A TECHNIQUE TO INTERPOLATE FREQUENCY CURVES BETWEEN FREQUENT EVENTS AND PROBABLE MAXIMUM EVENTS

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PREFACE

CRC Project D3 "Probability and Risk of Extreme Floods" was formulated to meet the needs of Australian dam owners with the assessment of spillway adequacy. The risk assessment for large reservoirs requires the computation of a flood frequency curve covering the range of events from "frequent" to "probable maximum"; the existing methodology for this tended to be very conservative, due to the uncertainties of extreme event estimation.

A key research outcome from Project D3 has been methodology to extract more information from daily rainfall data. With this methodology (CRC-FORGE), more reliable estimates of design rainfalls of duration 6-120 hours, and annual exceedance probabilities (AEP) in the range 1 in 100 to 1 in 2000 are now possible.

This report deals with the interpolation range for even rarer events, i.e. from the 1 in 2000 to the Probable Maximum Precipitation (PMP). Here, the authors address the issue of a consistent and acceptable way to construct frequency curves where no data are available.

The D3 project has made substantial contributions for the important task of extreme rainfall (and hence flood) estimation. This report by Lionel Siriwardena and Erwin Weinmann typifies the efforts, made in the CRC for Catchment Hydrology, to present research outcomes in a way designed to facilitate their implementation into practice. I commend it to you.

Russell Mein
Program Leader, Flood Hydrology
CRC for Catchment Hydrology

ABSTRACT

This report describes a pragmatic approach to extend directly derived rainfall frequency curves to the probable maximum precipitation (PMP) through a well defined transition function. Importantly, the slope of the interpolated curve is not constrained in approaching the PMP, unlike in the present Australian Rainfall and Runoff (ARR87) procedure.

The 'generalised procedure' to fill the 'gap' in the frequency curve was developed on the basis of work by Lowing (1995) for the UK; a simple 2-parameter parabolic function was fitted in log-log space to satisfy requirements at either end of the curve. Conditions for satisfactory behaviour of the function were stipulated. The report presents two specific adaptations of the generalised procedure; ie. (i) to extend the frequency curve from an **AEP of 1 in 2000** to the PMP, to apply with CRC-FORGE based design rainfall frequency curves and (ii) to extend the frequency curve from an **AEP of 1 in 100** to the PMP, with design rainfall quantiles estimated from the ARR87 procedures.

The proposed interpolation procedure has been shown to perform satisfactorily over a large range of theoretical conditions and design situations:

- different starting points for interpolation (AEP of 1 in 100 to 1 in 2000);
- different AEPs assigned to the PMP (ranging from 10^{-4} to 10^{-7} depending on the catchment size);
- different 'shape parameters' defined by the ratio of the slope of the upper end of the directly determined frequency growth curve, S_{gc}, and the slope between the two end points of the 'gap', S_{gap}, (the 'shape parameters' S_{gc}/S_{gap} ranging from 0.25 to 2.0).

For satisfactory application of the procedure in practical design situations, the directly determined part of the frequency curve and the PMP with its assigned AEP must be compatible. This condition places upper and lower limits on the acceptable shape parameter values. The application of the procedure can be extended to interpolate flood frequency curves; compatibility checks are particularly important when applying it directly to design floods, generally estimated to a lower AEP limit of 1 in 100.

Application of the new methodology to 25 Victorian catchments of diverse characteristics showed that the resultant rainfall frequency curves were plausible and well behaved; the new interpolation procedure can be successfully applied for diverse practical design situations. The methodology was also found to be satisfactory in application to shorter storm durations (< 24 hours), with design rainfalls for AEPs 1 in 200 to 1 in 2000 estimated by an approximate procedure; frequency curves were shown to be consistent over a practically significant range of storm durations.

Limited testing on rainfall frequency curves for Victorian catchments indicated that the new interpolation procedure resulted in a significant reduction of the design estimates in the interpolated range compared to those derived from the current interpolation procedure (Chapter 13, ARR87).

The proposed interpolation procedure is of purely empirical nature. While it is able to bridge significant gaps, the uncertainty of the interpolated values is expected to increase with the size of the gap. This uncertainty should be kept in mind when basing decisions on interpolated rainfall or flood data.

ACKNOWLEDGMENTS

The development of the procedure described in this report has benefited from discussions with R.G. Mein, R.J. Nathan and N. Nandakumar who were part of the project team. The permission of the Victorian Rural Water Authorities and Melbourne Water to use results of their spillway adequacy studies for comparisons is gratefully acknowledged. Sinclair Knight Merz assisted in testing the procedures on 25 Victorian catchments.

The authors also wish to thank representatives of the Bureau of Meteorology, the Department of Land and Water Conservation, NSW, the Department of Natural Resources, QLD, and the Hydro-Electric Corporation, TAS, for comments received on an earlier draft of this report.

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1. INTRODUCTION

1.1 Purpose of study

This report summarises outcomes of a research component of Project D3 "Probability and Risk of Extreme Floods" of the CRC for Catchment Hydrology (CRCCH). It details a pragmatic approach to interpolate frequency curves between "frequent events" and the "probable maximum event (PME)", and to thus allow the definition of complete frequency curves of design estimates. Specifically, this study aimed at deriving a well behaved transition function over the interpolated range, without making a restrictive assumption about the slope of the frequency curve at the PME.

1.2 Basic approaches to estimation of complete frequency curve

The problem of extending a fitted rainfall or flood frequency curve beyond the normal range of extrapolation (say to an AEP of 1 in 100 for estimates from at-site data and to an AEP of 1 in 1000 for rainfall estimates from regional data) can be approached in different ways. Firstly, the extrapolation/ interpolation procedure can be applied either to rainfall or to flood data. When the methods are applied to rainfall data, it is necessary to convert the complete rainfall frequency curve to a complete flood frequency curve by additional procedural steps. (Hybrid approaches using both rainfall and flood frequency data have also been proposed.) For both types of basic data the available methods can then be classified into three main groups.

(i) *Direct extrapolation of fitted theoretical probability distribution:*

This is the traditional method for estimating floods to AEPs of about 1 in 10000 from flood frequency analysis and is still the predominant method applied in many European countries (ICOLD, 1992). The GRADEX method of estimating extreme flood frequencies developed and applied in France (FRCOLD, 1994) can also be regarded as belonging to this approach. This method is based on extrapolation of the rainfall frequency curve (generally using a Gumbel distribution) and use of relatively simple runoff production and transfer functions to convert the extreme rainfall estimates to flood estimates. Its applicability to Australian conditions has only been investigated to a very limited extent. The AGREGEE method (FRCOLD, 1994) represents a further development of the GRADEX approach which places it closer to the methods described under (iii).

(ii) *Extrapolation of fitted probability distribution with the PME as an upper bound:*

There are a number of theoretical probability distributions that are bounded above (eg. Extreme Value Type III [Stedinger et al., 1993], Logistically Constrained Distributions [Brady, 1977]). It is also possible to specify not only the magnitude of the upper bound but also the AEP at which it will be reached. This approach has been proposed by Eliasson (1994, 1997), using a transformed distribution function or a cut-off distribution function based on the Extreme Value I (Gumbel) distribution.

(iii) *Empirical interpolation between fitted frequency curve and PME:*

Most of the methods investigated by Rowbottom et al. (1986) fall into this category, including the method recommended in Australian Rainfall and Runoff (ARR87, IEAust, 1987). There is no formal theoretical basis for the methods in this group, and their success is judged purely on a heuristic basis. The proposed CRCCH interpolation method introduced in this report fits into this group of methods. It could be regarded as an example of the 'Rainfall Extrapolation' method described by Rowbottom et al. (1986).

All three groups of methods share the common limitation that their theoretical or empirical basis cannot be rigorously tested against observed data in the range of interest, but they differ in the degree of flexibility for application in different situations. The more advanced methods from the third group can be regarded as being the most flexible and thus best suited to provide a common basis for application to the widely different flooding regimes encountered across Australia.

1.3 Background on proposed method

The currently recommended estimation method of rare events between an AEP of 1 in 100 and the AEP of the probable maximum event (PME) is described in Chapter 13 of ARR87 (IEAust, 1987). It is based on the criteria of being reasonable in a subjective sense, consistent and providing some degree of conservatism, rather than being based on rigorous analysis of data. In this method, the shape of the frequency curve for rare events in the intermediate range is based on the slope of the frequency curve between the AEPs of 1 in 50 and 1 in 100, and on the magnitude and assigned probability of the PME. *The frequency curve is made tangential to a horizontal line through the PME.* The assigned AEP of the PME is determined from criteria based on meteorological considerations and the shape of the frequency curve. The interpolation method can be applied to both rainfall and flood events.

Since then, there has been considerable development in regional rainfall frequency estimation methods in Australia, allowing estimation of more accurate design rainfall quantiles up to an AEP significantly lower than that stipulated in ARR87. The outcomes of CRCCH Project D3 "Probability and Risk of Extreme Floods" have made a major contribution to this development. The recently developed design components include:

- the CRC-FORGE method for estimation of *point* design rainfall quantiles up to an AEP of 1 in 2000 for any given location (Nandakumar et al., 1997, Nandakumar and Weinmann, 1998);
- the CRCCH methodology for derivation of areal reduction factors using daily rainfall data (Siriwardena and Weinmann, 1996);
- recommended annual exceedance probabilities for the probable maximum precipitation (PMP) estimated from generalised procedures (Laurenson and Kuczera, 1998).

The 'index variables' and 'growth factors' which allow for estimation of CRC-FORGE *point* design rainfall quantiles, and a set of CRC-developed areal reduction factors have been derived for Victoria. These are currently being derived for regions in other states of Australia, namely New South Wales, Queensland and Tasmania.

This report addresses the missing link in the determination of the 'complete' design rainfall frequency curve, ie. an appropriate procedure to interpolate the rainfall frequency curve from an AEP of 1 in 2000 (AEP of 1 in 100 in situations where CRC-FORGE estimates are not available) to the assigned AEP of the PMP.

