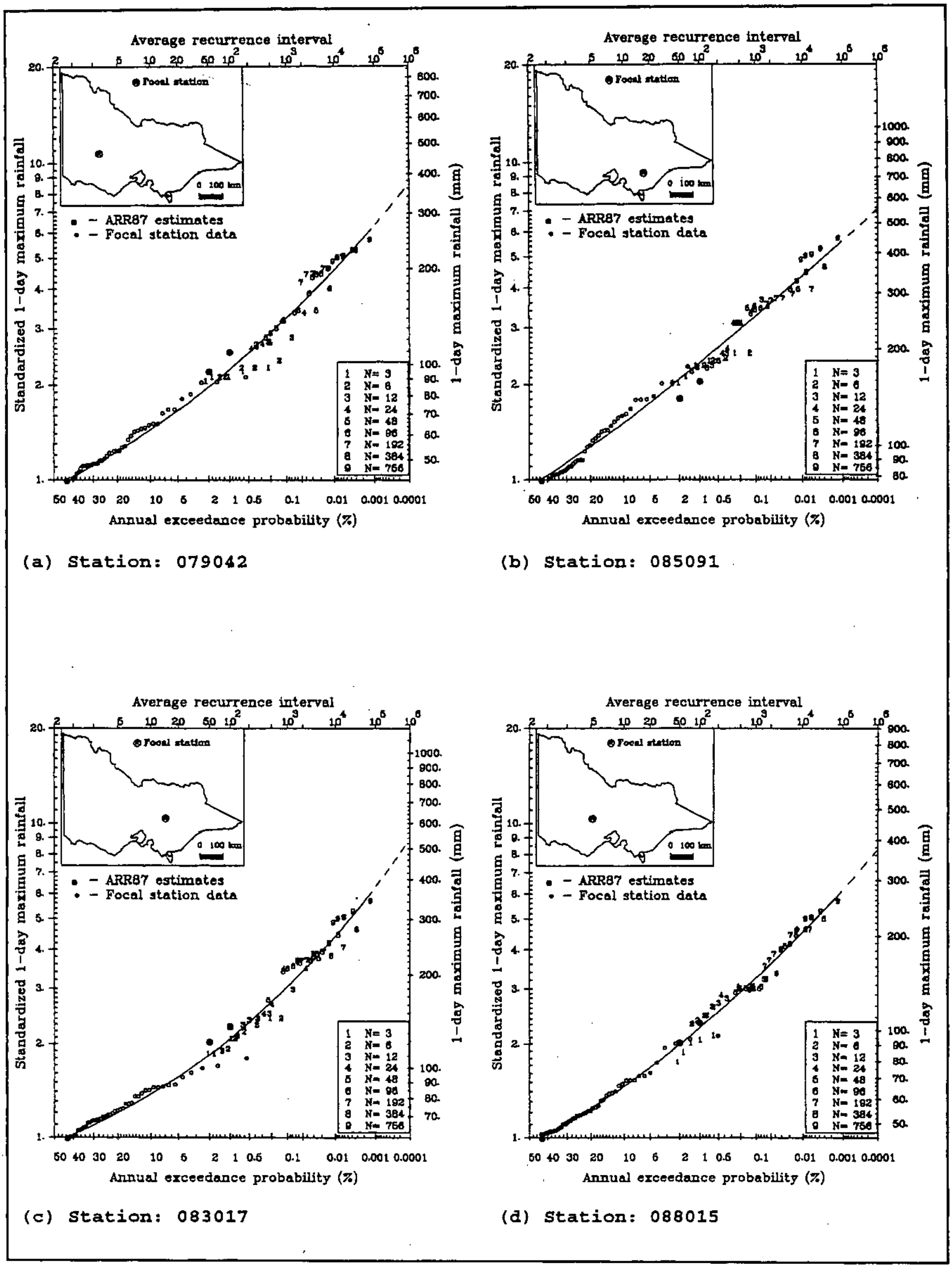


Figure 7.7 CRC-FORGE growth curves for 1-day rainfall maxima at selected Victorian stations (continued overleaf)

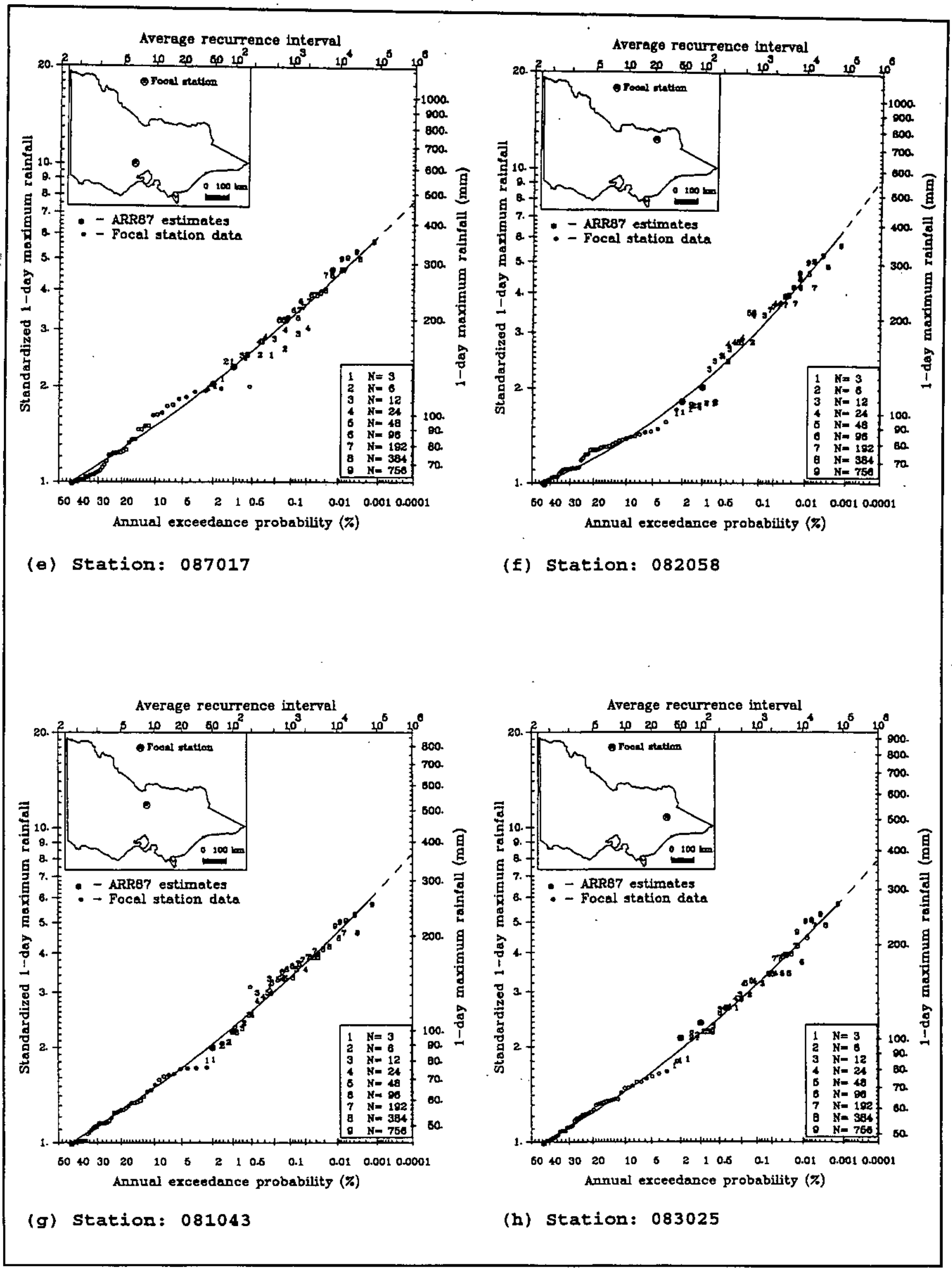


Figure 7.7 (continued) CRC-FORGE growth curves for 1-day rainfall maxima at selected Victorian stations

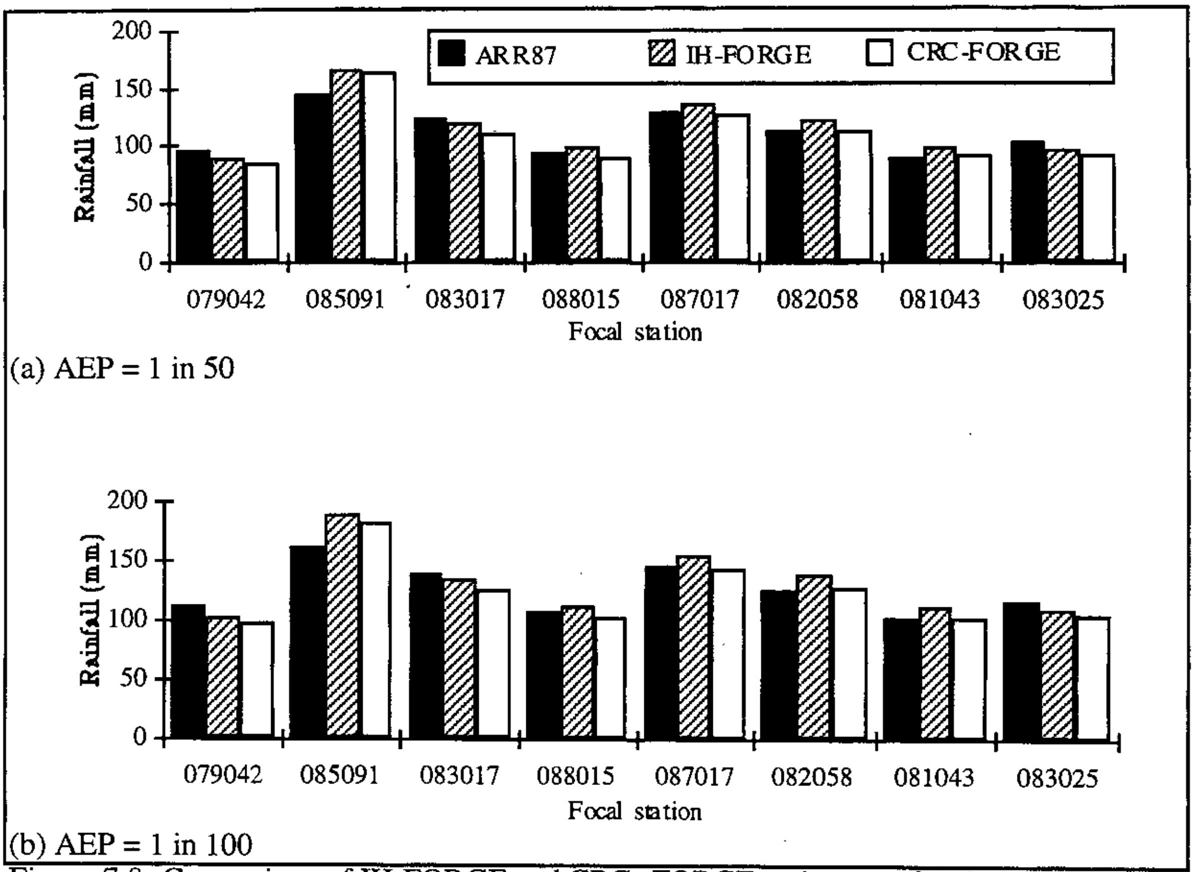


Figure 7.8 Comparison of IH-FORGE and CRC- FORGE estimates of 1-day maxima with ARR87 estimates

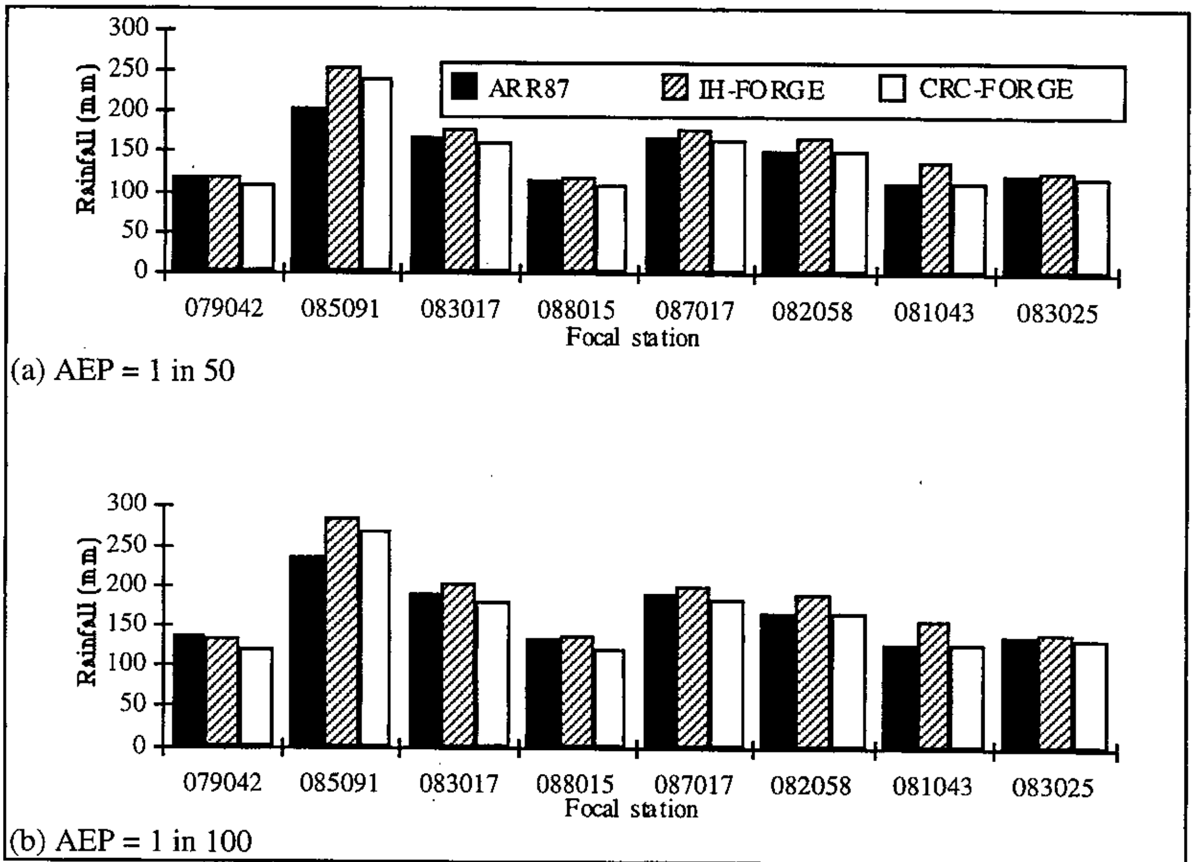


Figure 7.9 Comparison of IH-FORGE and CRC-FORGE estimates of 2-day maxima with ARR87 estimates

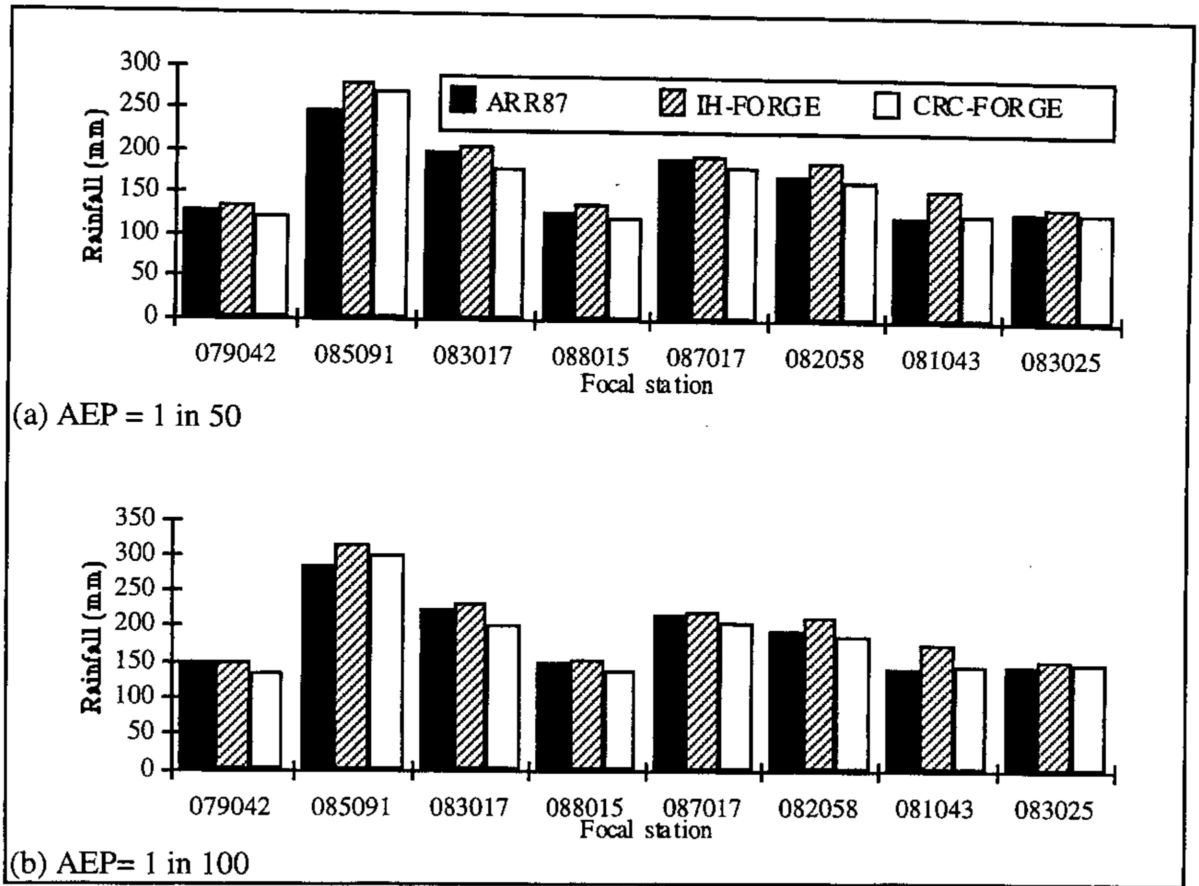


Figure 7.10 Comparison of IH-FORGE and CRC- FORGE estimates of 3-day maxima with ARR87 estimates

Comparison with Other Extreme Design Rainfall Estimates

McConachy (1996) applied the modified Schaefer method and three other regional rainfall frequency estimation methods to Victorian rainfall data (Data Set I). For the eight sites shown in Figure 7.7 the estimates of the AEP 1 in 2000 design rainfall obtained by the modified Schaefer method were all within ± 10% of the estimates from the CRC-FORGE method. The corresponding results from the three other methods, at the four sites they were applied to, were also quite close.

The results of the above comparisons confirm that the CRC-FORGE method is able to produce consistent design rainfall estimates for Victorian sites for an AEP range of 1 in 50 to 1 in 2000.

7.4 SENSITIVITY OF CRC-FORGE ESTIMATES TO DELINEATION OF REGION

For the application of the FORGE concept, the whole of Victoria was assumed to constitute a homogeneous region (Section 5.2.3). However, to estimate probable maximum precipitation, the Bureau of Meteorology divided the state into two homogeneous sub-regions, namely the GSAM Inland and Coastal Zones (BOM, 1996). This section examines the sensitivity of CRC-FORGE estimates to the assumed boundaries of homogeneous regions.

Figures 7.11 compares the growth curves obtained from a region covering the whole of Victoria (plus adjoining areas of NSW and SA), with those obtained from an appropriate GSAM Zone (either Inland or Coastal). From these figures it is evident that differences between the curves derived for the whole region and those derived for GSAM sub-regions are negligible compared to uncertainties in the estimates (Chapter 8). Accordingly, the assumption that Victoria is a homogeneous region is reasonable for extreme rainfall estimation purposes.

7.5 CONCLUSION

The IH-FORGE method was applied to the eight selected Victorian focal stations, and rainfall estimates of 1 in 50 and 1 in 100 AEP produced by it were found to be comparable with ARR87 estimates.

Using generated data, the IH-FORGE method was shown to overestimate extreme rainfalls for spatially correlated data. The use of the Variable N_e model in the IH-FORGE method was found to eliminate bias in its estimates. In addition, the introduction of focal point data also improved the quantile estimates. From the validation using synthetic data, the following modifications were made to the IH-FORGE method:

(i) use of the Variable N_e model instead of a constant N_e model to estimate plotting positions of FORGE data points (Section 7.2.2);

(ii) inclusion of focal point data together with FORGE data points (Section 7.2.2);

(iii) screening of outliers among the FORGE data points (Section 7.1.2);

(iv) fitting of a GEV distribution by a least squares method, instead of an eye-ball fit of a non-parametric distribution line (Section 7.1.1).

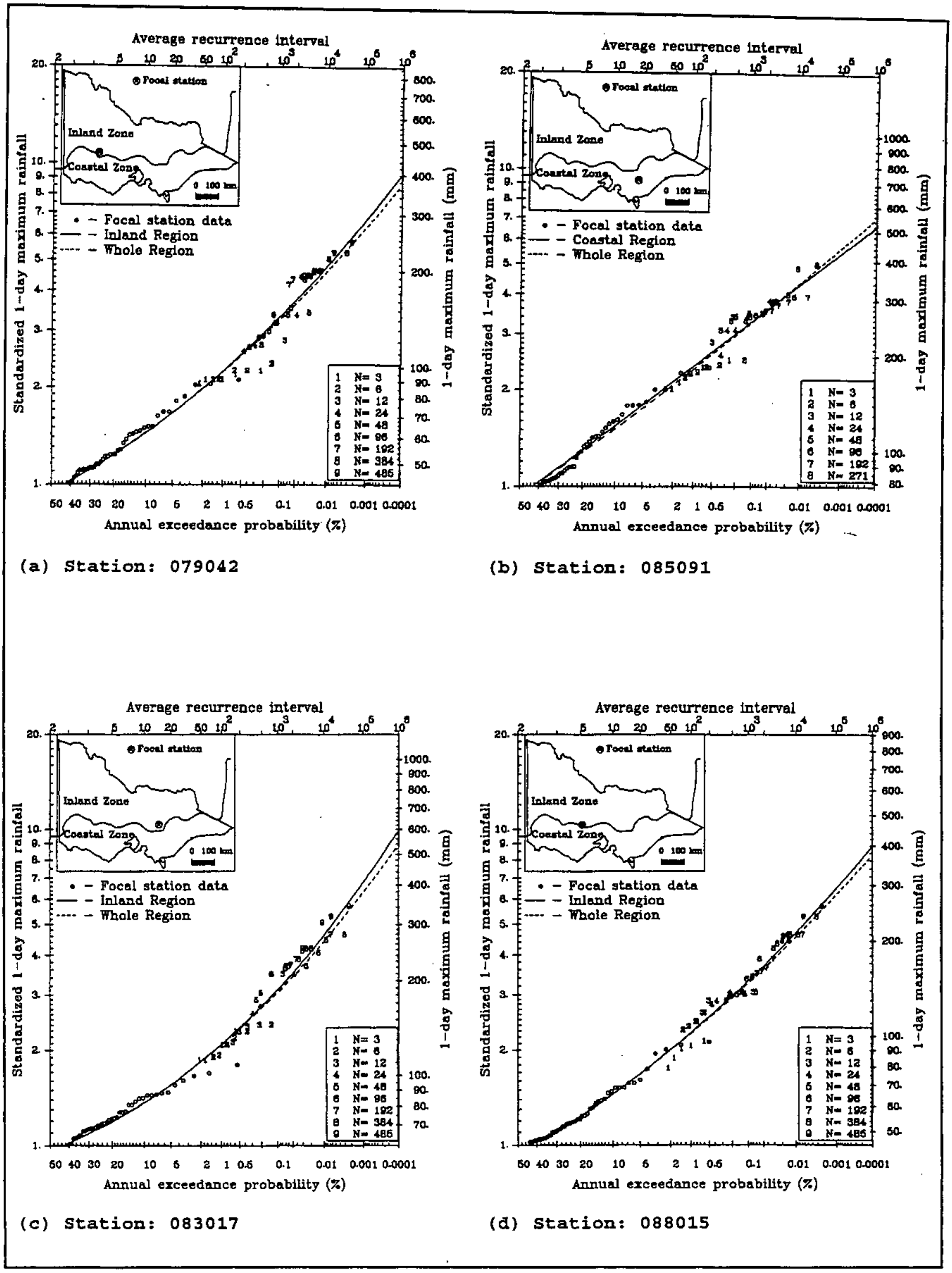
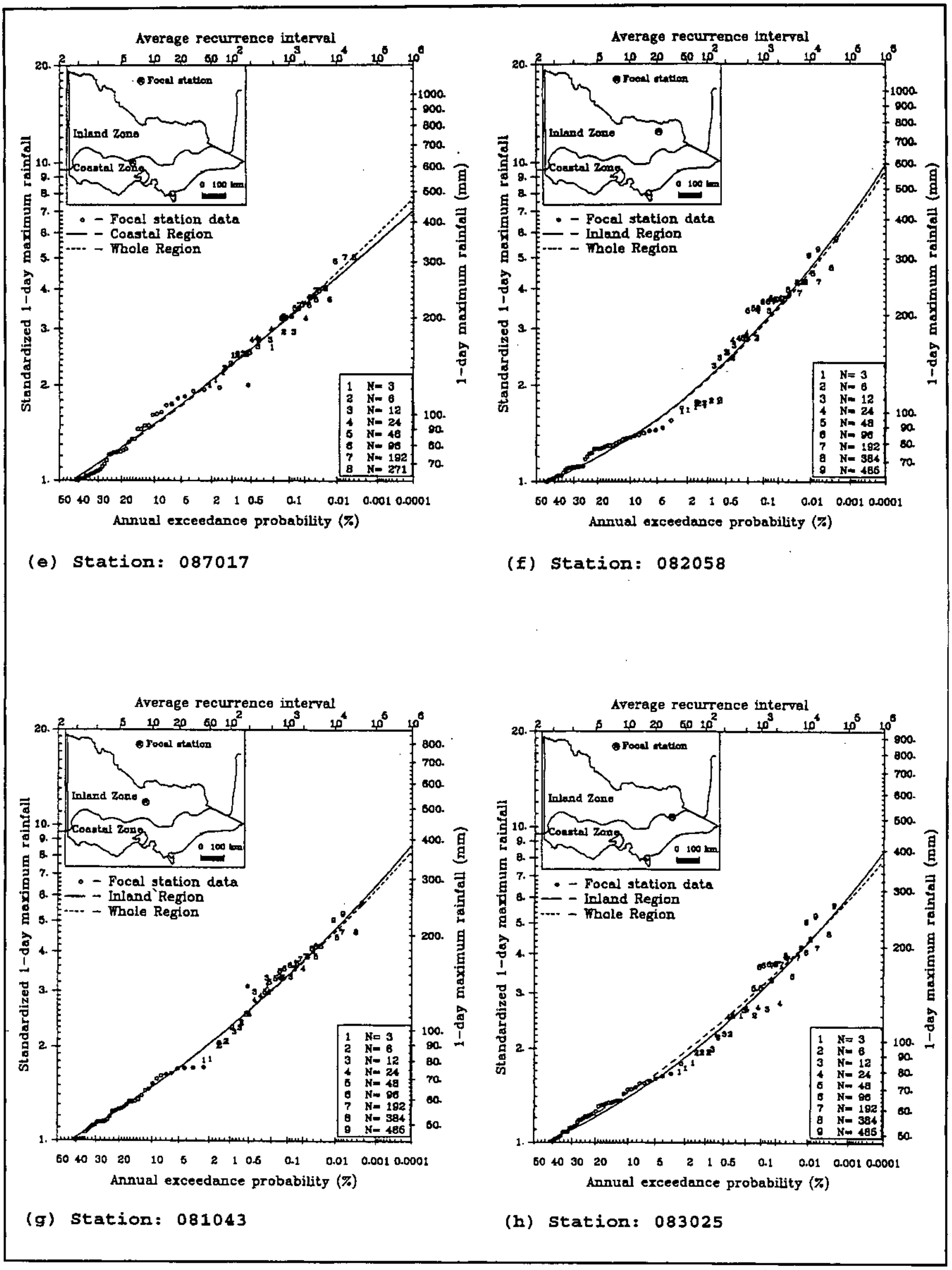


Figure 7.11 Comparison of CRC-FORGE growth curves for a region covering the whole of and for an appropriate GSAM Zone [plotted FORGE points for GSAM regions] (continued overleaf)



(e) Station: 087017

(f) Station: 082058

(g) Station: 081043

(h) Station: 083025

Figure 7.11 (continued) IH-FORGE growth curves for 1-day rainfall maxima at selected Victorian stations [plotted FORGE points for GSAM region]

The CRC-FORGE method's estimates were found to be (i) generally lower than the IH-FORGE estimates and (ii) comparable with estimates from ARR87. The CRC-FORGE estimates for 1 in 2000 AEP were also consistent with results reported by McConachy using four different regional rainfall frequency estimation methods.

8. ESTIMATION OF UNCERTAINTY IN CRC-FORGE ESTIMATES

8.1 INTRODUCTION

Rainfall frequency estimates are subject to errors from a number of sources. The aim of this chapter is to estimate uncertainty in the CRC-FORGE growth curve due to the most important of these errors.

The reliability of frequency estimates is generally measured by confidence limits to the frequency curve. The uncertainty in the quantile estimates depends on the underlying assumptions; larger uncertainty is generally associated with less restrictive assumptions (Rosbjerg and Madsen, 1995). Inappropriate assumptions have the potential to result in systematic errors (bias) especially in the region of extrapolation.

The two major assumptions of homogeneity and distribution choice involved in the application of the FORGE concept, were dealt with in Chapter 5. The implications of the distributional assumption on N_e and thus on the plotting position estimates were discussed in Chapter 6 (Section 6.5). This chapter presents the derivation of confidence limits for the CRC-FORGE growth curve to account for *sampling errors* in the estimate of N_e and in the estimated parameters of the fitted growth curve.

The next section presents the uncertainty estimation methods used in this study. The derivation of the 90% confidence limits for the rainfall frequency curves at the eight selected focal stations is described in Section 8.3.

8.2 METHODS OF UNCERTAINTY ESTIMATION

The models (or equations) for the variable N_e and for fitting the growth curve were calibrated using a least-squares method, after appropriate transformation of the model response. If the residuals satisfy the least-squares assumptions that they (i) are independent from each other, (ii) are normally distributed, and (iii) have constant variance, the posterior distribution of model parameters can be estimated from the observed data (Kuczera, 1983). Using the posterior distribution of parameters, a Monte-Carlo simulation approach can be used to derive the confidence limits for the estimated growth curves. The derivation method of the posterior distribution of model parameters and the Monte-Carlo simulation method are described in the following sections.

8.2.1 Posterior Distribution of Model Parameters

Assume that a model takes the following form

$$g_i = f(\mathbf{x}_i;\beta) + \varepsilon_i \tag{8.1}$$

where g_i is the i^{th} observed value, f( ) is the model function, $\mathbf{x}_i$ is a vector of inputs corresponding to i^{th} observed value, β is a parameter vector and ε_i is the i^{th} residual. The residual ε is a normally distributed random number with a mean of zero and a variance of σ^2.

Given that the residuals satisfy the least-squares assumptions, it can be shown that the probability distribution function of the parameter vector β for this model is well approximated by a multivariate normal distribution with mean β_o and covariance matrix $\sigma^2[\Gamma_o^T \Gamma_o]^{-1}$, where Γ_o is a matrix of derivatives evaluated at β_o (Kuczera, 1983).

8.2.2 Monte-Carlo Simulation

A Monte-Carlo simulation method can be used to determine the effects on model response of input or model errors. A parameter vector is randomly sampled for a required number of times (say N_{samp}) from the posterior distribution, and the performance of the model evaluated for each sampled parameter vector. A random error $[0,\sigma]$ is added to the response.

The N_{samp} responses for a given input are ranked and $100\alpha\%$ and $100(1-\alpha)\%$ percentiles are referred to as the $100(1-2\alpha)\%$ prediction limits. If the random error is not added to the response, the ranked responses result in the $100(12\alpha)\%$ confidence limits.

8.3 DERIVATION OF CONFIDENCE LIMITS FOR THE CRC-FORGE GROWTH CURVES

If basic assumptions on the distribution and homogeneity of data are satisfied and daily rainfall readings are assumed to be free from any reading or processing errors, uncertainties in the CRC-FORGE estimates are mainly attributed to the parameter estimation errors in the variable N_e model and in the growth curve fitting model. The following sections describe the derivation of the confidence limits for the selected focal stations due to these errors.

8.3.1 Posterior Distribution of Variable N_e Model Parameters

The variable N_e model relates N_e to the average correlation coefficient ($\bar{\rho}$), the number of stations in a network (N), and the annual exceedance probability via the Gumbel reduced variate (y) (Eq. 6.18, Chapter 6). The adopted form of this model assumes asymptotic independence of rainfall extremes at different sites in a region.

Using the method given in Section 8.2.1, the posterior distribution of parameters (Table 8.1) was obtained. For this, the model response ($\ln N_e / \ln N$) was subjected to a log-transformation in order to satisfy the constant variance assumption, as stated in Chapter 6 (Section 6.4.3). Figure 8.1 shows the scatter plots of residuals with the model responses, which illustrate that the constant variance assumption is reasonably satisfied.

8.3.2 Posterior Distribution of Growth Curve Parameters

Despite the GEV distribution having three parameters, only two *effective parameters* were fitted in the CRC-FORGE method, because the growth curve was forced through the theoretical point (0.57,1) on a Gumbel plot (ie. the mean of the standardised values).

Bardsley (1989) showed that a GEV distribution can be described by three different points on a Gumbel plot. As one of the points is fixed, the rainfall estimates for the Gumbel reduced variates 5 and 9 (corresponding to AEPs of 1 in 150 and 1 in 8100) were considered as the *effective parameters*. These effective parameters were fitted to the log-transformed standardised rainfall data points using a least squares method. The posterior distribution of the effective parameters was then obtained as described in the previous section.