1.4 Objectives and scope

There is now an increasing body of opinion that the frequency curve should not be asymptotic to a horizontal line at the PMP, but should extend smoothly through the notional AEP of the PMP point at a slope defined by the trend of the curve, as it is clear that a more extreme combination of factors could yield a larger event at a lower AEP (Laurenson and Kuczera, 1998). The currently used constraint (ARR, 1987) results in deliberately conservative estimates in the intermediate range.

To address the above concerns as well as to incorporate the latest developments in the design flood estimation procedures, the aim of this study was to develop a *generalised procedure* to objectively interpolate the rainfall frequency curve from the largest event that can be reliably determined from direct frequency analysis of data to the PMP. In this procedure, no constraint was placed on the manner in which the frequency curve reaches the PMP.

In line with the recent research developments, this report presents an adaptation of the generalised procedure to extend the frequency curve from an ***AEP of 1 in 2000*** to the AEP of the PMP. The report also illustrates a slight modification to the above procedure to interpolate the frequency curve from an ***AEP of 1 in 100*** to the AEP of the PMP, for situations where only ARR87 design quantiles are available.

The applicability of the proposed procedure was then evaluated by testing it on data for 25 Victorian catchments of diverse topographical and meteorological characteristics. Its applicability to short storm durations (6 - 18 hours) was also examined.

In this report, the proposed interpolation procedure is specifically developed for use with the design *rainfall* frequency curve rather than with the flood frequency curve. This is because in conventional design situations, 'complete' design rainfall frequency curves (for a range of storm durations) are first determined and these are then transformed to a 'critical' flood frequency curve. However, the proposed procedure could in principle be applied for interpolation of flood frequency curves, if the situation warrants this. Its applicability to floods is restricted because of greater uncertainty in the slope at the starting point of the interpolated flood frequency curve. Factors affecting the applicability with design rainfalls and floods are discussed in Section 3.3.

1.5 Outline of report

The remainder of this report is structured as follows. Chapter 2 presents a critical review of the currently adopted interpolation procedures of rainfall or flood frequency curves. In Chapter 3, the need for a change in philosophical concepts in procedures for deriving estimates for lower order AEPs is discussed; accordingly, a generalised procedure to derive the frequency curve in the extreme range of AEPs is fully described.

Chapter 4 presents an adaptation of the generalised procedure to extrapolate the frequency curve from an AEP of 1 in 2000 to the AEP of the PMP, applicable to regions where CRC-FORGE design rainfalls are determinable. Results of extensive testing of the methodology against 25 Victorian catchments are also presented in this chapter. An alternative approach for frequency curve interpolation, for application in regions where CRC-FORGE design quantiles are not available, is presented in Chapter 5. An extension of the interpolation procedure to short duration rainfalls, based on approximately estimated CRC-FORGE rainfall quantiles described in Appendix A, is introduced in Chapter 6; the consistency of the approach across a wide range of storm durations is examined. The summary and conclusions drawn from the study are presented in Chapter 7.

2. FREQUENCY CURVE FOR INTERMEDIATE PROBABILITIES: CURRENT PRACTICE AND OTHER RELEVANT APPROACHES

2.1 Outline of overall approach

Adoption of regionalised procedures for estimation of probable maximum precipitation (PMP) over the recent years has led to significantly higher values of probable maximum floods (PMF) than those used in the design of spillways previously. As a consequence, it has been necessary to review the spillway adequacy of many Australian dams. The "Guidelines on Design Floods for Dams" by the Australian National Committee on Large Dams (ANCOLD, 1986) provide the basis for determining the acceptable level of risk of failure of dams from flooding. These guidelines are currently in the process of substantial revision (ANCOLD, 1998). Risk assessment techniques for spillway adequacy require a continuous flood frequency curve to the AEP of the probable maximum flood (PMF); the current procedure of construction of 'complete frequency curves' of rainfalls and floods is described in Chapter 13 of ARR87 (IEAust, 1987).

In the *rainfall-based* approach, design values of rainfall intensity for specified storm durations and AEPs are used in combination with "typical values" of other relevant inputs such as spatial and temporal patterns of rainfall, and loss parameters. It is then assumed that the resulting design flood corresponds to the AEP of the design rainfall. In practical application, this procedure leads to determination of complete rainfall frequency curves for different storm durations and then transformation to a 'critical' flood frequency curve.

In the current procedure, the basic steps involved in the construction of 'complete frequency curve' are summarised below.

(i) *Frequency curve for AEP ≥ 0.01 (1 in 100):*
The *rainfall frequency curve* is based on the design intensity-frequency-duration (IFD) curves (Chapter 2, ARR87); this can then be transformed to a *design flood frequency curve* by applying a calibrated rainfall-runoff model. Alternatively, the flood frequency curve can be directly constructed by statistical treatment of recorded maximum (annual or partial series) flood data (Chapter 10, ARR87).

(ii) *Probable maximum precipitation (PMP) and its assigned probability:*
PMP estimates for short storm durations (≤ 6 hours) are determined using the Generalised Short Duration Method, GSDM (BoM, 1994), whereas PMP estimates for long durations are determined using the Generalised Southeast Australia Method, GSAM (Pearce and Kennedy, 1994) or Generalised Tropical Storm Model, GTSM

(Kennedy et al., 1988). The assigned AEP to the PMP is determined from criteria based on meteorological considerations and in conformity to the slope of the frequency curve (Section 13.5.3, ARR87); it is generally assumed that the AEPs of the PMP and PMF are identical.

(iii) *Frequency curve between AEP of 1 in 100 and AEP of the PMP/PMF*
This is constructed using the shape factors given in Section 13.5.4 of ARR87. These factors are based on a family of smoothly sketched curves which are made tangential to the frequency curve at the AEP of 1 in 100 and to a horizontal line through the PME plotted at its assigned AEP.

These steps are described in more detail in Sections 2.3 to 2.5.

2.2 Uncertainty in estimated frequency curve

Nathan and Weinmann (1995) state two main factors contributing to the uncertainty in design flood estimation, particularly in estimation of rare and extreme events. These are:

- uncertainty arising from the limitations of knowledge of meteorological and hydrological processes responsible for producing extreme floods (absolute uncertainty);
- uncertainty arising from the subjective elements involved in the flood estimation procedure followed by individual hydrologists (relative uncertainty).

Due to the above factors, it needs to be emphasised that the uncertainties associated with estimation of both PMP and its assigned AEP are very large. Laurenson and Kuczera (1998) indicated that the 75% confidence limits of the AEP of the PMP are estimated to be plus or minus one order of magnitude of AEP.

Figure 2.1 illustrates the uncertainties associated with the general procedure for constructing a complete design rainfall frequency curve (neglecting uncertainty in the directly determined frequency curve component to an AEP of 1 in 100); a range of plausible frequency curves can be constructed with different conditions imposed in approaching the PMP. The uncertainty associated with the determination of the PMP and its assigned AEP further compounds the uncertainty of the design quantiles in the interpolated range. It is recognised that the uncertainty in the estimation of design quantiles increases with decreasing AEP.

The uncertainty arising from the subjective elements involved with the frequency curve interpolation over the large range from the AEP of 1 in 100 to the assigned AEP of the PMP is considered to be very large. The assumption of horizontally approaching the PMP in the current interpolation procedure of ARR87 leads to deliberately conservative estimates over the interpolated range.

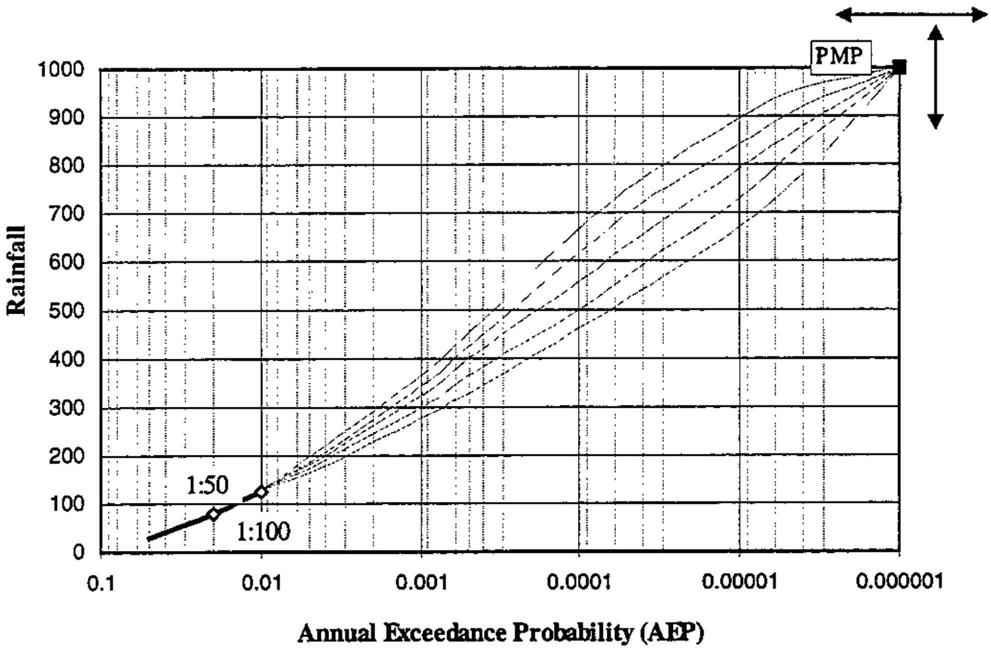


Figure 2.1 : Uncertainty in construction of frequency curve

2.3 'Directly determined' frequency curve

2.3.1 ARR87 procedure

(i) Rainfall-based design flood estimation:

Chapter 2 of ARR87 presents a procedure to derive spatially consistent IFD design rainfall curves for any desired location in Australia. The development of IFD curves is based on fitting a log-Pearson type III distribution to annual maximum point rainfall data allowing a positive skewness of up to 0.8. These IFD curves allow derivation of 'point rainfall frequency curves' for a *range of AEPs to a lower limit of 1 in 100* and for durations from 5 minutes to 72 hours. These point rainfall curves can then be transformed to 'areal (design) rainfall frequency curves' by applying areal reduction factors (ARFs) given in Chapter 2 of ARR87. These ARF values have been derived based on US data and are invariant with the AEP.