Table 8.1 Posterior distribution of Variable N_e model parameters

Duration	Parameter					
(days)	α		β		γ	
	mean	std	mean	std	mean	std
1	1.3×10^{-3}	3.2×10^{-5}	4.4	0.07	17.7	0.18
2	2.4×10^{-3}	9.6×10^{-5}	2.8	0.08	14.5	0.24
3	2.4×10^{-3}	8.0×10^{-5}	2.8	0.07	14.6	1.20

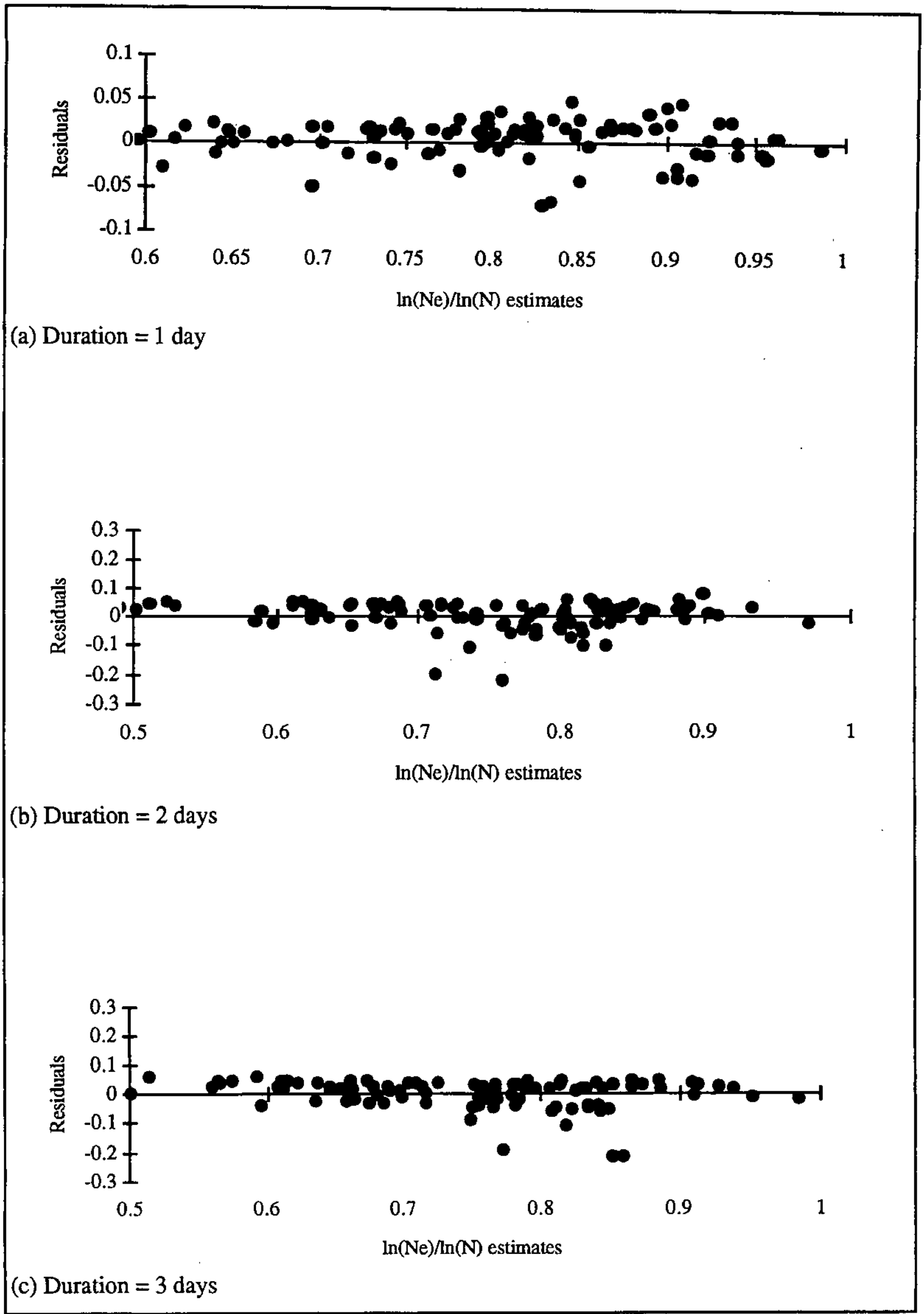


Figure 8.1 Scatter plots of 100 randomly selected residuals (after log-transformation) against estimated $\ln(N_e)/\ln(N)$

8.3.3 Derivation of Confidence Limits

The 90% confidence limits of the growth curve for a given focal station were derived using the following steps.

Step 1

Using the posterior distribution given in Table 8.1, 100 samples of the *Variable N_e model parameter* vector were generated. The generation procedure (Matalas, 1967) preserved the cross correlations between parameters.

Step 2

Using each *Variable N_e model parameter* vector sample, the FORGE points for the focal station were obtained. This resulted in 100 sets of the FORGE points.

Step 3

For each set of the FORGE points, a growth curve was fitted after including the focal point data, and the posterior distribution of *the growth curve parameters* was obtained as described in Section 8.3.2. This resulted in 100 sets of the posterior distribution of *growth curve parameters.*

Step 4

From each set of the posterior distribution of *growth curve parameters,* one sample of the parameter vector was generated. This resulted in 100 *growth curve parameter* vector samples.

Step 5

Using each sample of the *growth curve parameter* vector, rainfall quantiles were calculated for a set of annual exceedance probabilities (AEPs). From this, 100 rainfall values for each AEP were obtained.

Step 6

For each AEP, the 100 rainfall values were ranked, and the 5^{th} and 95^{th} percentiles were assigned as the 90% confidence limits.

It should be noted that the above steps only dealt with the parameter errors for the derivation of confidence limits. If the residuals of the model fittings (ε in Eq. 1) were included in the above steps, the resulting percentiles in *Step 6* would be the 90% prediction limits for the growth curve.

8.3.3 90% Confidence Limits of the Growth Curves for the Selected Focal Stations

Figures 8.2 (a) to (h) show the 90% confidence limits for the growth curves of 1-day rainfall for the eight selected stations. The confidence limits for durations of 2 and 3 days are given in Figures B.3 and B.4 in Appendix B. The 90% confidence limit curves are quite close to the mean growth curve. As expected, the width of the confidence limit band increases with decreasing AEP. The 90% confidence limits are within ±3% at the AEP of 1 in 2000. However, they increase to about ±10% at the AEP of 1 in 10^6, as this AEP is in the extrapolation region.

The very narrow confidence intervals imply a high degree of precision of the CRC-FORGE growth curve estimates. This was achieved by using data from a large number of stations for calibrating the variable N_e model and by the inclusion of the focal point data for the curve fitting. However, it should be noted that the variance of residuals was not stabilised even after the log-transformation. From Figure 8.2, it is obvious that the constant variance assumption is hard to satisfy because two sets of data were used: (i) the FORGE points and (ii) focal station data. The variability of focal station data is significantly lower than that of the FORGE points (Focal station data points are plotted in ranked order). Thus it is expected that for lower exceedance probabilities the true confidence intervals would be wider than those shown in Figure 8.2.

The confidence limits shown in these figures are based on the assumptions made about the distribution, the regional homogeneity and the functional form of the variable N_e model. Any violation of these assumptions would result in biased estimates of rainfall quantiles especially for lower exceedance probabilities. Considering the potential bias that may arise from partial violation of these assumptions, it is recommended to use the CRC-FORGE estimates to a lower exceedance probability limit of 1 in 2000.

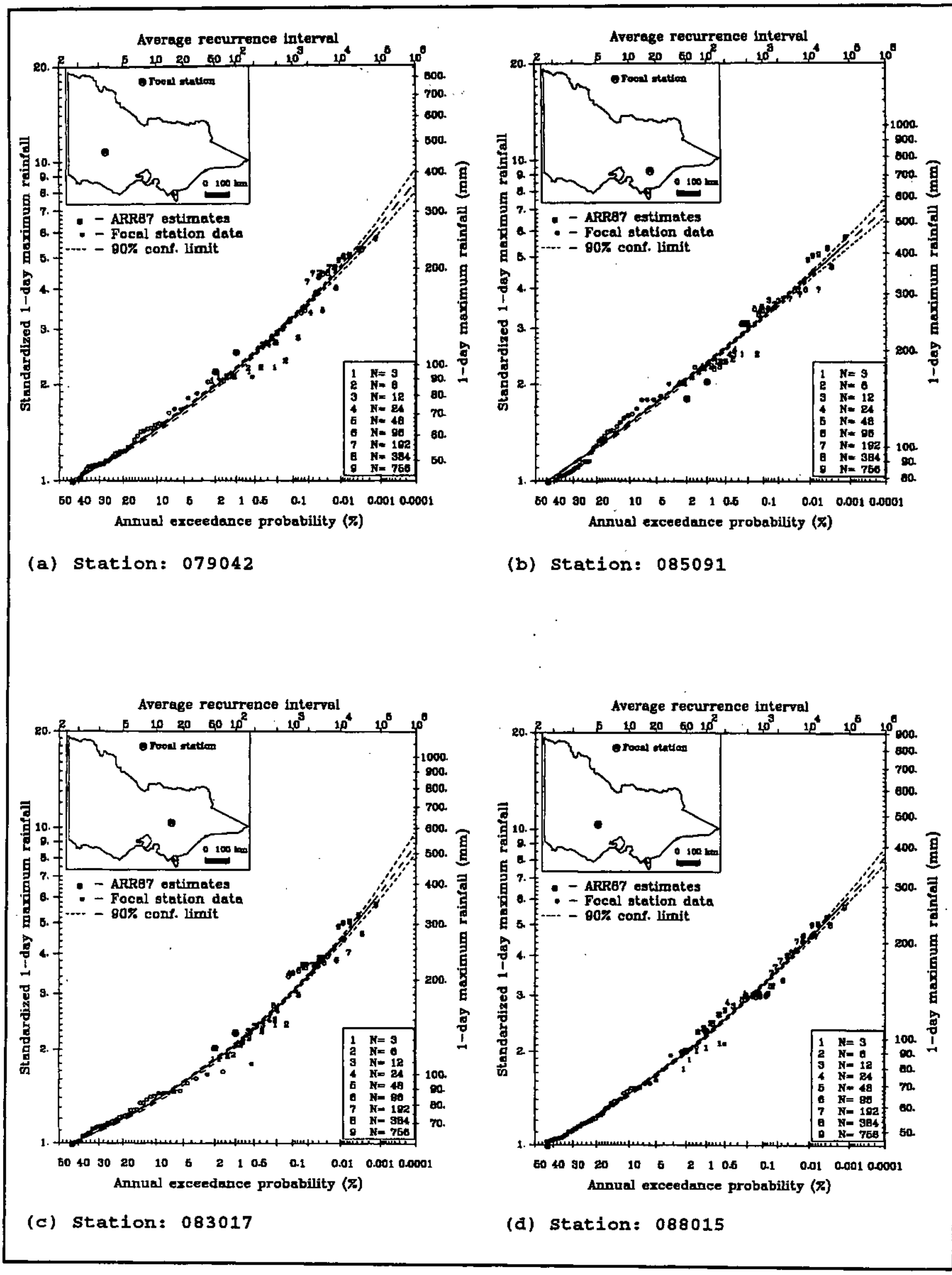
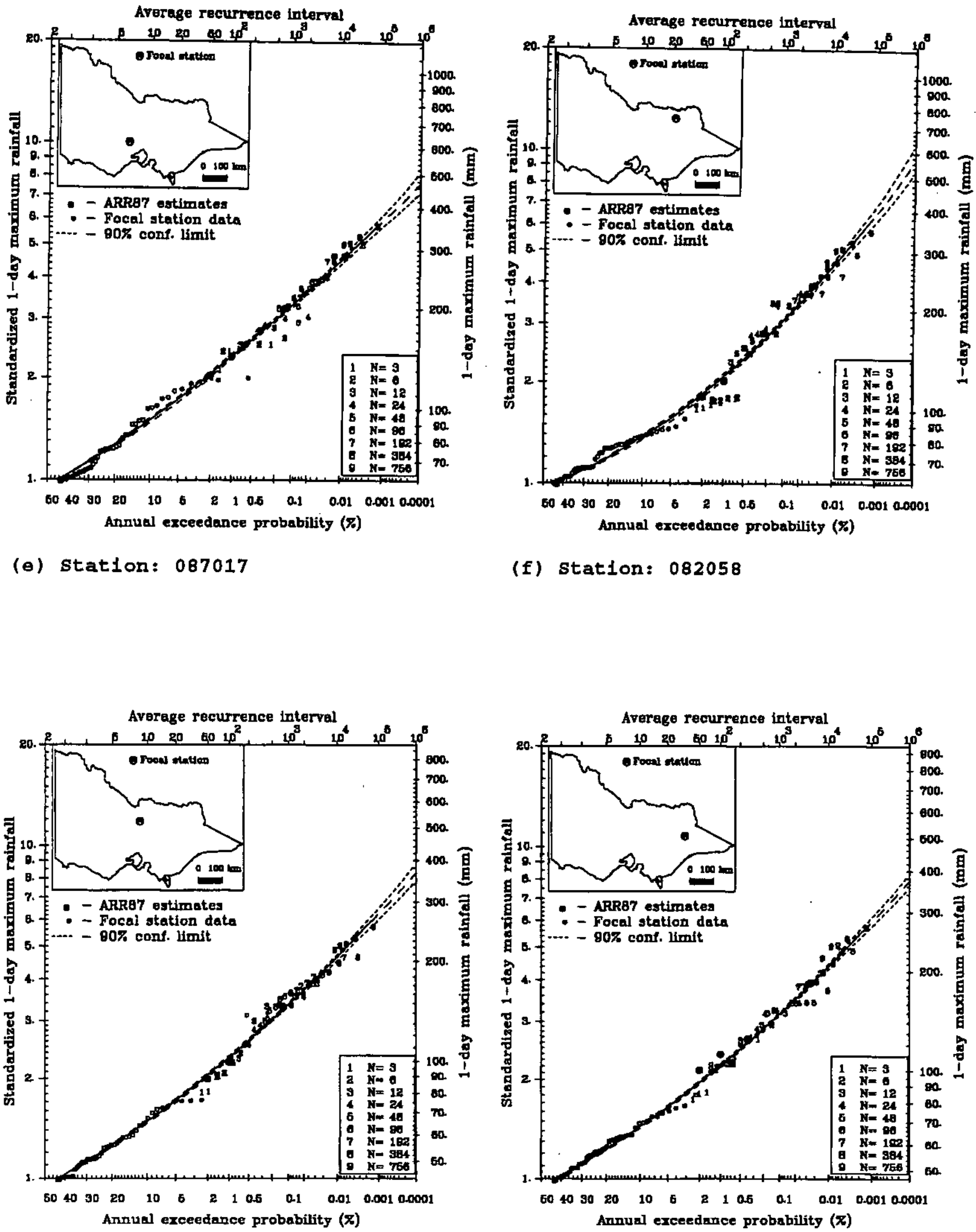


Figure 8.2 90% confidence limits for 1-day CRC-FORGE growth curves (continued overleaf)



(e) Station: 087017

(f) Station: 082058

(g) Station: 081043

(h) Station: 083025

Figure 8.2 (continued) 90% confidence limits for 1-day CRC-FORGE growth

8.4 CONCLUSION

This chapter has described the derivation of the 90% confidence limits of the estimated growth curves for the selected focal stations. It was assumed that the basic assumptions on distribution and homogeneity of annual rainfall maxima were satisfied, and that the adopted form of the variable N_e model was appropriate. The computed confidence limits thus express only the uncertainty attributed to parameter estimation errors in the variable N_e model and the fitting of the growth curve. The confidence limits are also based on a somewhat unrealistic assumption of constant variance of the plotted data points (after log transformation).

The derived confidence limits were found to be quite tight and were about ±3% of the growth curve estimates at the AEP of 1 in 2000. For lower AEPs, the confidence limits widen gradually and, more importantly, the effects of the model assumptions become more pronounced. Thus, the CRC-FORGE method is recommended for estimation of design point rainfalls to an AEP of 1 in 2000.

The derivation of design values for any site in Victoria, as described in the next chapter, involves a considerable degree of spatial smoothing of the growth factors calculated for the sites in Data Set II. This smoothing is expected to further reduce sampling variability but may introduce a small degree of bias in estimated growth factors.

9. DERIVATION OF EXTREME DESIGN RAINFALLS FOR VICTORIA

In Chapters 7 and 8, it was illustrated that the CRC-FORGE method can be used at rainfall station sites to derive design rainfalls up to an annual exceedance probability of 1 in 2000. This chapter describes the methodology to obtain estimates of extreme design rainfall for any location in Victoria.

9.1 CRC-FORGE ESTIMATES FOR SELECTED FOCAL STATIONS

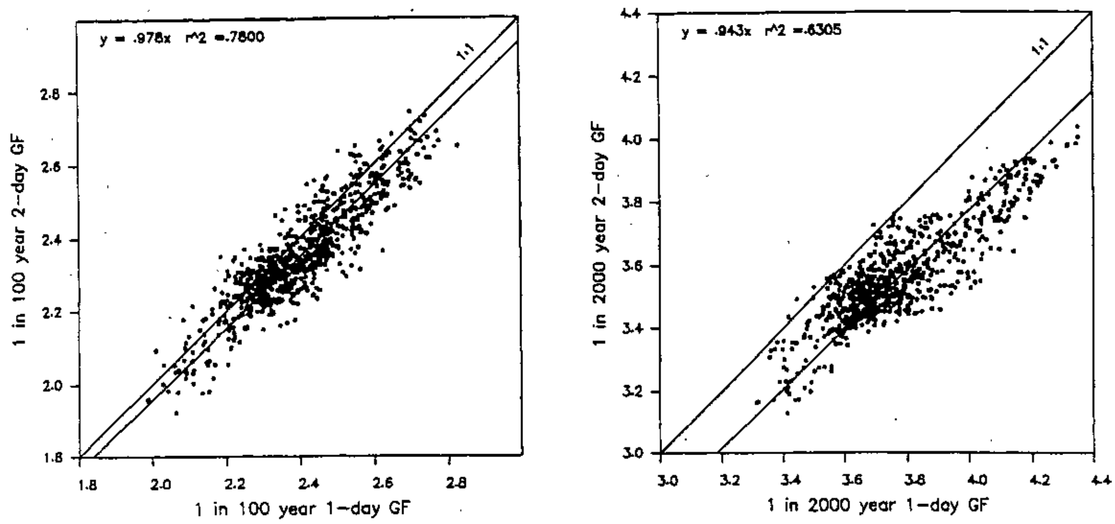
The CRC-FORGE method is a special form of the index frequency estimation method. It involves separate estimation of the *index variable* (the mean annual maximum rainfall depth) and the *growth factors* for selected AEPs. The following sections describe the adjustment of the CRC-FORGE estimates for regionalisation.

9.1.1 Adjustment of Growth Factor Estimates for Regionalisation

The CRC-FORGE method was applied to all the (more than 750) focal stations in Data Set II. For these focal stations, the growth factors for rainfall durations 1,2 and 3 days and for AEPs of 1 in 50, 1 in 100, 1 in 200, 1 in 500, 1 in 1000 and 1 in 2000 were obtained from the fitted growth curves.

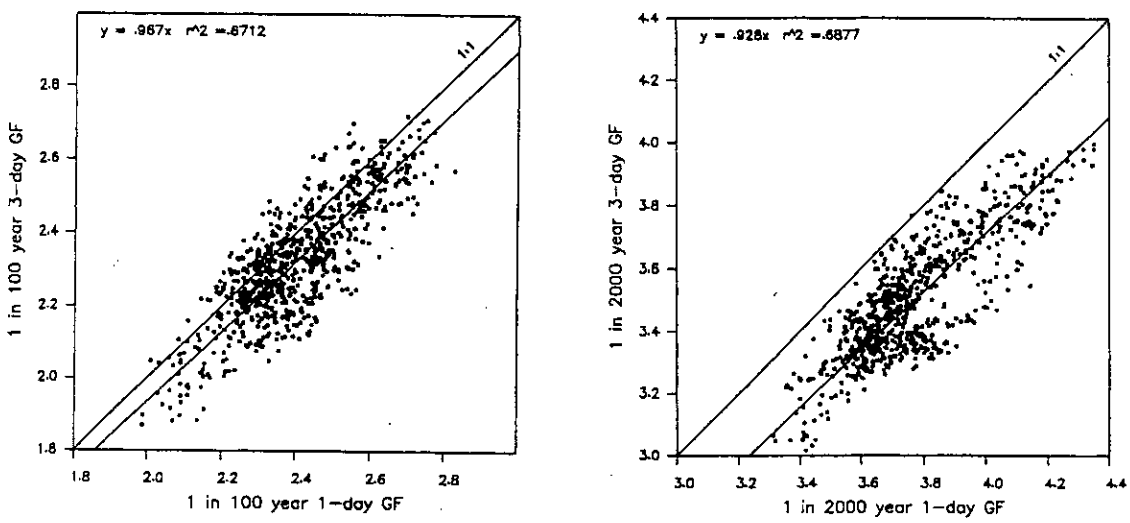
Figures 9.1 and 9.2 show the comparisons of 1 and 2 day growth factor estimates and 1 and 3 day growth factor estimates respectively for AEPs of 1 in 100, 1 in 1000 and 1 in 2000. These figures indicate that the growth factors for different durations within this range are closely related. The limited scatter around the regression line (generally less than 10%) can be mostly explained by sampling variation. The slopes of the linear regression lines (forced through the origin) for all AEPs are shown in Table 9.1.

To reduce the estimation errors in the growth factors and to eventually allow the estimation of design rainfalls for durations other than those considered here (see the next section for explanation), growth factors for 2 and 3 days were converted to 1-day growth factors using the ratios (regression coefficients) in Table 9.1. The 1-day growth factors obtained from 2 and 3-day growth factors and the original 1-day growth factors were then averaged to obtain a representative 1-day growth factor at each site.



(a) 1 in 100 year (b) 1 in 2000 year

Figure 9.1 Comparison of 2-day and 1-day growth factors for shown exceedance probabilities



(a) 1 in 100 year (b) 1 in 2000 year

Figure 9.2 Comparison of 3-day and 1-day growth factors for shown exceedance probabilities

Relationship between growth factors, duration and AEP

The values in Table 9.1 indicate a small but systematic reduction of the growth factors with both increasing duration and reducing AEP. However, so far no satisfactory mathematical relationship has been found to explain this variation in growth factors. The ratios in the table have therefore been used directly.

Table 9.1 Regression coefficients for growth factors

AEP	2 day vs 1 day*		3 day vs 1 day*	
1 in 50	0.985	(0.784)	0.976	(0.669)
1 in 100	0.978	(0.780)	0.968	(0.672)
1 in 200	0.971	(0.771)	0.959	(0.674)
1 in 500	0.960	(0.744)	0.947	(0.681)
1 in 1000	0.952	(0.704)	0.938	(0.685)
1 in 2000	0.943	(0.689)	0.928	(0.630)

* the values in parentheses are coefficients of determination for the fitted regression lines

9.1.2 Estimation of Index Variable for Regionalisation

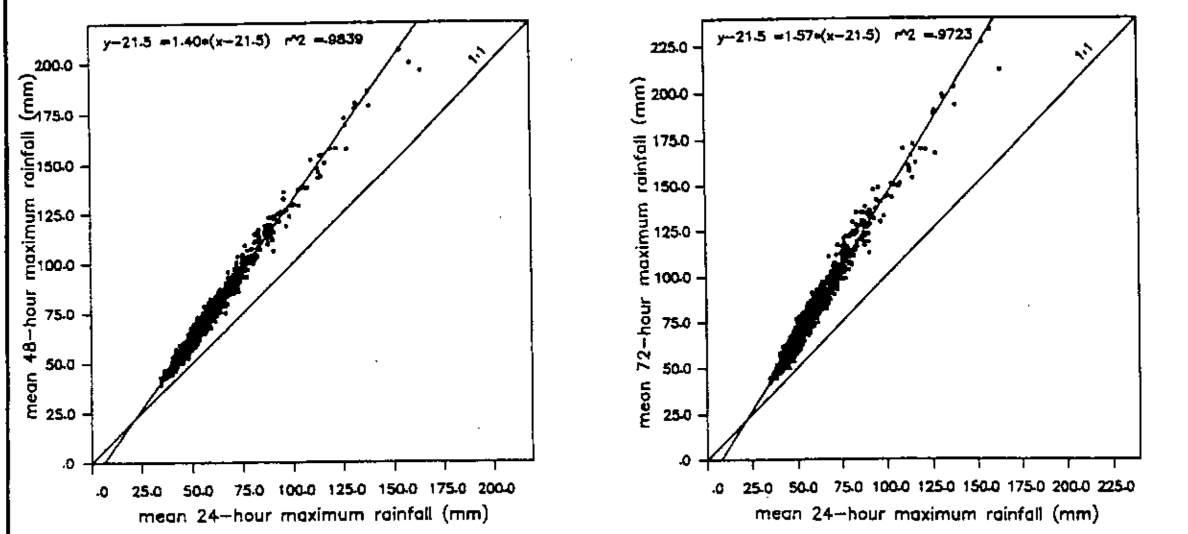
The spatial variability of the index variable (mean annual maximum value) is significantly higher than that of the growth factors. Thus a relatively uniform coverage of rainfall stations over the whole region is desirable to allow accurate estimation of the index variable. From Figure 4.6 in Chapter 4, it is clear that stations with 60 or more years of annual maxima do not cover the alpine area where the index variable is expected to be very high because of the strong correlation between storm rainfall and elevation. Thus the mean maxima for stations which have 25 or more years of annual maxima (Data Set III) were used for the regionalisation.

The 1, 2 and 3 day mean annual maxima were converted to 24, 48 and 72 hour unrestricted duration rainfalls using factors of 1.16, 1.11, 1.07. respectively. The derivation of these factors is described in Appendix C.

High correlations between 24, 48 and 72 hour mean annual maxima have indicated that 48 and 72 hour mean annual maxima can be estimated from the 24 hour maxima. The following investigation was carried out to establish the relationship between the mean annual maxima for different durations.

Figures 9.3 (a) and (b) show the relationship between 48 and 24 hour mean annual maxima and 72 and 24 hour mean annual maxima respectively. The linear regression lines were forced through a common point (21.5, 21.5) so that a relationship between the slope of the line (R) and the rainfall duration (D) could be derived. This relationship is expressed by the following equation which is considered applicable for durations in the range from 12 to 72 hours:

$$R = \frac{D}{24}(2.385 - 1.003 \log D) \qquad (9.1)$$



(a) 48 and 24 hour mean annual maxima (b) 72 and 24 hour mean annual maxima

Figure 9.3 Relationship between (a) 48 and 24 hour mean annual maximum rainfalls and (b) 72 and 24 hour mean annual maximum rainfalls

As for the growth factors, the representative 24-hour mean maxima for regionalisation were obtained by averaging for each site the 24-hour mean maximum, and the 48 and 72 hour mean maxima which were first converted to 24 hour equivalents using the fitted regression equations.

Uncertainty in index variable estimates

As stated previously, the index variable (the mean annual maximum rainfall) is much more spatially variable than the growth factors. Although the variation of the index variable is determined from a larger data set (Data Set III with nearly 1400 stations), there is a considerable degree of uncertainty in the estimation of this variable for ungauged sites.

This problem is common not only to all index methods of regional frequency estimation but to regional estimation methods in general. While this uncertainty has not been investigated in detail, the results presented in the next section give an indication of the order of magnitude of spatial variation, and likely errors in estimates for ungauged sites. In assessing this uncertainty, it should be kept in mind that in most applications the interest is on average values of design rainfalls for relatively large catchments. These average values are less sensitive to errors in interpolation between rainfall stations than the point estimates.

It is possible that the use of the *median* of the annual rainfall maxima as the index variable, instead of the mean, might lead to a small reduction in the sampling uncertainty of the index variable at gauged sites. However, any such improvement would be small, compared to the uncertainties involved in spatial interpolation for ungauged sites.

9.2 REGIONALISATION OF DESIGN RAINFALLS

The regionalisation of design rainfall values involves transfer of information from gauged to ungauged sites. This can be done using either a relationship between design rainfall values and measurable catchment characteristics or a geostatistical mapping method. The latter approach is used in this study.

The following sections describe use of a spatial interpolation method to regionalise the design rainfall estimates for Victoria.

9.2.1 Spatial Interpolation Methods

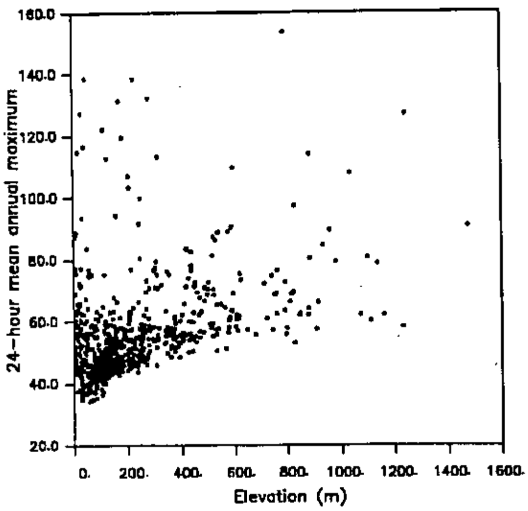
The geostatistical methods analyse the spatial structure of random variables and perform interpolations. They assume a continuous surface; an assumption which is likely to be satisfied for design rainfall depths. Two popular spatial interpolation methods are kriging and Laplacian or thin plate smoothing splines. Kriging depends critically on an assumption that the spatial covariance function is stationary with respect to space, whereas thin plate smoothing splines require the estimation of a smoothing parameter that determines an optimal balance between fidelity to the data and smoothness of the fitted function which can be automatically calculated (Hutchinson, 1990).

Stewart et al. (1995) used a kriging method to map median at-site maximum rainfall. Hutchinson et al. (1984) successfully applied the thin plate smoothing splines method to the spatial interpolation of monthly rainfall. In this study, the thin plate smoothing splines method was used to regionalise the design rainfall estimates.

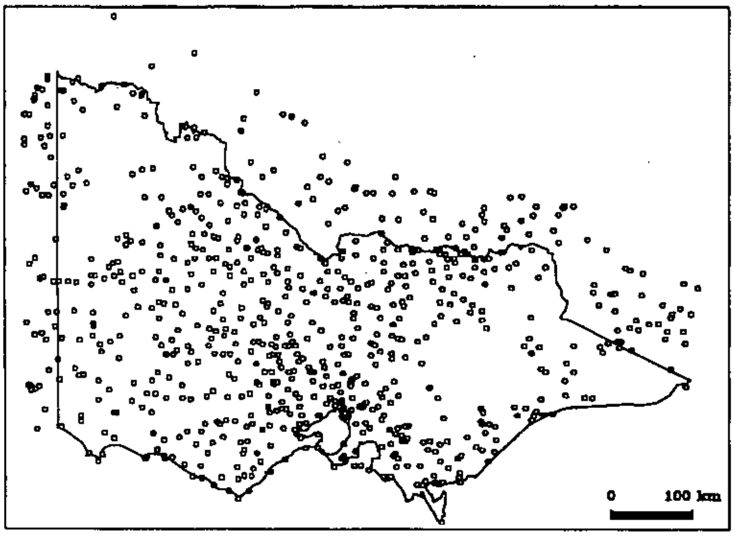
9.2.2 Correlation Between Design Values and Elevation

Rainfall amounts are generally influenced by elevation, along with other factors such as distance from coast. Figure 9.4 (a) shows a scatter plot of mean 24-hour maximum rainfall and elevation for stations shown in Figure 9.4 (b). It is evident from this figure that the mean 1-day maximum rainfalls are positively correlated with elevation. However, the relationship shown in this figure is not as strong as expected because of the influence of other factors affecting rainfall (eg. closeness to ocean, rain shadow effects).

Figure 9.5(a) shows a much better relationship between 24-hour mean annual maxima and elevation for stations shown in Figure 9.5(b). These stations were selected such that the influence of the ocean is similar and rain shadow effects are minimal. The growth factors appear to be negatively correlated, as shown in Figures 9.6 (a), (b), (c) and (d)for the 1 in 50, 1 in 100, 1 in 500 and 1 in 2000 AEP 24-hour rainfalls respectively. The negative correlation seems to increase with reducing exceedance probability. These correlations were taken into account in thin plate spline fitting by incorporating elevation as the third independent variable, in addition to latitude and longitude.

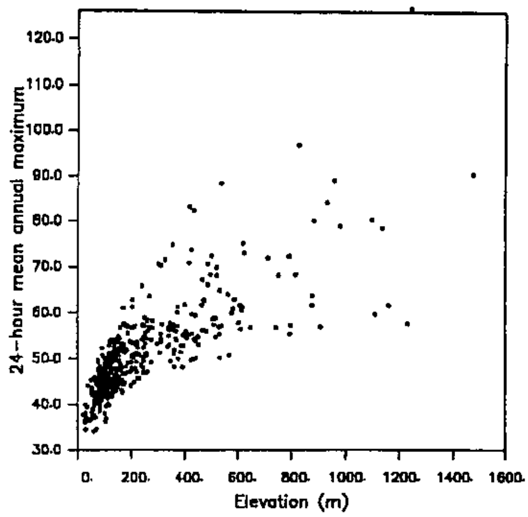


(a) mean annual maxima

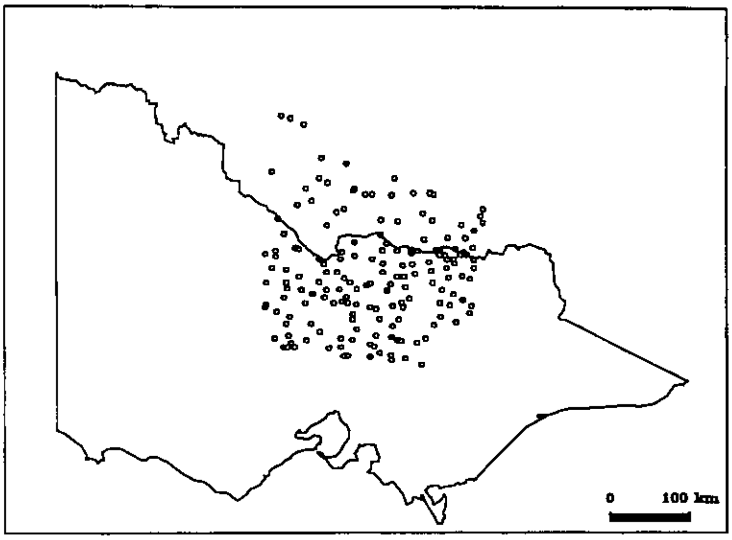


(b) stations

Figure 9.4 Relationship of 24 hour mean annual maximum rainfall with elevation for shown stations in (b)



(a) mean annual maxima



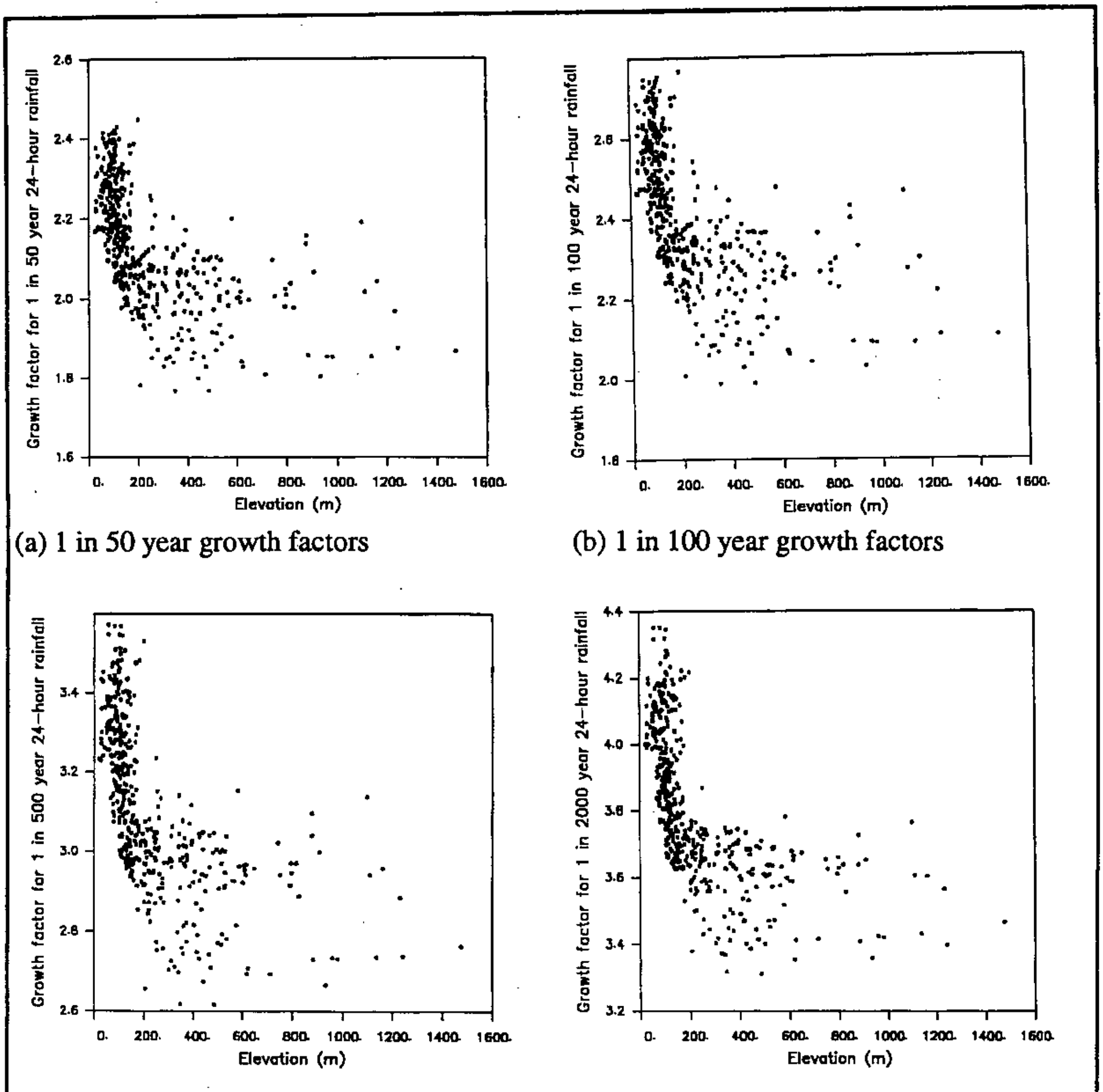
(b) stations

Figure 9.5 Relationship of 24 hour mean annual maximum rainfall with elevation for the selected stations shown in (b)