(ii) Flood frequency analysis:

This method involves derivation of flood frequency curves directly, using *at-site* or *regional* flood frequency methods, in situations where sufficient recorded streamflow data are available. In an at-site procedure, Chapter 10 of ARR87 recommends fitting a log-

Pearson type III distribution to an annual series of flood peaks. In general, satisfactory results can only be obtained to a minimum AEP of 1 in 100.

2.3.2 New developments in deriving rainfall frequency curve (CRCCH approach)

The CRC-FORGE method developed by Nandakumar et al. (1997) allows a 'growth curve' (standardised rainfall frequency curve) of *point rainfall depths* to be drawn for any site with long rainfall record, using regional rainfall data. The standardisation is based on the mean annual maximum rainfall (the "index variable") for the duration of interest, which can be readily calculated for any site with rainfall records of moderate length. 'Growth factors' express the ratio of rainfall quantiles for given AEPs in relation to this index variable or a specified rainfall quantile used as the scaling variable.

To allow estimation of point design rainfalls for any site within the region, the growth factors and the index variables (or the specified quantile used to "scale up" the growth factors) need to be generalised to cover the whole region. The results of this generalisation for Victoria allow the estimation of extreme rainfalls for durations from 12 to 120 hours and for *AEPs from 1 in 50 to 1 in 2000* for any point of interest in the state. Representative catchment point rainfall values can be determined as the average of values at selected points within the catchment boundary.

Design estimates of *areal average rainfall depths* can be obtained by multiplying the CRC-FORGE point rainfall estimates by the corresponding new areal reduction factors derived for Victoria (Siriwardena and Weinmann, 1996). These new ARFs are considered to be more appropriate for design flood estimation than those given in ARR87, as they were derived directly from daily rainfall data for Victoria. Furthermore, the new values make allowance for variation with AEP and cover a wider range of catchment areas.

In situations where rare and extreme events are required to be incorporated in the design approach, it is desirable to first derive the design rainfall frequency curve over the full range of AEPs (ie. up to the assigned AEP of the PMP) and then transform it to a 'critical flood frequency curve' using a rainfall-runoff modelling approach.

2.4 Probable maximum precipitation and its assigned probability

2.4.1 Estimation of probable maximum precipitation

Generalised methods of estimating PMP are now used in Australia, replacing 'in-situ maximisation' and 'storm transposition' methods. The generalised methods encompass all available data over a large region, and include adjustments for moisture availability and differing topographic effects on rainfall depth. The adjusted storm data is enveloped by smoothing over a range of areas and durations. Generalised methods also provide design spatial and temporal patterns of PMP for the catchment. These methods are relatively robust to sampling deficiencies and provide regionally consistent estimates.

The generalised methods currently available in Australia are:

(i) Generalised Southeast Australia Method (GSAM), for use in catchments in southeast Australia. This method is described by Pearce and Kennedy (1994) and Minty et al. (1996).

(ii) Generalised Tropical Storm Model (GTSM), applicable to those parts of Australia affected by tropical storms. The method is described by Kennedy and Hart (1984) and Kennedy et al. (1988).

Generally, The GSAM and GTSM methods involve :

- a dew point based moisture maximisation;
- an envelopment of maximised and standardised rainfall or, in the case of GSAM, of convergence rainfall;
- transposition from the recorded storm site to the catchment of interest;
- an adjustment for differences in various topographical factors.

Application of GSAM and GTSM requires a good understanding of hydrometeorological processes as well as of the procedures themselves, leaving it in the hands of specialised personnel.

(iii) Generalised Short Duration Method (GSDM), suitable for small catchments (≤ 1000 km^2) anywhere in Australia for deriving PMP estimates of short durations (≤ 6 hours). The method is based on the fact that the mechanisms of small and short storms are little dependent on topographic or other local physical factors other than maximum possible atmospheric moisture content. Consequently, storms of this kind that occurred in other countries can be transposed to anywhere in Australia and supplemented by similar Australian storms. These storms can then be adjusted to the appropriate local maximum moisture content with account taken of the type of local terrain and elevation. Bulletin 53 (BoM, 1994) with subsequent amendments describes the procedure in detail.

2.4.2 Assigned AEP to the probable maximum precipitation

Current Australian practice for assigning an AEP to the PMP or PMF for the purpose of plotting a complete frequency curve is set out in ARR87. It consists of determining the AEP by two methods, one being based on meteorological considerations, as determined by Kennedy and Hart (1984), the other on the shape of the frequency curve, as proposed by Rowbottom et al. (1986).

Kennedy and Hart (1984) proposed that the notional AEPs assigned to the PMPs estimated from generalised methods were directly proportional to the catchment area. For large area generalised methods (GSAM and GTSM), notional AEPs ranged from 10^{-7} to 10^{-4} for areas of 10^2 to 10^5 km^2. For the small area generalised method (GSDM), notional AEPs were 10^{-8} and 10^{-7} for areas of 100 and 1000 km^2 respectively.

The Rowbottom et al. (1986) method of assigning an AEP to the PME was based mainly on achieving a reasonable shape (without excessive curvature) for the rainfall or flood frequency curve between the 1 in 100 AEP event, and the PME, when plotted to log-Normal scale. They allocated values of AEP ranging from 10^{-4} to 10^{-7}, depending on the value of the ratio:

$$\frac{\log(X_{PM} / X_{100})}{\log(X_{100} / X_{50})}$$

where X_{PM}, X_{100} and X_{50} are the magnitudes of the probable maximum, 1 in 100 and 1 in 50 AEP events respectively.

Chapter 13 of ARR87 recommends to select the larger value of AEP determined from the above two methods.

Three recent Australian studies carried out to address the problem of assigning an AEP to the PMP are briefly mentioned here:

- Pearce (1994), based on an analysis similar to the work by Kennedy and Hart (1984), proposed that notional AEP values should vary between 10^{-6} and 10^{-7} for all generalised methods of PMP estimation, irrespective of catchment area.
- Nathan et al. (1997), based on a method similar to one proposed by Schaefer (1994) and using the BoM's GSAM storm database, obtained notional AEPs close to those proposed by Kennedy and Hart (1984) for the Inland Zone of the GSAM region.
- Pearse and Laurenson (1997), applying their methodology to an area near Sydney (1700 km^2), estimated an AEP of the PMP of 10^{-5}.

Based on a critical review of all Australian work on assigning an AEP to the PMP, Laurenson and Kuczera (1998) recommended that the AEP values proposed by Kennedy and Hart (1984) for the GTSM be adopted for all current generalised methods, including the GSDM for short durations. These values were found to be not greatly different from those derived by Nathan et al. (1997) for the GSAM Inland Zone. The recommended AEPs are directly proportional to the catchment area and vary linearly from 10^{-7} to 10^{-4} for areas 10^2 to 10^5 km^2 on a semi-log plot (area in logarithmic scale). For areas less than 100 km^2, the AEP of the PMP was conservatively taken as 10^{-7}. Recommended values by Laurenson and Kuczera (1998) are shown in Table 2.1.

Table 2.1 : Recommended AEP of the PMP by Laurenson and Kuczera (1998) for all generalised methods of PMP estimation

	Catchment area (km^2)				
	10	100	1000	10,000	100,000
AEP of the PMP	10^{-7}	10^{-7}	10^{-6}	10^{-5}	10^{-4}

2.5 Current approaches for interpolation of frequency curve between frequent events and extreme events

The current rainfall based design flood estimation procedures used in Australia are based on the assumption that the assigned AEP of the PMF is the same as that of the PMP; this assumption is made for other design flood estimates as well. Hence, the procedure of interpolation for either design rainfalls or design floods from 'frequent' to 'maximum possible' events is the same. Implications of this assumption on practical aspects of design flood estimation are worth for investigating.

2.5.1 *ARR87 recommended procedure for practical applications*

The recommended procedure for interpolation of frequency curve from 1 in 100 event to the AEP of the PME by ARR87 for use in Australian practice was based on a study carried out by Rowbottom et al. (1986). The procedure is based on the criteria that the design estimates should be regionally consistent and reasonable in a subjective sense, and should provide some degree of conservatism. Types of events to which the procedure can be applied include rainfall, peak discharge and flood volume.

In this procedure, the shape of the frequency curve on a log-Normal probability plot between the AEP 1 in 100 event and the PME is constrained by the following criteria:

(i) The frequency curve should be drawn tangential to the horizontal at the PME, implying that the curve is upper-bound and approaches the PME horizontally. (The validity of this criterion is discussed in Section 2.6.)

(ii) The assigned AEP of the PMP is to be based on achieving a reasonable shape of the rainfall or flood frequency curve over the interpolated range. Values of AEP ranging from 10^{-4} to 10^{-7} were allocated to the PME depending on the value of a 'shape index ratio' (see Section 2.4). The larger value of AEP from the AEP corresponding to this 'shape index ratio' and that determined from meteorological considerations (Kennedy and Hart, 1984) should be used.

(iii) The slope of the frequency curve at AEP of 1 in 100 should match with the slope between the estimates at AEPs of 1 in 50 and 1 in 100.

As the approach was based on sketched curves, rather than fitting mathematical functions to satisfy the criteria mentioned above, the following procedure was developed to reduce the subjectivity in sketching curves by individuals.

The development of the procedure for interpolation of event magnitudes between 1 in 100 AEP event and the PME involved the drawing of a set of frequency curves on a log-Normal dimensionless plot. An index of $\frac{\log(X_{PM} / X_{100})}{\log(X_{100} / X_{50})}$ representing the relative slopes of the upper and lower segments of the frequency curve was used to encompass all possible design situations. For an assigned AEP of the PMP, a set of curves was drawn for a range of index values. Figure 2.2 illustrates the case with an AEP of 1 in 10^5 for the PME. Corresponding to each index value, smooth curves were sketched to be tangential to the frequency curve at an AEP of 1 in 100 and to the horizontal line at the AEP of 1 in 10^5. Sets of curves were developed for assigned probabilities of 1 in 10^3, 10^4, 10^5 and 10^6 for the PME.