9.2.3 Spatial Interpolation of Design Values Using ANUSPLIN

ANUSPLIN, a program suite developed by Hutchinson (1996), was used to map representative values of the index variable (mean annual maximum for 24-hour rainfall) and representative values of the design growth factors for the AEPs of 1 in 50, 1 in 100, 1 in 200, 1 in 500, 1 in 1000 and 1 in 2000 over the state of Victoria

The following steps are involved in the fitting of these surfaces:



(a) 1 in 50 year growth factors

(b) 1 in 100 year growth factors

(c) 1 in 500 year growth factors

(d) 1 in 2000 year growth factors

Figure 9.6 Relationship of 24 hour growth factors for shown AEPs with elevation for the stations shown in Figure 9.5(b)

(i) prepare a file of the dependent data points (design rainfall characteristics) to be fitted, together with the corresponding latitudes and longitudes plus, if appropriate, an additional independent variable (in this case the elevations of the focal point stations);

(ii) use the SELNOT program to select a subset of data points (knots) to be used in the fitting of the surface;

(iii) run the spline fitting program SPLINBB and optimise the results by:
- varying the number of knots,
- varying the degree of smoothing (the order of derivatives being

considered),

- including additional data points with large residuals as knots;

The appropriateness of the fitted surfaces can be assessed by a range of error statistics. Apart from a check of the remaining degrees of freedom, to ensure that the surface is not overfitted, the assessment is mainly based on the magnitude of the average residuals and the largest residuals.

In the case of the index variable, the root mean square residual was 2.3 mm or about 4% of the mean value, with the largest residual from 1491 values being 25 mm, or about 27% of the data value at that location. For the growth factors, the root mean square residual was in the order of 0.03, or about 1% of the mean value, with the largest residual being about 5% of the data value.

These error statistics and spot checks for selected areas indicate that the fitted surfaces provide a reliable basis for estimating design rainfalls at ungauged sites.

It is of interest to note that the performance of the *ANUSPLIN* method in preparing a map of mean annual precipitation for a mountainous area in Montana (USA) has recently been assessed on the basis of a comparison with hand-drawn precipitation maps (Custer et al., 1996). The study concluded that overall the maps produced by the two methods are similar, with some systematic differences in areas where there are few input data points.

9.2.4 Estimation of Complete Rainfall Frequency Curve for a Catchment

The steps in estimating catchment average values of *point rainfalls* for the six selected AEPs are:

(i) determine catchment average values of the 24h duration design values for:
- the index variable (mean annual maximum rainfall depth)
- the six growth factors for the selected AEPs

by averaging the computed spline surface values for all the grid points that fall within the catchment boundary;

(ii) apply the appropriate ratios to convert the 24h values of the index variable and the growth factors to the corresponding values for durations of 48h and 72h;

(iii) multiply the index values by the growth factors to obtain rainfall quantile estimates for the three basic durations and six AEPs;

(iv) Determine rainfall quantiles for other durations D in the range from 12h to 72h by first converting the rainfall depths (mm) into rainfall intensities I(D) (mm/h) and then fitting a linear regression of ln[I(D)] vs ln[D].

These point rainfall estimates are then converted into *areal rainfall estimates* for the catchment using the appropriate areal reduction factors from Siriwardena and Weinmann (1996).

An *interpolation procedure* developed by Weinmann and Siriwardena (1997) is finally used to determine *areal rainfall estimates* for the intermediate range between the AEP of 1 in 2000 and the AEP assigned to the PMP (usually 1 in 10^6, based on Pearce, 1994).

A complete summary of all the steps in the application of the CRC-FORGE method is given in Appendix D.

9.3 COMPARISON OF CRC-FORGE ESTIMATES AND GSAM PMP ESTIMATES FOR SELECTED CATCHMENTS

As discussed in Chapter 8, the effect of the basic assumptions in the CRC-FORGE method and the larger uncertainties in design rainfall estimates for low AEPs led to the adoption of an AEP of 1 in 2000 as the recommended limit for its application. However, as design rainfall estimates in the intermediate AEP range, between 1 in 2000 and the AEP assigned to the PMP, are to be established by interpolation, it is of interest to check how consistent the extrapolated CRC-FORGE estimates are with the PMP estimates.

For this purpose, the 24-hour growth factors for an AEP of 1 in 10^6 were also established as catchment average values for the five benchmarking study catchments described in Section 4.6. The corresponding point rainfall quantiles were determined by multiplying the growth factors by the average index value for the catchment These point rainfall estimates were then converted to areal design rainfall values for the catchment by multiplication with the appropriate areal reduction factor (ARF) from Siriwardena and Weinmann (1996). The ARF values for the lowest AEP (1 in 2000) were used for this purpose. Table 9.2 shows the comparison of extrapolated CRC-FORGE estimates with the PMP values.

Table 9.2 Comparison of Extrapolated CRC-FORGE Values with PMPs (24-hour duration)

Reservoir/ Dam Site	Catchm. Area (km^2)	PMP Areal Estimate (mm)	CRC-FORGE (Extrapol.) Point Est. (mm)	ARF	CRC-FORGE (Extrapol.) Areal Est. (mm)	Diff. (%)
Lake Bellfield	100	610	538	0.91	490	-20
Lake Buffalo	1145	760	723	0.81	590	-22
Dartmouth Dam	3564	520	592	0.75	450	-13
Rosslynne Res.	90	880	488	0.91	450	-49
Thomson Res.	487	850	720	0.85	610	-28

The results indicate that the extrapolated values are lower than but generally consistent with the PMP values, allowing for the fact that the two estimates are based on very different assumptions and different data sets. For this comparison, it should also be kept in mind that the AEP assigned to the PMP is a notional value, associated with the application of a particular methodology to a given data set, rather than a direct probability estimate for the PMP rainfall depth (Pearce, 1994).

The consistency of the CRC-FORGE estimates with the PMP estimates was further confirmed by the fact that in the benchmarking studies conducted for the five dam sites, the application of the interpolation method by Weinmann and Siriwardena (1997) resulted in a smooth frequency curve between the AEP 1 in 2000 design rainfall estimate and the PMP estimate.

9.4 CONCLUSION

This chapter has described the regionalisation of the CRC-FORGE design rainfall estimates using a geostatistical method. The ANUSPLIN package was used to spatially interpolate the index variable (mean annual maximum rainfall) and the growth factors for any site in Victoria.

When applied to five dam catchments in Victoria, ranging in size from about 100 to more than 3000 km^2, the method produced extrapolated results that are somewhat lower than but generally consistent with PMP estimates.

10. CONCLUSION

This chapter provides a summary of the work performed, the conclusions drawn from this study, and recommendations for future work.

10.1 SUMMARY

This report described the estimation of rainfall frequency curves using the CRC-FORGE method. The main objective was to derive design rainfalls of durations 1, 2 and 3 days up to an annual exceedance probability of 1 in 2000 for Victoria.

Annual maximum rainfall data for stations in Victoria and neighbouring regions were extracted from daily rainfall records provided by the Bureau of Meteorology and were checked for any recording and processing errors. For the development and application of the CRC-FORGE method, three data sets were used: (i) stations with 100 or more years of annual maximum rainfalls, (ii) stations with 60 or more years of annual maximum rainfalls and (iii) stations with 25 or more years of annual maximum rainfalls.

Although the FORGE concept is a non-parametric, the estimated plotting positions of extreme rainfall data points are dependent on the assumed distribution for annual maxima. Using the L-moment ratio diagram and the probability plot correlation coefficient techniques, the GEV distribution was identified as the most appropriate distribution of the annual maximum rainfalls for the Victorian region (Chapter 5). Based on results from other studies, the rainfall extremes for the Victorian and neighbouring regions were deemed to be homogeneous.

The spatial dependence in annual maximum rainfall data reduces the net information available in regional frequency analysis. This effect is normally measured by the effective number of independent stations (N_e) in a raingauge network. A model for a constant N_e (CRC Constant N_e model) based on the average correlation coefficient in the network was developed. This model was found to perform better than the Dales and Reed (1989) spatial dependence model (Dales and Reed Constant N_e model) used in the IH-FORGE method. A model for variable N_e was also developed based on an investigation of generated data sets (Chapter 6).

Using generated data, the IH-FORGE method was shown to overestimate extreme rainfalls for spatially correlated data. From the validation using synthetic data, the following modifications were made to the IH-FORGE method (Chapter 7):

(i) use of a variable N_e model instead of a constant N_e model to estimate the plotting positions of FORGE data points,

(ii) inclusion of focal point data together with FORGE data points,

(iii) screening of outliers among the FORGE data points, and

(iv) fitting of a GEV distribution by a least squares method rather than an eye-ball fit of a non-parametric distribution line.

The uncertainties in the CRC-FORGE method were estimated using a Monte-Carlo simulation method. The main sources of errors in the modified estimation were assumed to be parameter estimation errors in the Variable N_e model and in the fitting of the growth curve. It was assumed that the basic assumptions on distribution and homogeneity of annual rainfall maxima were closely satisfied, and that the adopted form of the Variable N_e model was appropriate.

The index variable (mean annual maxima) and design growth factors estimated for the selected focal stations were regionalised using a geostatistical spatial interpolation technique based on a thin plate spline smoothing method. In this method, elevations of stations were treated as an independent variable along with latitudes and longitudes of the stations. The results of this regional smoothing now allow the estimation of extreme rainfalls for any site in Victoria for the range of durations from 12 to 72 hours and for AEPs from 1 in 50 to 1 in 2000.

Appendix D gives a summary of all the steps in the application of the CRC-FORGE methodology to estimate design values of extreme catchment rainfalls.

10.2 CONCLUSION

The conclusions drawn from this study are:

(i) the CRC-FORGE method provides consistent extreme point rainfall estimates for Victoria to an annual exceedance probability (AEP)of 1 in 2000, and establishes a better basis for interpolation of design rainfalls in the range between that event and the probable maximum precipitation;

(ii) the CRC-FORGE rainfall estimates derived for rain gauge sites can be regionalised to allow estimation of extreme design rainfalls for any site in Victoria, with some loss of accuracy in the sparsely gauged mountainous regions;

(iii) the CRC-FORGE methodology and software developed in this project lend themselves for application to design rainfall estimation in other regions of Australia;

(iv) the IH-FORGE method, which assumes no variation of the effective number of independent stations with rainfall magnitude, tends to overestimate very extreme rainfalls;

(v) the functional form of the model used in the CRC-Forge method to estimate the variable effective number of independent stations produces satisfactory results when fitted to Victorian data;

(vi) the Generalised Extreme Value (GEV) distribution satisfactorily describes the annual maximum rainfall data for the Victorian region; and

(vii) the Victorian annual maximum rainfall data can be assumed to be homogeneous for the purpose of estimating design rainfalls in the extreme range.

10.3 SUGGESTIONS FOR FUTURE WORK

The following recommendations are made to further improve the CRC-FORGE method estimates:

(i) homogeneity tests for very extreme rainfalls should be developed, as the CRC-FORGE method considers only the highest observations in a region, and the available tests mainly check the homogeneity in relation to higher exceedance probabilities;

(ii) the appropriate form of the Variable N_e model to estimate the effective number of independent stations should be identified based on theoretical investigations;

(iii) the influence of the constant inter-station correlation assumption in the Monte Carlo simulations, used to identify the appropriate functional form of the Variable N_e model, should be examined;

(iv) the sensitivity of the CRC-FORGE estimates to the number of selected highest rainfalls in each FORGE step should be investigated;

(v) further data generation studies should be undertaken to determine the effects on the estimated design rainfall values of any violations of the basic assumptions on homogeneity and distribution; and

(vi) the spatial variation of the index variable (mean maximum annual rainfall) should be further investigated to identify options for more reliable extrapolation into sparsely gauged areas, including the use of data from short record stations and the effects of non-concurrent records.

11. REFERENCES

Aitken, A.P. (1973). Assessing systematic errors in rainfall-runoff models. *J. Hydrol.*, 20, pp. 121-136.

Alexander, G.N. (1954). Some aspects of time series in hydrology. *J. Inst. Eng. Aust.*, 26, pp. 188-198.

ANCOLD, (1986). Australian National Committe on Large Dams. *Guidelines on design floods for dams.*

Bardsley, W.E. (1989). Graphical estimation of extreme value prediction functions. *J. Hydrol.*, 110: 315-321.

Bobee and Rasmussen (1995). Recent advances in flood frequency analysis. *Reviews of Geophysics*, Supplement to Vol 33, 1111-1116.

BOM [Bureau of Meteorology] (1994).The estimation of probable maximum precipitation in Australia: Generalised short-duration method. *Bulletin 53*, Aust. Gov. Publ. Service, December 1994.

BOM [Bureau of Meteorology] (1996). Development of the generalised Southeast Australia Method for estimating probable maximum precipitation. *HRS Report No. 4*, Hydrology Unit, Melbourne. 44p.

Buishand T.A. (1984), Bivariate extreme-value data and the station-year method. *J. Hydrol.*, 69, pp. 77-95.

Cunnane, C. (1989). Statistical distributions for flood frequency analysis. *Operational Hydrology Report No. 33*, World Meterological Organisation, WMO - No. 718, Geneva, Switzerland.

Custer, S.g., Farnes, P., Wilson, J.P. and Snyder, R.D. (1996). A comparison of hand- and spline-drawn precipitation maps for mounainous Montana. *Water Resources Bulletin*, 32/2, pp. 393-405.

Dales, M.Y. and Reed, D.W. (1989). Regional flood and storm hazard assessment. *Rep. No. 102*, Institute of Hydrology, Wallingford, Oxon, UK.

Dalrymple, T. (1960). Flood frequency analysis. *US Geol. Surv. Water Supply Paper*, 1543-A: 11-51.

Fill, H.D. and Stedinger, J.R. (1995). Homogeneity tests based upon Gumbel distribution and a critical appraisal of Dalrymple's test. *J. Hydrol.*, 166, pp. 81-105.

Galambos, J. (1978). *The asymptotic theory of extreme order statistics*. Wiley.

Greenwood, J.A., Landwehr, J.M., Matalas, N.C. and Wallis, J.R. (1979). Probability weighted moments: definitions and relation to parameters of several distributions expressible in inverse form. *Water Resour. Res.* 15(6): 1049-1054.

Hosking, J.R.M. (1990). L-moments: analysis and estimation of distributions using linear combinations of order statistics. *J. Royal Statist. Soc., B.*, 52(2), pp. 105-124.

Hosking J.R.M. and Wallis J.R. (1988), The effect of intersite dependence on regional flood frequency analysis. *Water Resour. Res.*, 24(4), pp. 588-600.

Hosking J.R.M. and Wallis J.R. (1990), *Regional frequency analysis using L-moments.* (RC 15658). IBM Research Division, Yorktown Heights, NY.

Hosking J.R.M. and Wallis J.R. (1993), Some statistics useful in regional frequency analysis. *Water Resour. Res.*, 29(2), pp. 271-281.

Hutchinson, M.F. (1990). Climatic analysis in data sparse regions. *Climatic Risk in Crop Production* (R.C. Muchow and J.A. Bellamy, Eds.). Proceedings of the International Symposium on Climate Risk in Crop Production: Models and Management for the semiarid Tropics and Subtropics, pp. 55-71.

Hutchinson, M.F. (1996). *ANUSPLINE Version 3.0.* Centre for Resource and Environmental Studies, The Australian National University, Canberra.

Hutchinson, M.F. and Bischof, R.J. (1983). A new method for estimating the spatial distribution of mean seasonal and annual rainfall applied to the Hunter Valley, New South Wales. *Aust. Meteor. Mag.*, 31, 179-184.

I.E. Aust. (1987). *Australian rainfall and runoff.* Ed. D.H. Pilgrim, Volume 1, (Institution of Engineers: Australia.)

Irish, J. (194). *Review of methodology for estimation of flood frequency curves for extreme events*. Report prepared for the Cooperative Research Centre for Catchment Hydrology. Water Resources Consulting Service, 35p.

Kuczera, G. (1983). Improved parameter inference in catchment models:1. Evaluating parameter uncertainty. *Water Resour. Res.*, 19(15): 1151-1162.