This set of curves was used to derive a table of factors representing intermediate event magnitudes for different design situations. For the designer to reconstruct these curves in practice, ordinates of the curves at two AEPs between 1 in 100 and the AEP of the PME are read from this table. The ordinates for an intermediate AEP of 1 in Y are given as values of the ratio $\frac{\log(X_Y / X_{100})}{\log(X_{PM} / X_{100})}$. This can be solved for the value of X_Y. Plotting on a log-Normal probability paper of these values at the two relevant intermediate AEPs, the PME at its assigned AEP, the 1 in 100 AEP event and the slope of the frequency curve at this point, will enable the complete frequency curve to be drawn in a consistent manner. The quantile value for any intermediate AEP can then be read from the curve.

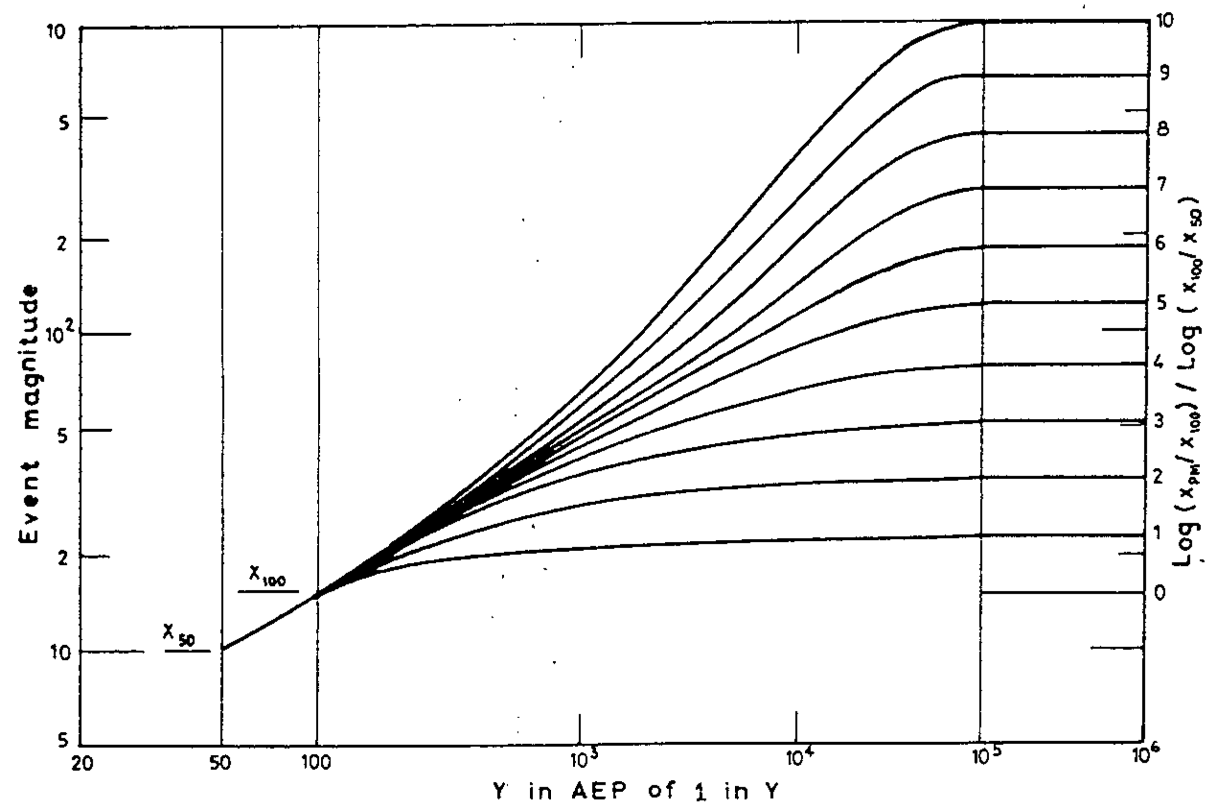


Figure 2.2 : Set of frequency curves for assigned AEP of 1 in 10^5 for probable maximum event (after Rowbottom et al., 1986)

2.5.2 *Recommended procedure for United Kingdom*

Lowing (1995) and Lowing and Law (1995) describe a technique which has been devised for smoothly linking a flood frequency curve, defined up to the AEP of 1 in 1000, with a probable maximum flood (PMF) value. The method was developed, in particular, for use in the United Kingdom in compliance with the Flood Studies Report (NERC, 1975).

The technique is an adaptation of the methodology developed by Rowbottom et al. (1986) and recommended in ARR87 for use in Australia. The criteria for deriving the curve are the same as that of Rowbottom et al. (1986) but the form of the curve is determined by *cubic spline interpolation*, rather than being based on a series of (subjectively) sketched curves. This fitting procedure will construct a smooth curve objectively between the two points where the gradients are known.

Some of the features of this adopted technique for UK, in relation to the methodology of Rowbottom (1986), are:

- As evidence for the UK points to the PMF being approached less frequently than the PMP, the AEPs assigned to the PMP and PMF are not assumed identical. Hence, the proposed interpolation procedure is valid for deriving the flood frequency curve only.

- The UK Flood Studies Report (NERC, 1975) and subsequent supplementary reports provide guidance on the construction of a flood frequency curve to the AEP of 1 in 1000, based on analysis of data. In compliance with this upper limit, the slope of the interpolated curve at the lower end is assumed to match the slope between estimates for AEPs of 1 in 100 and 1 in 1000.

- The gap in the frequency curve between the AEP 1 in 1000 and PMF event is filled using a fitted mathematical function (cubic spline interpolation) to satisfy requirements at the two ends of the curve. This fitting procedure is carried out in semi-log space, whereas Rowbottom et al. (1986) sketched curves on a log-Normal plot.

- The procedure of assigning an AEP to the PMF is similar to that adopted by Rowbottom et al. (1986) in concept, but using a slightly different criterion in satisfying the geometry of the curve. The assigned AEP of the PMF would be decided based on experience and published guidance, as given in the Flood Studies Report (NERC, 1975), subject to considerations such as seasonal effects, catchment size, snowmelt conditions etc. A criterion for 'geometrical consistency' of the frequency curve would then be applied; the AEP is determined in relation to the magnitude of the ratio $\dfrac{Q_{PMF} / Q_{1000} - 1}{1 - Q_{100} / Q_{1000}}$. The higher of the AEPs determined from the two criteria (the methodology-based and geometry-based) is adopted.

Lowing (1995) gives a detailed description of how the segment of the frequency curve from the AEP 1 in 1000 event to the PMF is determined by fitting a third degree polynomial. The coefficients of the polynomial are evaluated to satisfy the requirements of:

- the curve passes through the 1 in 1000 and PMF event magnitudes;
- the slope of the curve at the lower end matches the slope between 1 in 100 and 1 in 1000 events;
- the upper end of the curve is tangential to the horizontal line drawn through the PMF value.

Figure 2.3 illustrates flood frequency curves determined using the proposed interpolation procedure for various extreme combinations (for UK). In this figure, GCa and GCb are two different derived frequency distributions expressed as dimensionless growth curves; PMF 30, PMF 15 and PMF 5 are three PMF magnitudes, and n6, n7 and n9 corresponds to plotting positions of the PMF of 10^{-6}, 10^{-7} and 10^{-9} respectively. The figure illustrates that the procedure produces smooth frequency curves over a wide range of conditions.

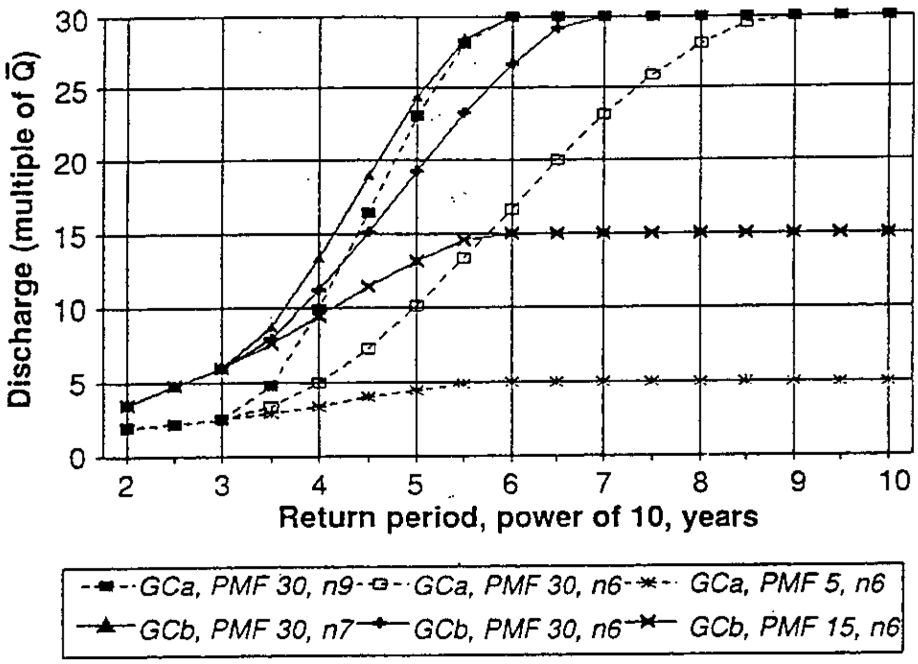


Figure 2.3 : Illustration of frequency curve interpolation (after Lowing, 1995)

2.6 Evaluation of current approaches

Current practice of design flood estimation requires that design estimates for extreme probabilities be obtained by first estimating the probable maximum event (rainfall or flood), assigning an AEP to it, and then drawing a frequency curve between the 1 in 100 year event and the PME.

With such a design practice, the uncertainty associated with a highly subjective interpolation procedure can be very large, noting the absence of data to verify the estimates. Hence, an approach that leads to *deliberately conservative estimates* is recommended by ARR87 for practice. This conservatism in the design approach may have large economic implications for water resources projects.