Lall, U. (1995). Recent advances in nonparametric function estimation: hydrologic applications. (1995). *Reviews of Geophysics*, Supplement to Vol 33, 1093-1102.

Lettenmaier, W.B., Wallis, J.R. and Wood, E.F. (1987). Effect of regional heterogeneity on flood frequency estimation. *Water Resour. Res.*, 23(2), pp. 313-324.

Lu, L.H. and Stedinger, J.R. (1992). Sampling variance of normalized GEV/PWM quantile estimators and a regional homogeneity test. *J. Hydrol.*, 138, pp. 223-245.

McConachy, F.L.N. (1995). Estimation of extreme rainfall for Victoria - application of Schaefer's method. *Working Document 95/6*, CRC for Catchment Hydrology.

McConachy, F.L.N. (1996). *Estimation of the rainfall frequency curve in the extreme range using regional methods*. PhD Thesis, Dept. of Civil Eng., Monash University, Australia.

McGilchrist, C.A. and Woodyer, K.D. (1975). Note on a distribution free CUSUM technique. *Technometrics*, 17(3), pp. 321-323.

Nandakumar (1996). *The modified 'FORGE' method for the estimation of the frequency of extreme point rainfalls for Victoria.* Unpublished report, CRC for Catchment Hydrology, Monash University, Australia.

Nandakumar, N. (1995). Estimation of extreme rainfall for Victoria - application of FORGE method. *Working Document 95/7*, CRC for Catchment Hydrology, Monash University, Australia.

Nandakumar, N., Siriwardena, L. and Weinmann, P.E. (1996). *Software for rainfall data preparation and checking*. Notes for workshop on 17 Dec. 1996, CRC for Catchment Hydrology, Monash University, Australia.

Nathan, R.J. and Weinmann, P.E. (1995). The estimation of extreme floods - the need and scope for revision of our guidelins. *Aust. J.Water. Resour*, 1(1): 40-50.

Pearce, H.J. (1994). Notional probabilities of estimated PMP events. *International Hydrology & Water Resources Symposium.* The Institution of Engineers, Australia, Preprints of Papers, Nat. Conf. Publ. No. 94/15, pp 55-61.

Pescod, N. (1990). *The district extreme point-to-area method of assigning probabilities to estimates of rainfall intensities approaching the probable maximum precipitation.* Unpublished report for Hydrology Branch, Australian Bureau of Meteorology.

Pescod, N. (1991). Estimating rainfall intensities for recurrence intervals of several thousand years using the district extremes point-to area method. International *Hydrology and Water Resources Symposium*, I.E. Aust., Nat. Conf. Publ. No: 91/22: 400-408.

Reed D.W., and Stewart E.J. (1989), Focus on rainfall growth estimation, *Proc. 2nd National Hydrology Symposium*, British Hydrological Society.

Rosbjerg and Madsen (1995). Uncertainty measures of regional flood frequency estimators. *J. Hydrol.*, 167: 209-224.

Schaefer, M.G. (1990). Regional analyses of precipitation annual maxima in Washington State. *Water Resour. Res.*, 26(1), pp 119-131.

Siriwardena, L. and Weinmann, P.E. (1996). Derivation of areal reduction factors for design rainfalls in Victoria (for rainfall durations 18-120 hours). Cooperative Research Centre for Catchment Hydrology, Report 96/4, October 1996.

Srikanthan, R. and Stewart, B.J. (1991). Analysis of Australian rainfall data with respect to climate variability and change. *Australian Meteorological Magazine*, 39 (1), pp. 11-20.

Stedinger, J.R. (1983). Estimating a regional flood frequency distribution. *Water Resour. Res.*, 19(2), pp 503-510.

Stewart, E.J., Faulkner, D.S. and Reynard, N.S. (1995). *Rainfall frequency estimation in England and Wales Phase 1b: Pilot Study*. Institute of Hydrology, Wallingford, R&D Note 478, 122 pp.

Wang, Q.J. (1996a). LH-moments for analysing hydrological extremes. *Preprints 23rd Hydrology & Water Resources Symposium, Hobart.* I.E.Aust., Nat. Conf. Publ. 96/05, Vol. 2, 601-607.

Wang, Q.J. (1996b). Direct sample estimators of L-moments. Water Resour. Res. (In press).

Weinmann, P.E. and Siriwardena, L. (1997). *An interpolation procedure for the estimation of extreme rainfalls and floods in the intermediate range.* Cooperative Research Centre for Catchment Hydrology, Technical Note, (in preparation).

Wiltshire, S.E. (1986). Regional flood frequency analysis I. Homogeneity statistics. *Hydrol. Sci.*, 31(3), pp 321-333.

APPENDIX A VARIATION OF N_e WITH AEP

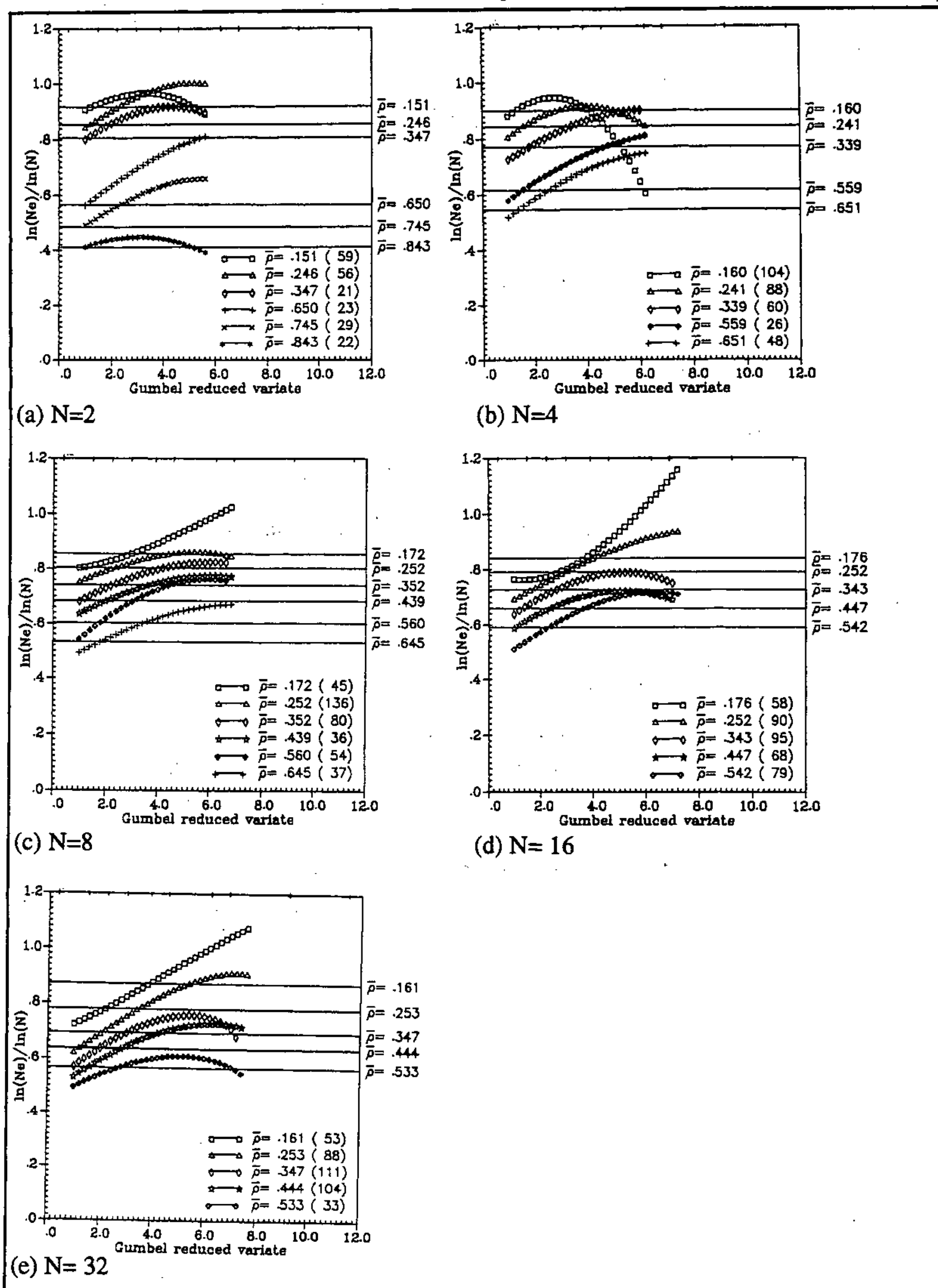


Figure A.1 Variation of $\ln N_e / \ln N$ calculated from GEV distribution fit for Victorian 2-day rainfall maxima (horizontal lines indicate the average constant N_e values for given $\overline{\rho}$ group)

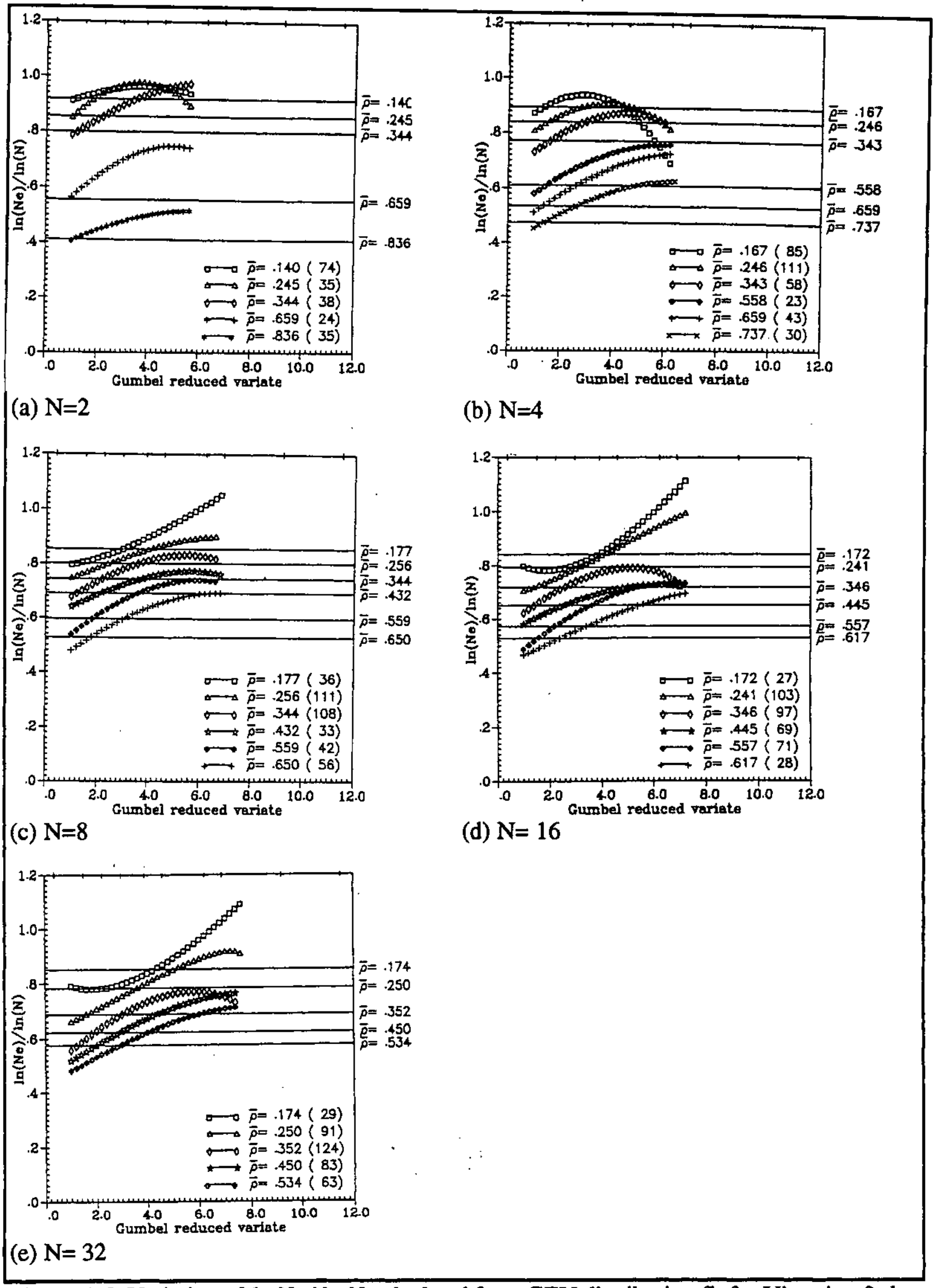


Figure A.2 Variation of $\ln N_e / \ln N$ calculated from GEV distribution fit for Victorian 3-day rainfall maxima (horizontal lines indicate the constant N_e values for given $\bar{\rho}$ group)

APPENDIX B IH-FORGE GROWTH CURVES

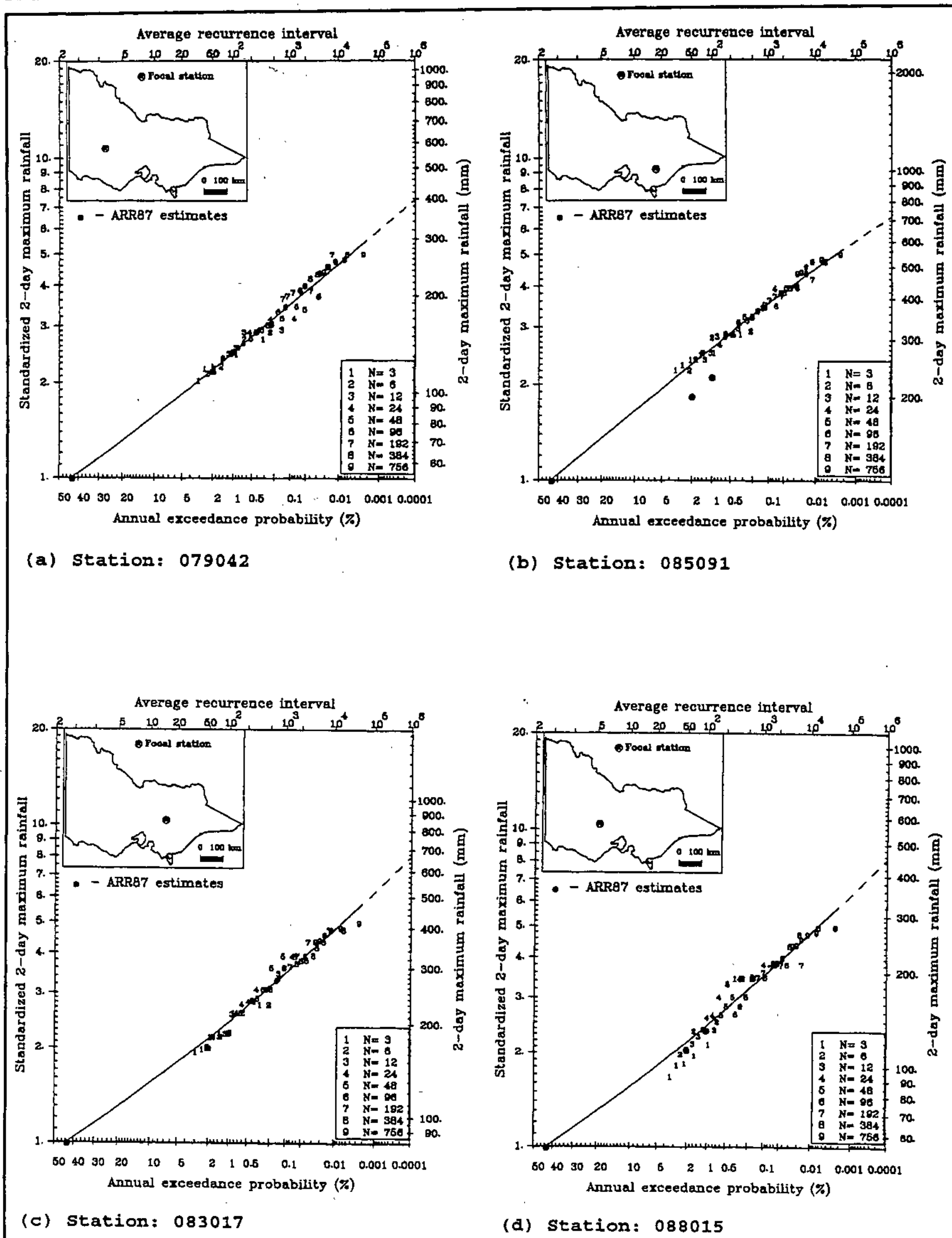


Figure B.1 IH-FORGE growth curves for 2-day rainfall maxima at selected Victorian stations (continued overleaf)