Since then, a considerable amount of work has been carried out to improve the reliability of the PMP estimate and the AEP assigned to it. Australian PMP estimation procedures are now based on generalised PMP methods using more extensive storm data bases and more liberal assumptions about transposability of storms. This resulted in the PMP estimates being more consistent and reliable with reduced uncertainty. Nevertheless, the estimates from generalised methods were found to be significantly greater than those derived from 'old' methods for many dam sites.

It has been shown that the CRC-FORGE method (Nandakumar et al., 1997; Nandakumar and Weinmann, 1998) yields consistent design rainfall estimates to an AEP of 1 in 2000, and thus establishes an improved basis for the estimation of design floods in the extreme range. This method has been applied to Victorian data to produce design rainfall frequency curves for any site in Victoria for rainfall durations from 12 to 120 hours and for AEPs in the range from 1 in 50 to 1 in 2000. It is currently being applied in other regions of Eastern Australia.

Following these new developments, the range of the frequency curve that requires interpolation has become smaller, so that subjectivity and uncertainty associated with such a procedure will now be comparatively less. While it is not possible to derive rigorously a "correct" interpolation procedure, a more pragmatic approach can now be adopted, based on more realistic assumptions that aim at achieving a solution which will not be overly conservative.

In the light of these developments, the following concepts in the current interpolation procedure (ARR87) are considered to be no longer appropriate, both in terms of their philosophical background and the excessive conservatism introduced by them:

- The assumption that the frequency curve approaches the PME horizontally, ie. curve to be tangential to the horizontal line drawn through the PME.
- The increase in the assigned AEP of the PME due to considerations of geometrical consistency of the frequency curves, where necessary.

There is a growing concern among hydrologists that the frequency curve should not approach the PME horizontally (Laurenson and Kuczera, 1998). Among the factors leading to this concern are:

- The horizontal slope concept (upper limit concept) would be consistent with a definition of Maximum Possible Event (MPE), an unknown true limiting value for the catchment. In contrast, the Probable Maximum Event (PME), being the best operational estimate of a quantity which should not be exceeded, inherits a degree of uncertainty in its derivation; hence, it is associated with an annual exceedance probability. It is also evident that a more extreme combination of factors could yield a larger event at a lower AEP, therefore, there is no reason to believe that an upper limit has been reached with the concept of PME being dealt with.
- There is no apparent justification for the complicated reverse curvature that this assumption may cause in the frequency curve. There is no obvious reason to believe that

there exist differences between the processes causing floods in the two extreme ranges of AEP.

- The approach leads to conservatively high rainfall or flood quantiles in the extreme range of AEPs.

In the absence of knowledge of the shape of the frequency curve in the range of AEPs approaching the PME, and giving due regard to the factors discussed above, it is proposed that:

- The upper limit concept of the frequency curve should be dispensed with. A simpler smooth curve should be drawn to extend to the PME value and beyond, without undue constraints in approaching the PME.
- The assigning of an AEP to the PME should be based entirely on procedures that follow meteorological/statistical considerations. In the present context, the values recommended by Laurenson and Kuczera (1998) should be adopted.

There is also some concern about the degree of subjectivity in the currently recommended procedure (Section 13.5, ARR87). It is based on a subjectively sketched family of interpolation curves and allows direct estimation at two intermediate AEPs only; the construction of the complete frequency curve is still ambiguous and subject to personal interpretation. This can be carried out objectively, using a curve fitting procedure to satisfy the desired constraints. Lowing (1995) defined the frequency curve more specifically using a cubic spline interpolation procedure.

Finally, there is some concern that the option provided in ARR87 to apply the interpolation procedure to either design rainfalls or floods can lead to ambiguity in interpolated flood estimates for a specific site. To achieve greater consistency in the estimation, the definition of flood frequency curve between the AEP of 1:100 to the AEP of the PMF should preferably be determined from the rainfall frequency curve, and not from direct interpolation using design flood quantiles and the PMF (Nathan and Weinmann, 1995). This can be justified on the basis that the slope of the frequency curve between the 1 in 50 and 1 in 100 AEP events can be more reliably determined for design rainfalls than for floods.

3. PROPOSED NEW INTERPOLATION METHOD

3.1 Desirable attributes of a new approach

This chapter describes a modified procedure for interpolation of frequency curves based on a more 'liberal concept' that is easy to apply. To be consistent with the current design practice, the procedure is specifically developed for interpolation of design rainfall frequency curves rather than flood frequency curves. However, the procedure is deemed valid for use with flood quantiles in appropriate design situations.

Desirable characteristics to be considered in the development of the procedure are:

- The curve should represent a well behaved smooth transition over the interpolated range.
- It would be desirable that the slope approaching the PMP is determined by the trend of the frequency curve at the lower end. No constraint should be required at the PMP end unless in the case of an ill-conditioned curve. At the start of the transition, the slope of the interpolated curve should match the slope defined by design estimates from the uppermost segment of the directly defined frequency curve (eg. estimates of 1:50 and 1:100 AEPs with present design practice).
- A simple mathematical function may be fitted to define the curve over the range of interpolation.
- The interpolation procedure should be applicable (or adaptable) to varying design situations:
  - incorporating recent CRCCH outcomes; ie. range of interpolation from an AEP of 1 in 2000 to the assigned AEP of the PMP;
  - using with present design practice; ie. range of interpolation from an AEP of 1 in 100 to the assigned AEP of the PMP.

However, its application should be restricted to situations where it produces plausible curves, rather than forcing an interpolation between design estimates that are based on clearly incompatible data or assumptions.

3.2 Methodology for a 'generalised procedure'

Considering the fact that the fitted mathematical function should desirably be unconstrained in approaching the PMP (no slope constraint), it was considered that the frequency curve in the interpolated range can adequately be represented by a simple 2-parameter function. Coefficients of the function need to be determined to satisfy requirements at either end of the curve. A form of parabolic function was considered acceptable to suit the above requirements. Fitting other forms of functions, such as a power function, was found to be unfavourable due to difficulty in determining coefficients explicitly.

In some situations, a parabolic function fails to approach the PMP in a reasonable manner; this situation particularly arises when the initial slope of the curve is so large, that the parabolic function climbs to a maximum and then downwardly approaches the PMP. Hence, a limiting condition for satisfactory behaviour of the function needs to be defined; this limiting condition is rarely reached in practical design situations. It should also be noted that the new approach does not allow for adjustments to the AEP of the PMP due to considerations of 'geometrical acceptability'; design values and associated AEPs critical to the development of the frequency curve in the interpolated range are determined independently.

The adopted procedure for functional representation of the frequency curve was developed on the basis of the work by Lowing (1995) for the UK. The development was in log-log space, so that a smooth, well behaved transition can be obtained when design quantiles are plotted against AEPs on logarithmic scales. The choice of this development space from possible alternatives (eg. semi-log, log-Normal) was considered to be of minor importance in relation to the uncertainty and subjectivity associated with the interpolation procedure.

The fitting procedure was developed on standardised variables, thereby allowing the determination of shape factors to be used for AEPs in the interpolated range in compliance with the variable AEP assigned to the PMP depending on the catchment area. The computation steps involved in the development of a 'generalised procedure' are described in the following section. Figure 3.1 illustrates the development procedure.

In this 'generalised procedure' it is assumed that the slope of the frequency curve at the commencement of the transition is determined by the slope of the two design values at AEPs of $1{:}Y_1$ and $1{:}Y_2$ (See Figure 3.1). $1{:}Y_{PMP}$ denotes the assigned AEP of the PMP which needs to be determined from the recommended values given by Laurenson and Kuczera (1998) depending on the catchment area. P_{Y1} and P_{Y2} represent design values at AEPs of $1{:}Y_1$ and $1{:}Y_2$, respectively, and P_{PMP} denotes the magnitude of the PMP with an assigned AEP of $1{:}Y_{PMP}$.

The *inverse* annual exceedance probabilities (ie. the average recurrence intervals) are log transformed, and the corresponding ordinate values (design rainfall quantiles) are log transformed and then standardised by the quantity corresponding to the AEP of $1{:}Y_2$ (see Figure 3.1).

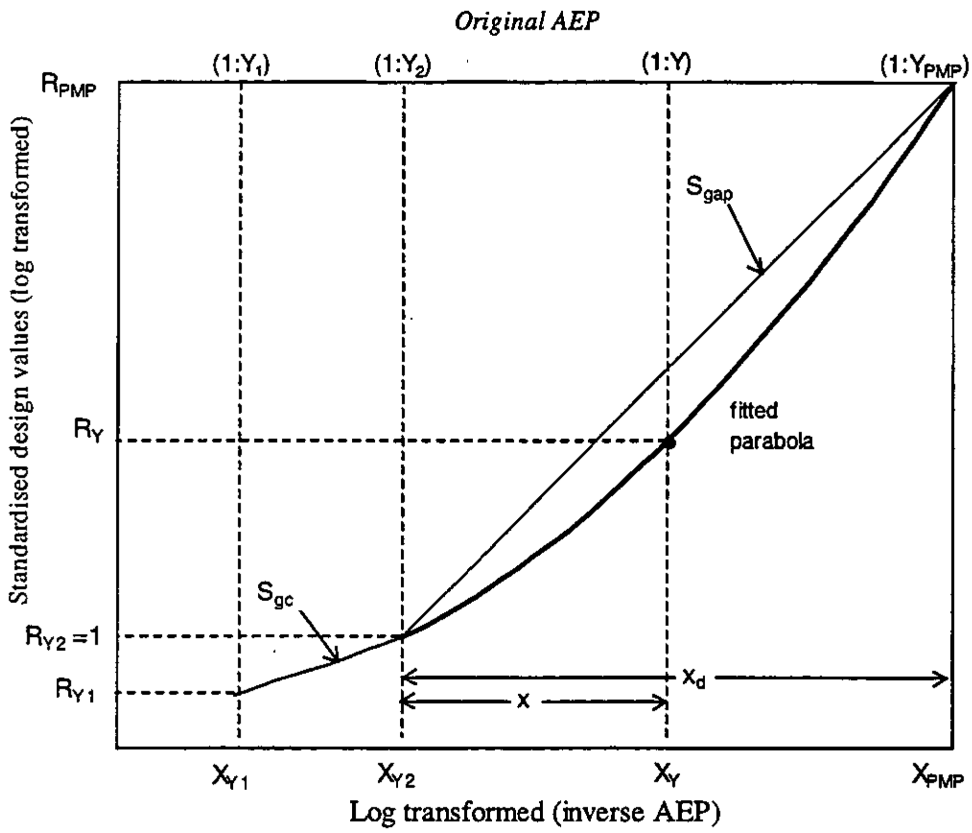


Figure 3.1 : Symbols used in generalised procedure for interpolation of frequency curve

Log transformed *inverse* AEPs are computed as:

$$X_{PMP} = \log(Y_{PMP})$$

$$X_{Y1} = \log(Y_1)$$

$$X_{Y2} = \log(Y_2)$$

Corresponding standardised values of log transformed design quantiles are obtained from:

$$R_{PMP} = \frac{\log(P_{PMP})}{\log(P_{Y2})}$$

$$R_{Y1} = \frac{\log(P_{Y1})}{\log(P_{Y2})}$$

$$R_{Y2} = 1$$

With linear scales for both standardised design values (log transformed) and log transformed *inverse* AEPs, the slope between the AEP of 1:Y_1 and 1:Y_2 events, S_{gc}, is computed as;

$$S_{gc} = \frac{(1 - R_{Y1})}{(X_{Y2} - X_{Y1})}$$

Similarly, the slope of the line joining the points (X_{Y2}, 1.0) and (X_{PMP}, R_{PMP}), S_{gap}, is computed as:

$$S_{gap} = \frac{(R_{PMP} - 1)}{x_d}$$

where $x_d = (X_{PMP} - X_{Y2})$

If R_Y represents the standardised design value corresponding to an AEP of 1:Y (with a log transformed *inverse* AEP of X_Y) in the interpolated range, where

$$R_Y = \frac{\log(P_Y)}{\log(P_{Y2})}$$

$$X_y = \log(Y)$$

$$x = (X_Y - X_{Y2})$$

then, R_Y can be represented by a *parabolic function with dimensionless variables,* with its origin at $(X_{Y2},1)$, as:

$$R_Y = 1 + a_1\left(\frac{x}{x_d}\right) + a_2\left(\frac{x}{x_d}\right)^2$$

where a_1 and a_2 are coefficients to be determined to satisfy given boundary conditions.

The above equation can be rearranged as:

$$R_Y = 1 + \frac{a_1}{x_d}(x) + \frac{a_2}{x_d^2}(x)^2 \quad \text{.......} \quad (3.1)$$

The gradient of the interpolated curve is given by the first derivative of equation (3.1):

$$\frac{d(R_Y)}{dx} = \frac{a_1}{x_d} + 2\frac{a_2}{x_d^2}\cdot x \quad \text{.......} \quad (3.2)$$

Boundary conditions

(i) at $x = x_d$: $R_Y = R_{PMP} = S_{gap}.x_d + 1$

substituting in equation (3.1)

$$a_1 + a_2 = S_{gap}.x_d \quad \text{.......} \quad (3.3)$$

(ii) at $x = 0$: $\frac{d(R_Y)}{dx} = S_{gc}$ (slope of the curve)

substituting in equation (3.2)

$$a_1 = S_{gc}.x_d \quad \text{.......} \quad (3.4)$$

solving equations (3.3) and (3.4)

$$a_2 = (S_{gap} - S_{gc})\, x_d \quad \text{.......} \quad (3.5)$$

Equations (3.1), (3.4) and (3.5) define the parabolic function for determination of quantile estimates (in the log-transformed domain) over the range from the upper end of the directly determined frequency curve (AEP of 1:Y_2) to the AEP of the PMP (1:Y_{PMP}). Quantile P_Y can then be obtained from:

$$P_Y = 10^{R_Y \cdot \log(P_{Y2})} \quad \text{.......} \quad (3.6)$$

For satisfactory behaviour of this parabolic function, the slope of the function at the AEP of the PMP should not be less than zero; ie. in the limiting case the function becomes tangential to a horizontal line drawn through the PMP.

From equation (3.2)

$$\text{at } x = x_d: \quad \frac{d(R_Y)}{dx} = \frac{a_1}{x_d} + \frac{2a_2}{x_d} = \frac{1}{x_d}(a_1 + 2a_2) \geq 0$$

substituting for a_1 and a_2, the condition for a satisfactory relationship is;

$$S_{gc} \leq 2S_{gap} \quad \text{.......} \quad (3.7)$$

The limiting case, that the frequency curve approaches the PMP tangential to a horizontal line drawn through it, occurs when the initial slope, S_{gc}, becomes twice the slope S_{gap}. No mathematical solution for situations exceeding this limiting case is presented in this report as it is considered that most practical design situations can be represented by the parabolic function. However, in a design situation where this limiting condition is exceeded, it is recommended that the design data and assumptions should be checked for compatibility. If the data is found to be compatible, a curve should be drawn subjectively to be tangential to the horizontal line at the PMP, and to be consistent with the curve representing the mathematical function for the limiting case.

Although no limiting condition for the lowest acceptable slope of S_{gc} is recommended, it is the responsibility of the practitioner to evaluate the compatibility of the basic quantile estimates adopted for the interpolation procedure, before accepting the results.

3.3 Discussion

For successful application of this interpolation procedure, the directly determined frequency curve should be compatible with the PMP (or the corresponding flood) estimate and its assigned AEP. Hence, it is desirable to assess the suitability of the proposed interpolation procedure in relation to the type and quality of data available and the methods used in quantile estimation.

Design rainfall frequency curves (derived from analysis of rainfall data) are considered to be better behaved than flood frequency curves as they are usually based on long records of reliable data and have lower skews. ARR87 design rainfall values (derived up to an AEP of 1 in 100) have been determined from an analysis of a large number of rainfall records and been smoothed for spatial consistency. CRC-FORGE design rainfall estimates (recommended up to an AEP of 1 in 2000), based on a *regional procedure* using rainfall data from long record stations, result in a more stable and reliable rainfall frequency curve. Hence, a directly determined rainfall frequency curve would generally be compatible with the PMP estimate, provided this is plotted at an appropriate AEP.

On the other hand, *flood frequency curves* are usually derived from relatively short at-site records, resulting in highly variable skews and sometimes inappropriate estimates of skew. Frequency analysis of flood data with inappropriate skew values would result in a derived flood frequency curve incompatible with the flood estimate corresponding to the PMP. Moreover, there is a greater uncertainty in the slope at the starting point of the interpolated flood frequency curve and its estimation is generally possible over a smaller range than would be desired (eg. to an AEP limit of 1 in 50).

It has been found in practice that, given the greater potential for non-linearities in the transfer between rainfalls and floods, the flood frequency curve tends to exhibit greater curvature than the rainfall curve (Rory Nathan, pers. comm.). Hence, it is more appropriate to interpolate rainfall quantiles rather than transformed flood quantiles.

After consideration of the above factors, the proposed procedure is deemed more suitable for interpolation of rainfall frequency curves than flood frequency curves. In situations where the flood frequency curve has to be extrapolated directly, additional care needs to be taken.

4. APPLICATION OF PROCEDURE WITH NEW CRC-FORGE DESIGN RAINFALLS

The new CRC-FORGE method (Nandakumar et al., 1997; Nandakumar and Weinmann, 1998) allows the definition of the design rainfall frequency curve to an AEP of 1:2000. This chapter describes the methodology for the interpolation of the rainfall frequency curve from an AEP of 1:2000 to the assigned AEP of the PMP, adapted from the generalised procedure described in Chapter 3.

4.1 Functional representation of interpolated frequency curve

In applying the generalised procedure described in Chapter 3, the slope S_{gc} of the curve at the design estimate of 1:2000 is taken equal to the slope determined between CRC-FORGE estimates of 1:1000 and 1:2000. The use of these estimates will reduce the 'effect of linearisation' in defining the limiting slope (at the AEP of 1 in 2000) of the directly determined frequency curve. A sensitivity analysis using the slope defined by estimates corresponding to AEPs of 1:500 and 1:2000 indicated only marginal differences in the interpolated curves.

Following the procedure described in Section 3.2, the computational steps involved in the interpolation of the design rainfall frequency curve from an AEP of 1 in 2000 to the assigned AEP of the PMP are outlined below.

For the catchment concerned and a selected storm duration, let P_{1000} and P_{2000} denote the design rainfall quantiles for AEPs of 1:1000 and 1:2000 respectively. These are determined by first deriving representative CRC-FORGE point rainfall quantiles (Nandakumar et al., 1997) for the catchment and then multiplying them by corresponding CRCCH areal reduction factors (Siriwardena and Weinmann, 1996). Let P_{PMP} and $1{:}Y_{PMP}$ denote the magnitude of the probable maximum precipitation for the relevant storm duration and the AEP assigned to it (Laurenson and Kuczera, 1998) respectively.