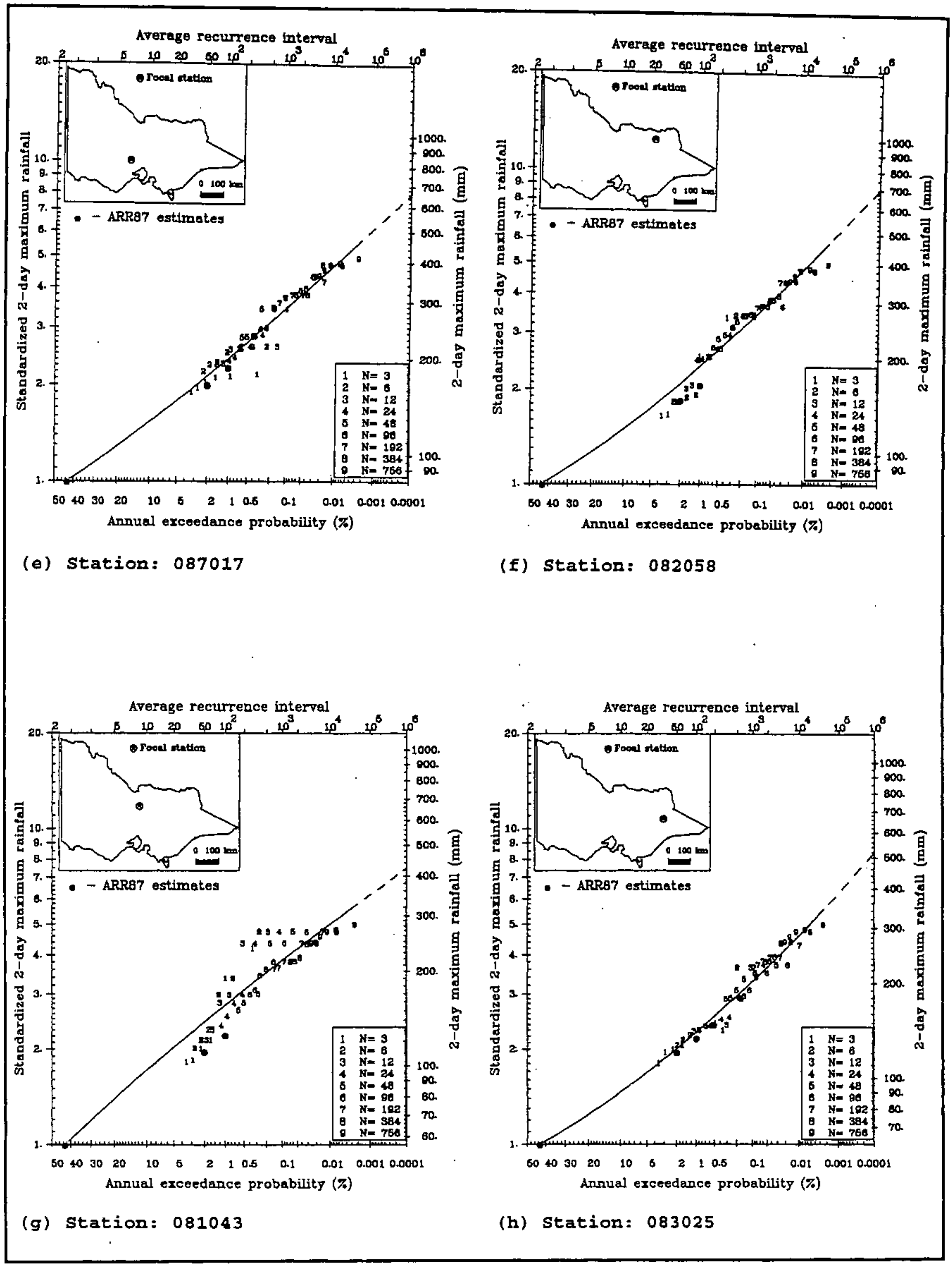


Figure B.1 (continued) IH-FORGE growth curves for 2-day rainfall maxima at selected Victorian stations

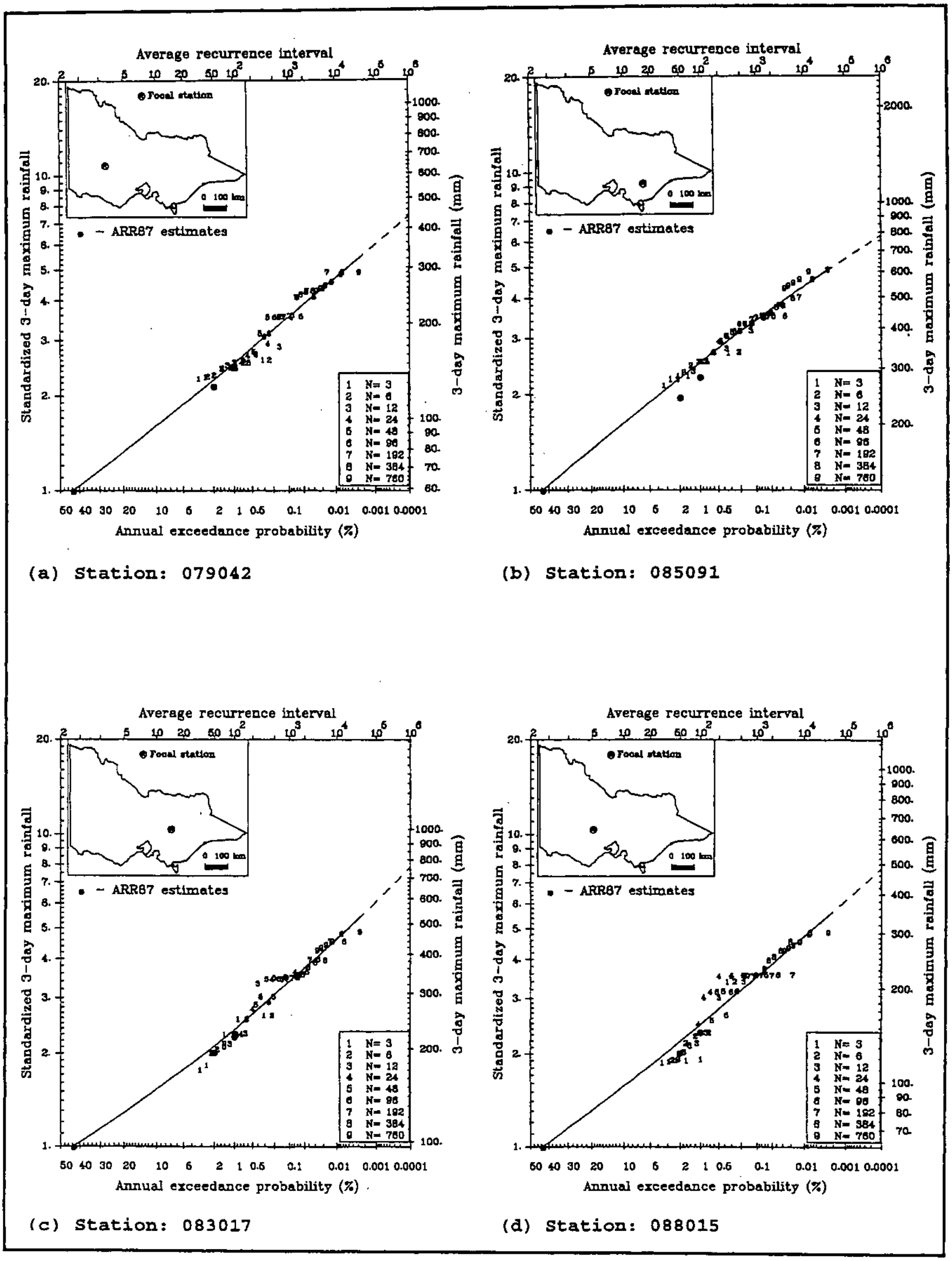


Figure B.2 IH-FORGE growth curves for 3-day rainfall maxima at selected Victorian stations (continued overleaf)

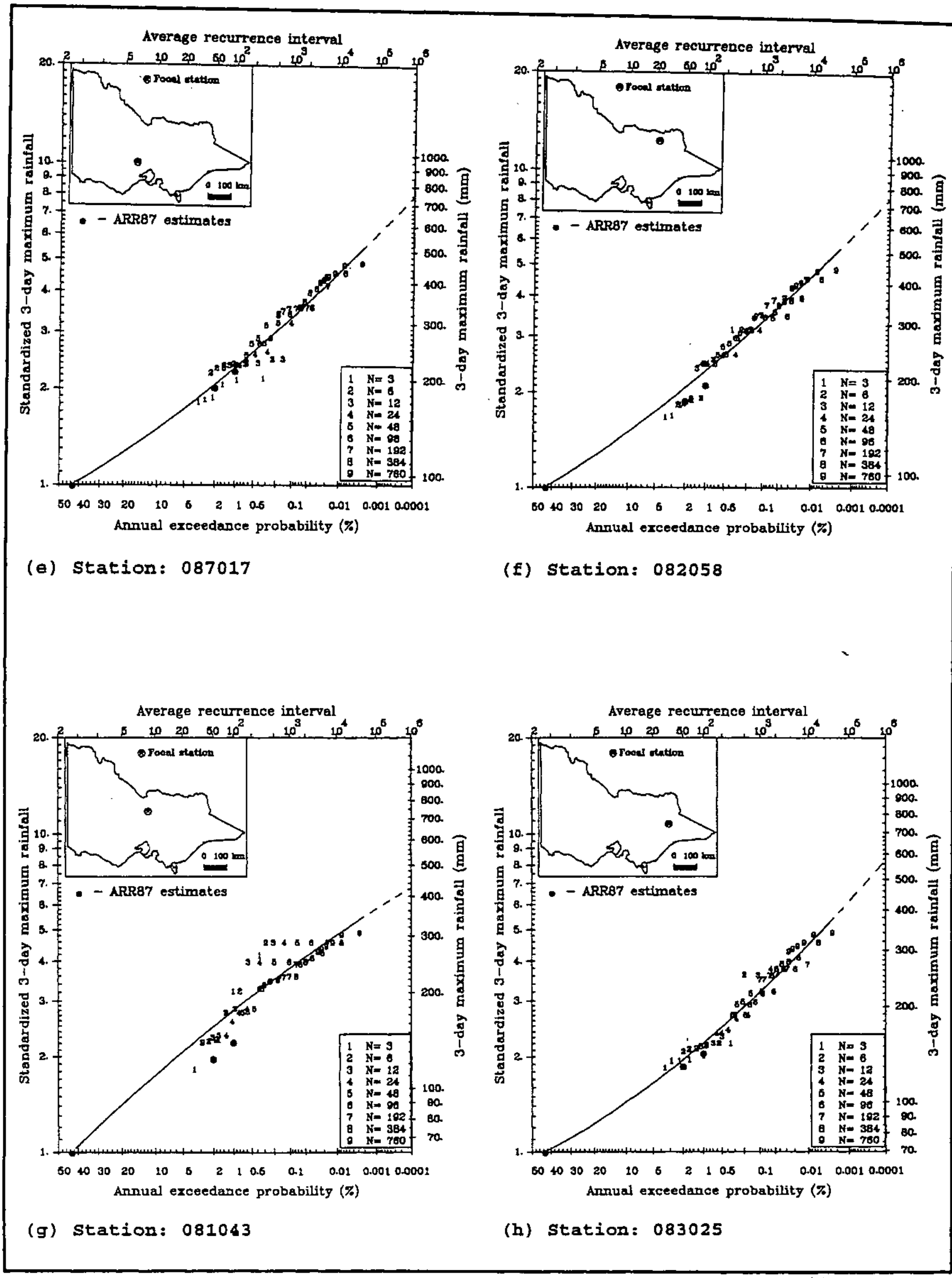


Figure B.2 (continued overleaf) IH-FORGE growth curves for 3-day rainfall maxima at selected Victorian stations

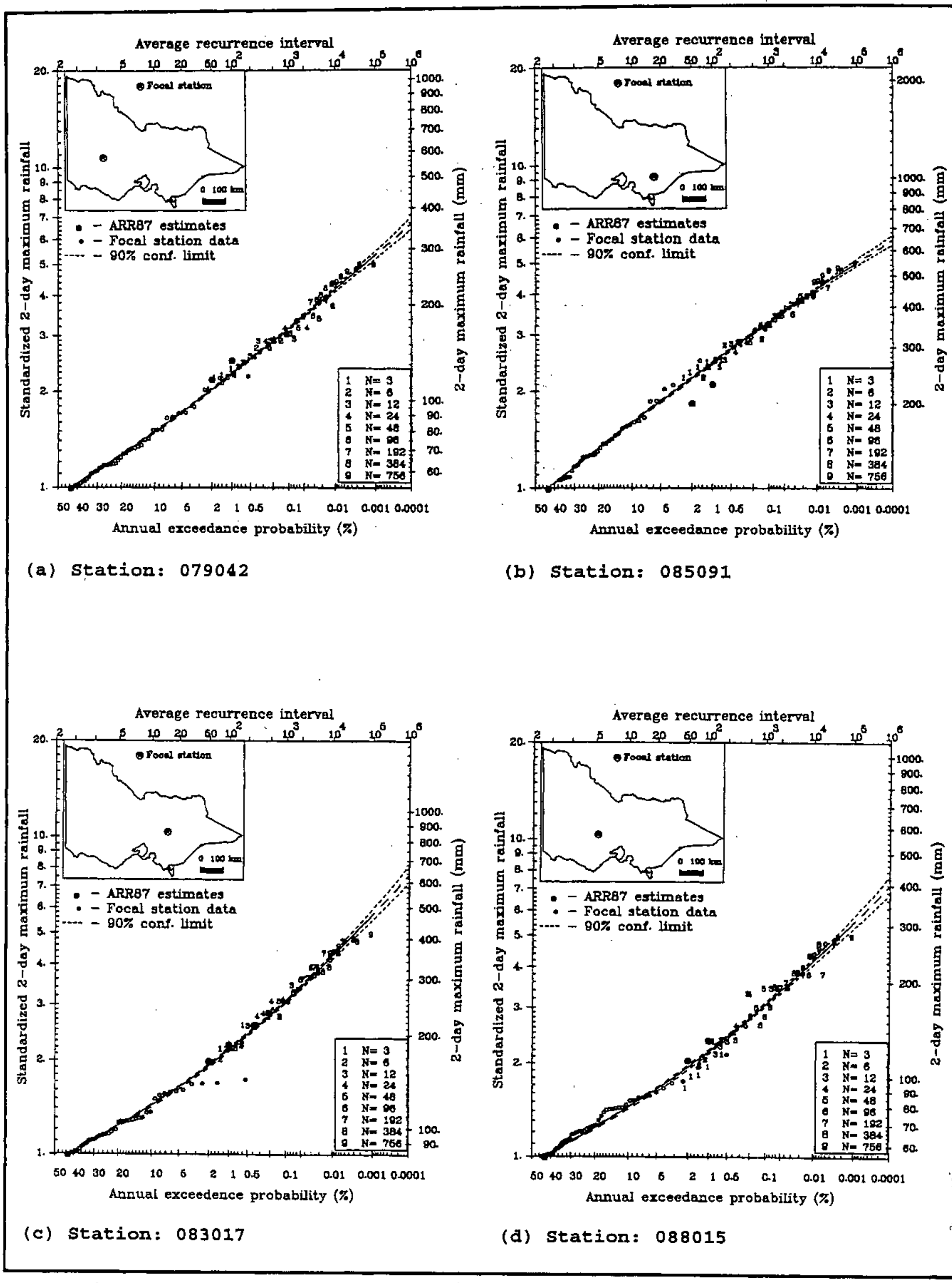
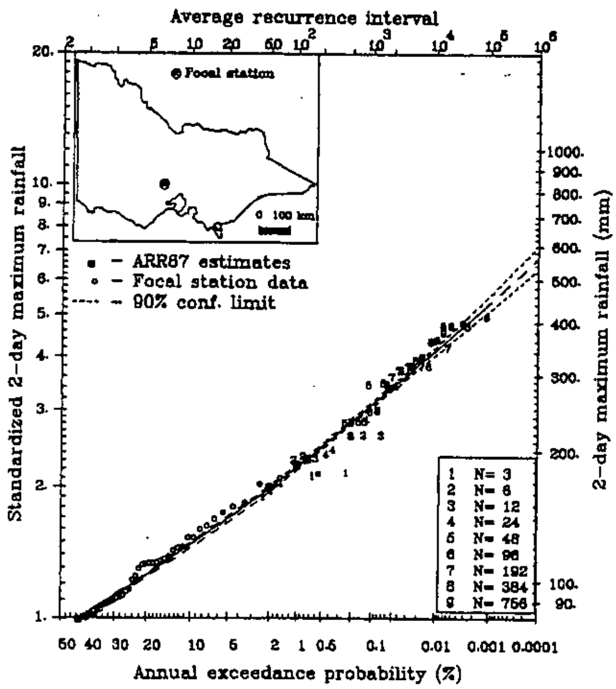
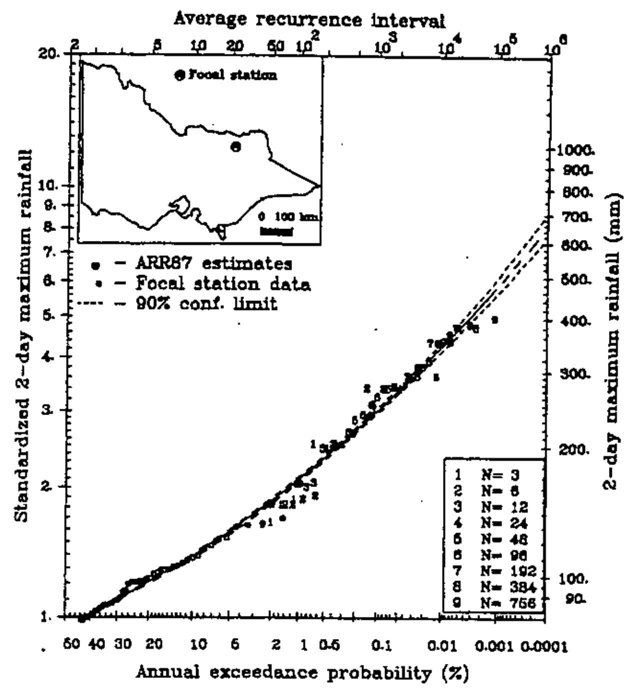