1. *Compute Y_{PMP} from the catchment area (A) in km^2.*

$$A \le 100\ km^2; \qquad Y_{PMP} = 10^7$$

$$A \ge 100{,}000\ km^2; \qquad Y_{PMP} = 10^4$$

$$100\ km^2 < A < 100{,}000\ km^2; \quad Y_{PMP} = \frac{1}{10^{[\log(A)-9]}}$$

2. *Calculate transformed variables.*

$$R_{1000} = \frac{\log(P_{1000})}{\log(P_{2000})}$$

$$R_{PMP} = \frac{\log(P_{PMP})}{\log(P_{2000})}$$

$$x_d = \log(Y_{PMP}) - \log(2000)$$

3. Calculate parameters of the fitted function.

$$S_{gc} = \frac{(1 - R_{1000})}{\log(2000) - \log(1000)}$$

$$S_{gap} = \frac{(R_{PMP} - 1)}{x_d}$$

$$a_1 = S_{gc}.x_d$$

$$a_2 = (S_{gap} - S_{gc})\, x_d$$

4. Determine design rainfall quantiles in the interpolated range.

For an AEP of 1:Y in the interpolated range, let P_Y denotes the design rainfall quantile that needs to be determined from the *parabolic function.*

$$x = \log(Y) - \log(2000)$$

$$R_Y = 1 + \frac{a_1}{x_d}(x) + \frac{a_2}{x_d^2}(x)^2$$

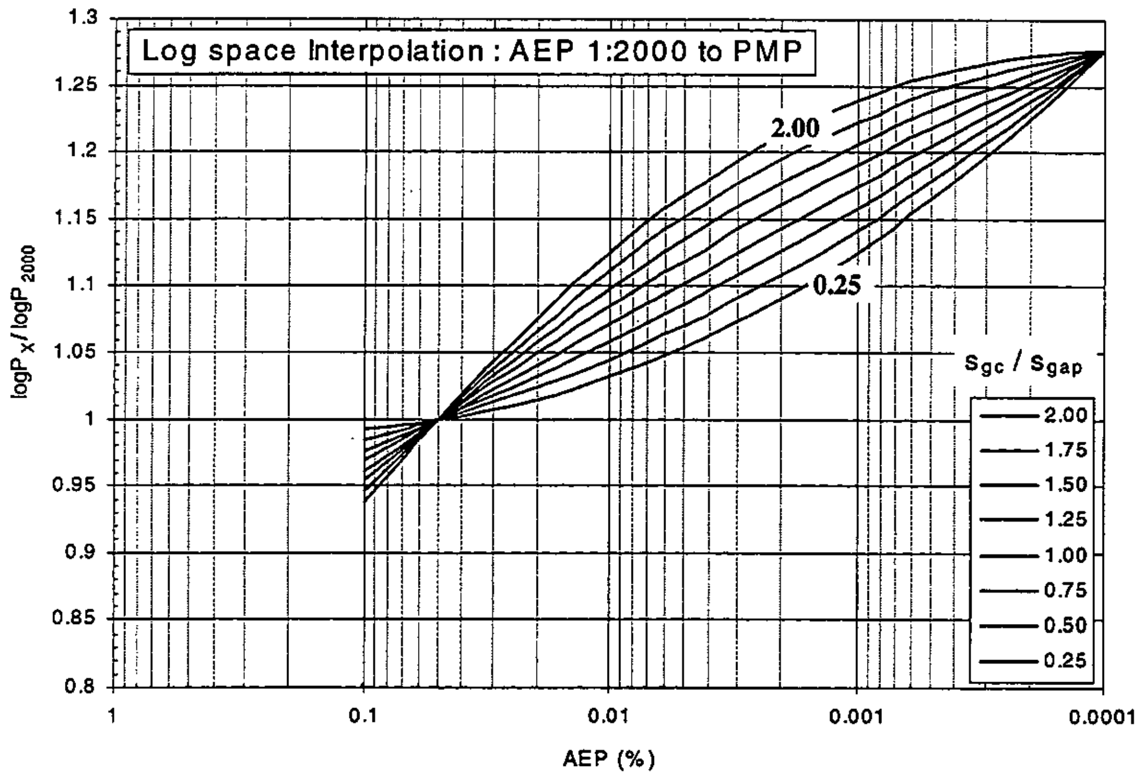
$$P_Y = 10^{R_Y \cdot \log(P_{2000})}$$

Step 4 is repeated for other AEPs of interest in the interpolated range, allowing the derivation of a 'complete' frequency curve .

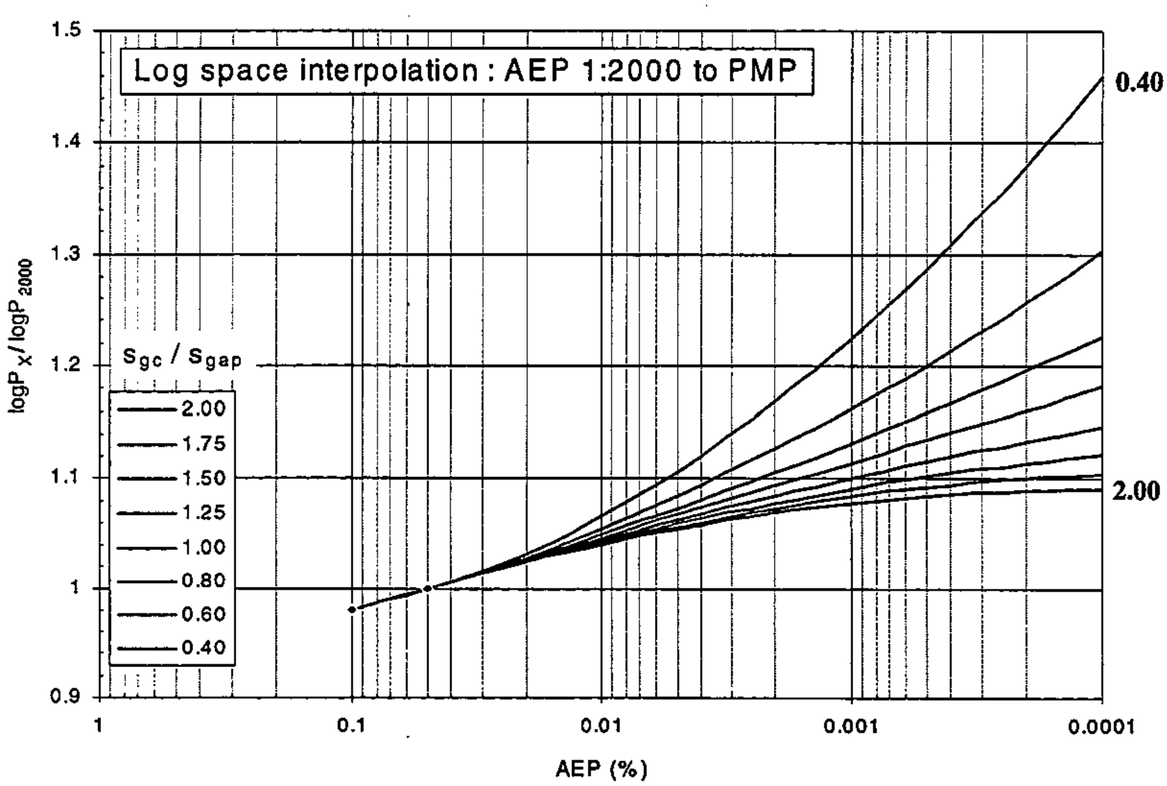
4.2 Range of application

A 'shape parameter', defined by the ratio of the slope at the upper end of the directly determined frequency growth curve, S_{gc}, and the slope between the two end points of the 'gap', S_{gap}, is used to assess the applicability of the interpolation procedure. The condition that $S_{gc} \leq 2S_{gap}$ determines the upper limit in which the function becomes tangential to the horizontal line drawn through the PMP. The range of S_{gc}/S_{gap} from 0.25 to 2.00 is recommended for satisfactory application, with the lower limit estimated from practical considerations and compatibility of design data.

Typical frequency curves encompassing a range of possible design situations are illustrated in Figure 4.1. In this figure, behaviour of the interpolated curves for different design situations with varying slopes of S_{gc} and S_{gap} is investigated; the ratio of S_{gc}/S_{gap} ranges from a recommended minimum of 0.25 to a upper limit of 2.00. In Figure 4.1(a) arbitrarily selected 1:2000 AEP and PMP design estimates determine a 'fixed' slope of S_{gap}, while the slope S_{gc} is varied by selecting a range of 1:1000 AEP design estimates such that the ratio



(a) Fixed S_{gap} and varying S_{gc}



(b) Fixed S_{gc} and varying S_{gap}

Figure 4.1 : Typical frequency curves derived from CRCCH interpolation procedure for interpolation from the estimate of 1:2000 to the PMP (applicable with CRC-FORGE design rainfall estimates)

of S_{gc}/S_{gap} ranges from 0.25 to 2.00. In Figure 4.1(b) arbitrarily selected 1:1000 AEP and 1:2000 AEP design estimates determine a 'fixed' slope of S_{gc}; the slope S_{gap} is varied by selecting a range of PMP estimates such that the ratio of S_{gc}/S_{gap} ranges from 0.40 to 2.00.

The results indicate a satisfactorily smooth transition over the interpolated range within the recommended range of S_{gc}/S_{gap} values.

4.3 Evaluation of new approach for design application in Victoria

The performance of the proposed interpolation procedure has been evaluated for practical design situations by applying it to 25 Victorian dam sites with catchments of diverse topographical and meteorological characteristics. The test catchments ranged from as small as 25 km^2 to a very large catchment of about 15,000 km^2 with a good spatial coverage over the state.

Average *point* rainfall quantiles (for 24 hours duration) for the selected catchments were determined from the CRC-FORGE method (Nandakumar et al., 1997). To achieve greater consistency with ARR87 estimates, adjusted CRC-FORGE 'growth factors' were applied to the 1 in 100 AEP scaling value derived from the ARR87 procedures. (It should be noted that in the subsequent preparation of a CRC-FORGE design rainfall database for Victoria, a 1 in 50 AEP scaling variable was used). These point estimates were then converted to *areal* rainfall quantiles by applying the new areal reduction factors derived for Victoria (Siriwardena and Weinmann, 1996). The design rainfall frequency curves thus determined to an AEP of 1 in 2000 were then extended to the PMP using the new interpolation procedure, with the AEP of the PMP determined according to the recommendations by Laurenson and Kuczera (1998).

The design rainfall frequency curve for each catchment was standardised with respect to the 1 in 2000 AEP design rainfall estimate, to enable direct comparison of the curves derived for the 25 dam sites. The 'dimensionless' curves are shown in Figure 4.2 with AEP on a log scale (as in the development of the procedure).

It was shown that the rainfall frequency curves are *plausible and well behaved* over the interpolated range for all catchments tested. The values of the 'shape parameter', S_{gc}/S_{gap}, are well within the recommended range of 0.25 - 2.0, as the values ranged from 0.53 to 1.44 with an average value of 0.90. The results indicate that the CRC-FORGE design quantiles (with FORGE 'growth factors' applied to the standardising value from ARR87 procedures), generalised PMP estimates and assigned AEPs to them form a consistent design basis; the new interpolation procedure can be successfully applied for diverse practical design situations. The reverse curvature in the frequency curve exhibited for some catchments is within acceptable limits.