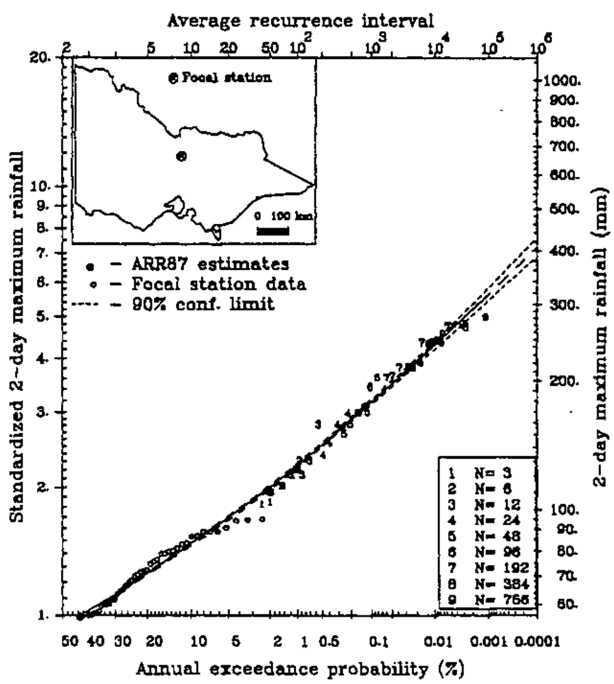
Figure B.3 CRC-FORGE growth curves with 90% confidence limits for 2-day rainfall maxima at selected Victorian stations (continued overleaf)



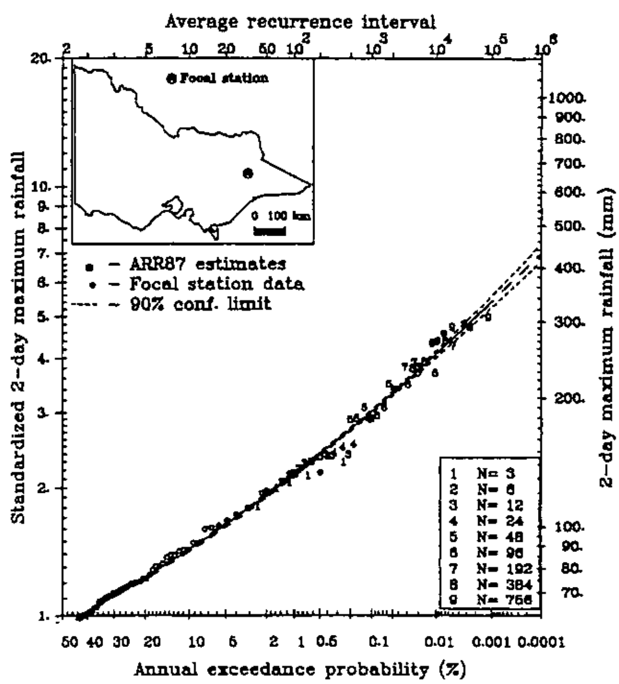
(e) Station: 087017



(f) Station: 082058



(g) Station: 081043



(h) Station: 083025

Figure B.3 (continued) CRC-FORGE growth curves with 90% confidence limits for 2-day rainfall maxima at selected Victorian stations

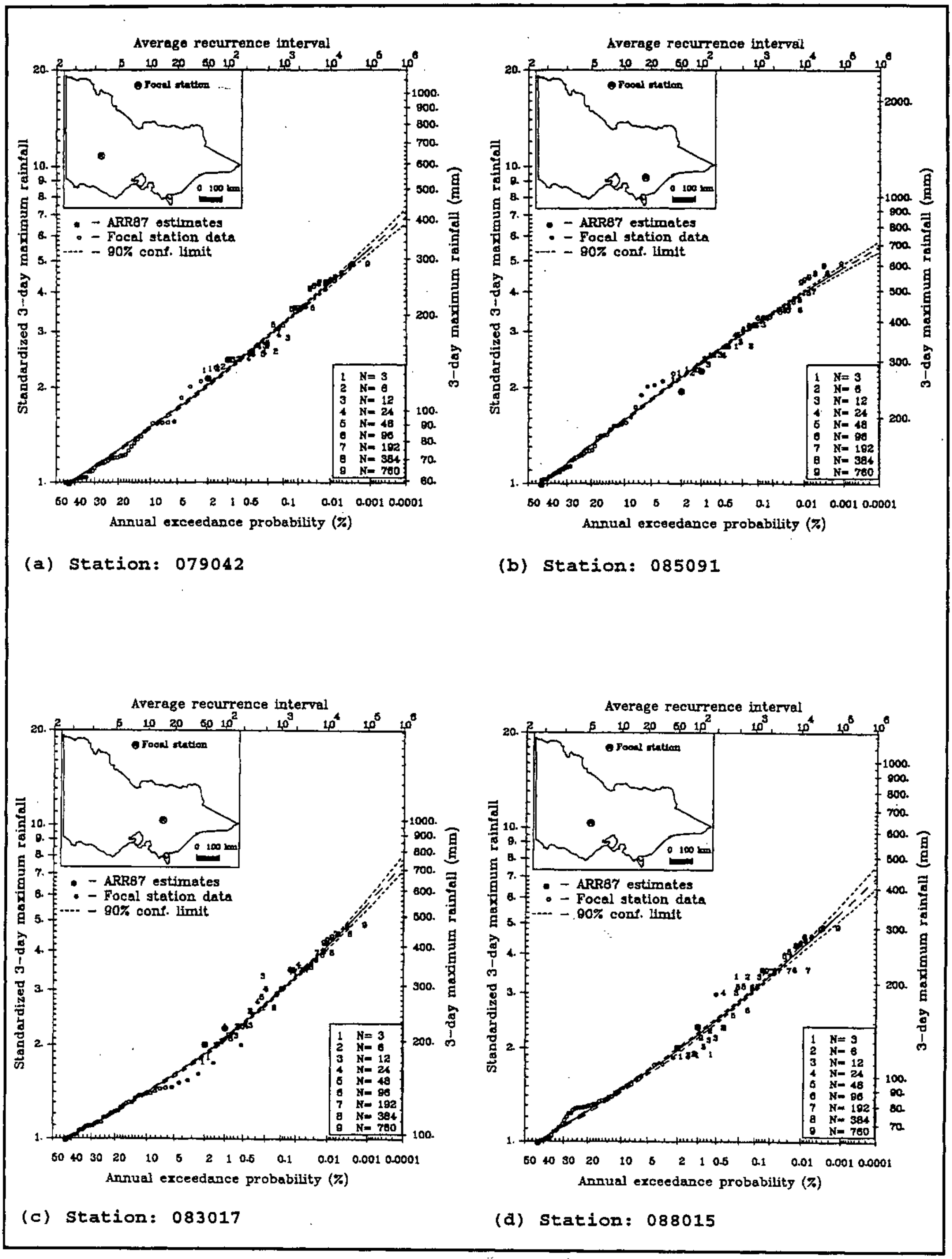


Figure B.4 CRC-FORGE growth curves with 90% confidence limits for 3-day rainfall maxima at selected Victorian stations (continued overleaf)

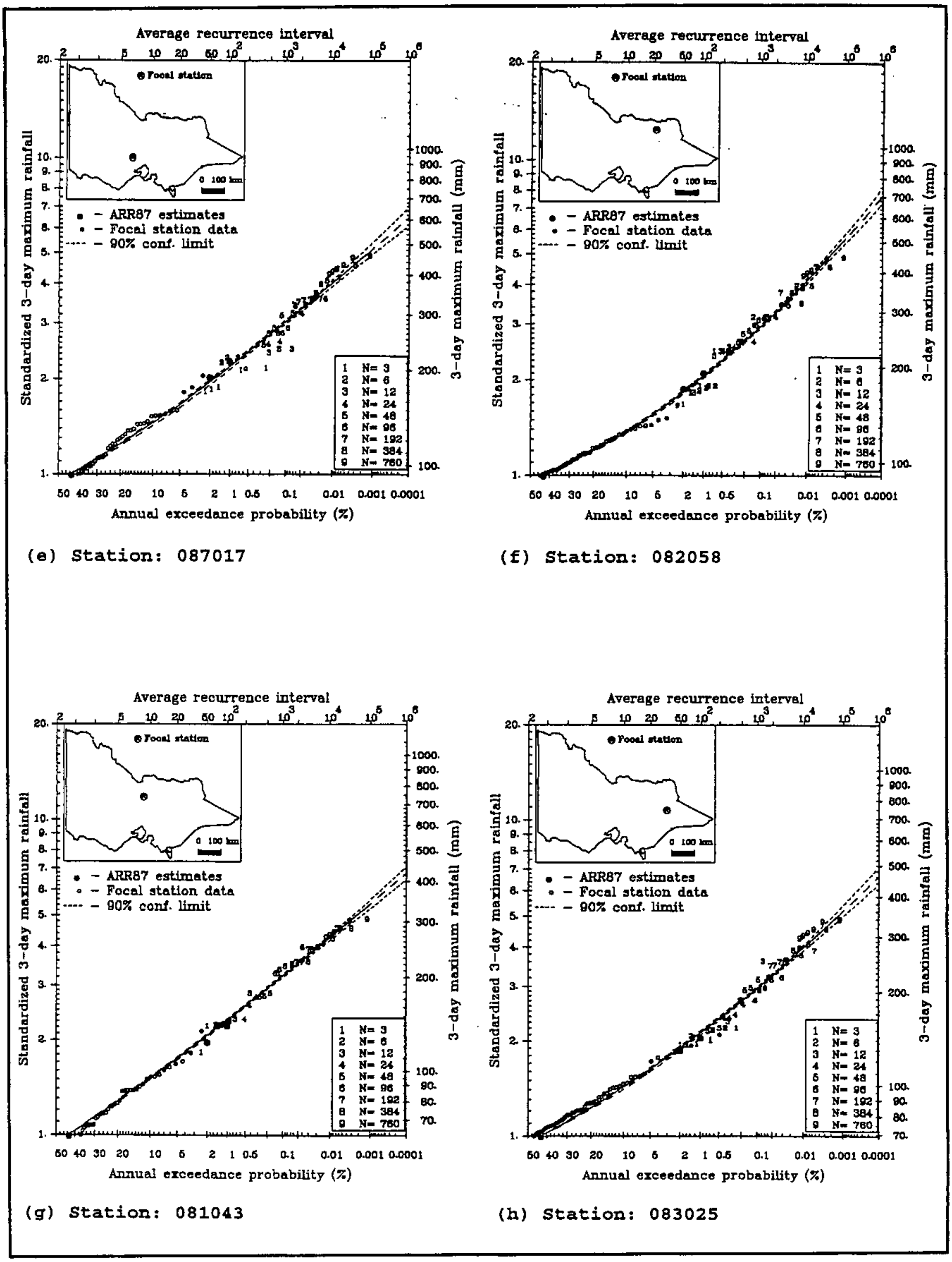


Figure B.4 (continued) CRC-FORGE growth curves with 90% confidence limits for 3-day rainfall maxima at selected Victorian stations

APPENDIX C - CONVERSION OF RAINFALL DATA FOR FIXED PERIODS TO RAINFALLS FOR UNRESTRICTED DURATIONS

C.1 Introduction

In flood design, the interest is on design rainfalls for time periods of various durations, irrespective of the starting time of those periods. However, daily rainfall observations are available for fixed periods of time, usually starting at 9.00 am the previous day and ending at 9.00 am on the day against which the rainfall is recorded. Design rainfall estimates based on the analysis of fixed period daily rainfalls underestimate the amount of rainfall in an unrestricted period of the same duration and thus require correction.

The correction factors used in the analysis of rainfall data to obtain the design values for Chapter 2 of Australian Rainfall and Runoff (IEAust, 1987) are referenced to Miller et al. (1973), but these factors probably go back to Hershfield and Wilson (1958) and Weiss (1964). A recent comprehensive investigation by the UK Institute of Hydrology (Dwyer and Reed, 1995) found correction factors that are slightly higher than the ones previously recommended. The derivation of these new factors for daily rainfall data is summarised below.

C.2 Correction factors for daily rainfall

From the detailed analysis of rainfall records at six stations (including Melbourne, Sydney and Brisbane) Dwyer and Reed (1995) recommended a factor ρ^* in the range of 1.15 to 1.17 to convert *calendar day maxima* to true (24h) maxima. For the Victorian rainfall regime, which includes a mixture of events from thunderstorms and frontal systems, a value of $\rho^* = 1.16$ has been adopted.

For the conversion of *cumulative rainfall totals over D days*, the factor ρ^* needs to be divided by $\rho(D)$. This is the ratio between a 'fixed maximum', computed from fixed intervals of length D days, to the 'sliding maximum', computed from daily data over D sub-intervals. Dwyer and Reed (1995) derived the following formula for the computation of $\rho(D)$ with D measured in days:

$$\rho(D) = 1 + 0.16[1-\exp\{-0.36(D-1)\}]$$

The resulting factors $\alpha(D) = \rho^*/\rho(D)$ are given in Table C.1 These factors were used in Chapter 9 to convert I-day rainfalls to 24-hour values, 2-day values to 48-hour values, etc.

Table C.1 Correction Factors for Unrestricted Rainfall (Based on Eqs. 8.1 and 8.3 in Dwyer and Reed, 1995)

Duration		ρ(D)	α(D)
(days)	**(hours)**		
1	24	1.000	1.160
2	48	1.048	1.106
3	72	1.082	1.072
4	96	1.106	1.049
5	120	1.122	1.034
6	144	1.134	1.023
7	168	1.142	1.016
8	192	1.147	1.011
9	216	1.151	1.008
10	240	1.154	1.005

C.3 References

Dwyer, I.J. and Reed, D.W. (1995). Allowance for discretization in hydrological and environmental risk estimation. Institute of Hydrology, Wallingford, UK; Report No. 123, January 1995.

Hershfield, D.M. and Wilson, W.T. (1958). Generalising of rainfall-frequency-intensity data. IUGG/IAHS Publication No. 43, 499-506.

IEAust (1987). Australian Rainfall and Runoff - A guide to flood estimation. The Institution of Engineers Australia.

Miller, J.F., Frederick, R.H. and Tracy, R.J. (1973). Precipitation frequency atlas of the western United States. Vol. III: Colorado. NOAA Atlas 2, National Weather Service, US Dept. of Commerce.

Weiss, L.L. (1964). Ratio of true to fixed-interval maximum rainfall. J. Hydraul. Div. Proc. ASCE, Vol. 90, 77-82.

APPENDIX D - STEP-BY-STEP PROCEDURE FOR CRC-FORGE METHOD

D.1 Preparatory Steps

1.1 Prepare a data base of daily rainfall data for all stations in the region(s), using a common format and data structure.

1.2 For each station whose length of record exceeds a nominated minimum record length, extract annual maximum rainfall depths for the durations of interest (see Section 4.2).

1.3 Check the annual maximum rainfall series for data errors, inconsistencies and non-stationarities (see Sections 4.4 and 4.5).

1.4 Standardise the annual maximum rainfall data (divide by index variable) and adjust the data from restricted to unrestricted durations (see Appendix C).

1.5 Test for homogeneity of the rainfall region(s) and identify the appropriate probability distribution(s), as described in Chapter 5.

D.2 Computation of Growth Factors at a Gauged Site

2.1 Define a subset of rainfall gauge sites with sufficient record length to give confident at-site estimates of rainfall frequency.

2.2 Select a site from this subset as the focal point for a CRC-FORGE analysis.

2.3 Form a sub-region defined by the focal point station and the two rainfall stations closest to it (total of 3 stations), then pool the standardised annual maximum rainfall data from this sub-region.

2.4 Select the six highest independent values from the pooled data and determine their plotting positions, using the Cunnane plotting position formula with the value of N_e determined from the CRC Variable N_e model.

2.5 Plot these six FORGE data points on probability paper at their appropriate plotting positions.

2.6 Double the total number of stations included in the next sub-region around the focal point and repeat Steps 2.3 to 2.5 until all the stations in the homogeneous region are included in the pooled data set.

2.7 Screen the plotted FORGE data points for outliers (using the procedure described in Section 7.1.2).

2.8 Plot focal station data points at their single-site plotting positions (include only the largest n_f data points, where n_f is the total number of FORGE data points after elimination of outliers).

2.9 Fit a GEV distribution to all the data points, using the least squares fitting procedure described in Section 8.3.2 (or use an appropriate method to fit the alternative distribution found in Step 1.5).

2.10 Compute growth factors for the nominated AEP values.

D.3 Computation of Point Rainfall Quantiles at Gauged Sites

3.1 Compute rainfall quantiles for the specified gauge site and nominated AEP values by multiplying the index value (mean annual maximum rainfall) for that site by the growth factors computed in Step 2.9.

D.4 Estimation of Point Rainfall Quantiles for Ungauged Sites

The following steps can be undertaken separately for each rainfall duration or in a combined fashion, allowing for the relationship between the index values and growth factors for the different durations.

4.1 Select all rainfall stations in the region with a sufficient record length to allow confident estimation of the *index variable*, and compute the value of the index variable for each site.

4.2 Prepare a map showing the value of the index variable at each selected site.

4.3 Use a manual or computer-based spatial interpolation procedure to draw isolines of the index variable covering the whole region.

4.4 Compute point rainfall *growth factors* for each site selected in Step 2.1 by repeating Steps 2.2 to 2.10 with the selected site as the focal point.

4.5 Prepare for each nominated AEP a map showing the growth factor values at the focal point locations.

4.6 Use a manual or computer-based spatial interpolation procedure to draw isolines of the growth factors covering the whole region.

4.7 Compute point rainfall quantile estimates for any site within the region by multiplying the interpolated growth factor values by the corresponding value of the index variable.

The Cooperative Research Centre for Catchment Hydrology
is a cooperative venture formed under the Commonwealth CRC Program between:

Bureau of Meteorology

CSIRO Division of Water Resources

Department of Natural Resources and Environment

Goulburn-Murray Water

Melbourne Water

Monash University

Murray-Darling Basin Commission

Southern Rural Water

The University of Melbourne

Wimmera-Mallee Water

Associates

CSIRO Division of Soils
State Forests of NSW

COOPERATIVE RESEARCH CENTRE FOR
CATCHMENT HYDROLOGY

Centre Office
Department of Civil Engineering, Monash University, Clayton, Victoria 3168, Australia.
Telephone: (03) 9905 2704 Facsimile: (03) 9905 5033 International: +61 3 9905 2704
http://www-civil.eng.monash.edu.au/centres/crcch/