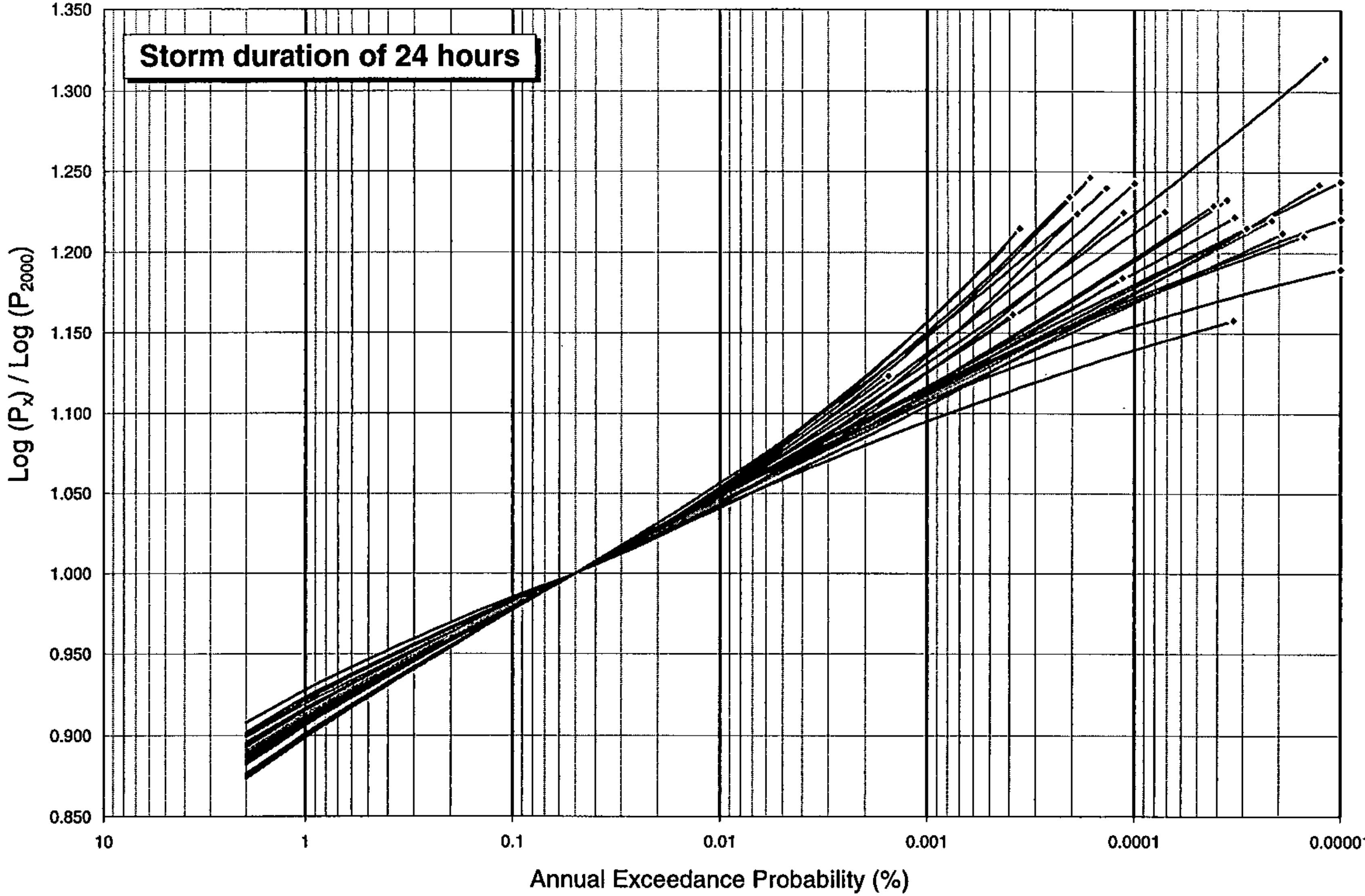


Figure 4.2 : Design rainfall frequency curves derived for 25 Victorian dam sites using CRCCH interpolation procedure based on CRC-FORGE design rainfall estimates for AEPs from 1:50 to 1:2000

For large catchments, the consistency between the directly determined frequency curve (up to 1 in 2000 AEP) and the PMP estimate at its assigned AEP, ie. the relationship between S_{gc} and S_{gap}, is very important to effect a satisfactory transition within only about two log cycles. It has been shown that, with the new design procedure, there is no need for adjustments to the assigned AEP of the PMP for geometrical consistency.

Design rainfall frequency curves are generally presented as log-normal probability plots rather than log-log plots. The behaviour of the interpolated frequency curves on a log-normal probability plot is examined in Figure 4.3 for curves representing extremes of 'shape parameters', S_{gc}/S_{gap}, (values of 0.53 and 1.44) and an average value of S_{gc}/S_{gap} (0.96). The curves were shown to be satisfactorily behaved. In comparison to a log-log plot, rising curves exhibit a pronounced curving effect on a log-normal probability plot, where as, falling curves appear to be straightened up.

It needs to be emphasised that regional rainfall estimation methods such as the CRC-FORGE method allow a significant reduction of uncertainty in design rainfall estimates in the interpolated range. This is because they provide an extension of the directly estimated rainfall frequency curve from an AEP of 1:100 to 1:2000, thus significantly reducing the gap over which interpolation is applied.

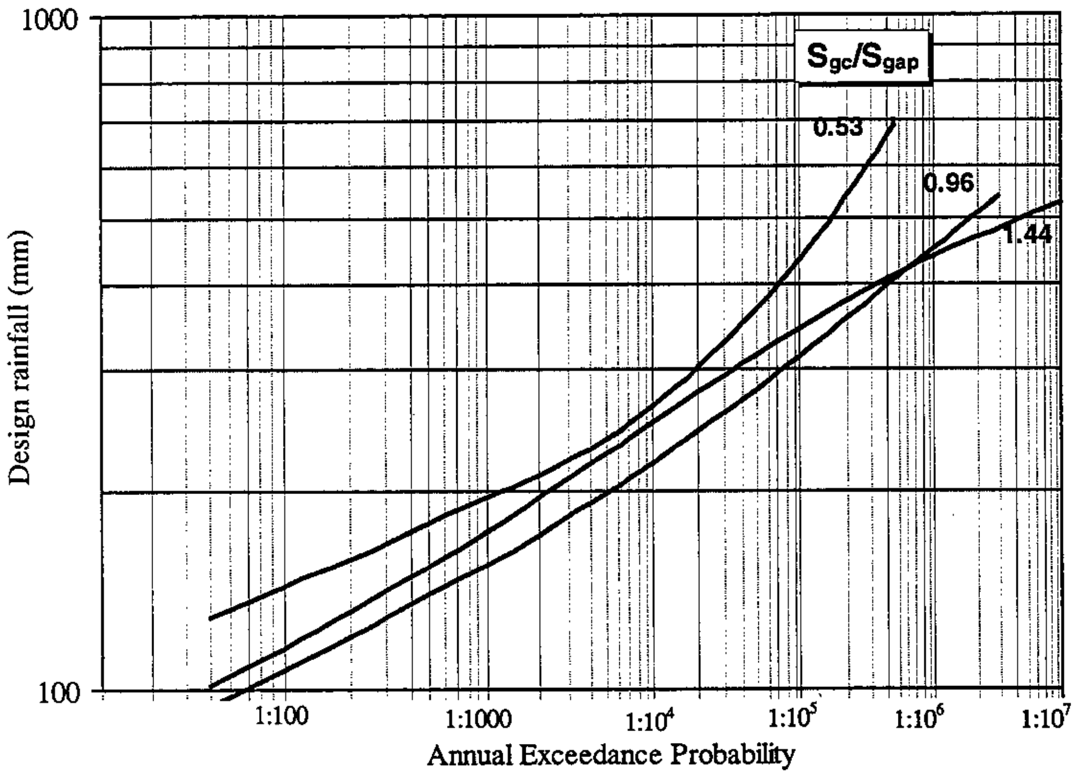


Figure 4.3 : Log-normal probability plot of interpolated rainfall frequency curves

4.4 Numerical example

This worked example illustrates the application of CRCCH interpolation procedure to a sample catchment of 360 km^2.

For the 24 hour storm duration, the design rainfall quantities for AEPs of 1:1000 and 1:2000, and the probable maximum precipitation were determined as:

$$P_{1000} = 207.1 \text{ mm}$$
$$P_{2000} = 228.8 \text{ mm}$$
$$P_{PMP} = 810.0 \text{ mm}$$

Following the computational steps in Section 4.1:

1. Compute Y_{PMP} knowing the catchment area (A) in km^2.

$$\text{AEP of the PMP} = 10^{[\log(360)-9]} = 3.60\times10^{-7}$$

$$Y_{PMP} = \frac{1}{\text{AEP}} = \frac{1}{3.60\times10^{-7}} = 2.778\times10^{6}$$

2. Calculate transformed variables.

$$R_{1000} = \frac{\log(207.1)}{\log(228.8)} = 0.9817$$

$$R_{PMP} = \frac{\log(810.0)}{\log(228.8)} = 1.2327$$

$$x_d = \log(2.778\times10^{6}) - \log(2000) = 3.1427$$

3. Calculate parameters of the fitted function.

$$S_{gc} = \frac{(1-0.9817)}{\log(2000)-\log(1000)} = 0.0608$$

$$S_{gap} = \frac{(1.2327-1)}{3.1427} = 0.0740$$

$$a_1 = 0.0608\times3.1427 = 0.1911$$
$$a_2 = (0.0740 - 0.0608)\,3.1427 = 0.0415$$

4. Determine design rainfall quantiles in the interpolated range.

Sample calculation for an AEP of 1:200,000:

$$x = \log(200{,}000) - \log(2000) = 2.00$$

$$R_Y = 1+\frac{0.1911}{3.1427}(2.00)+\frac{0.0415}{(3.1427)^2}(2.00)^2 = 1.1384$$

$$P_Y = 10^{1.1384 \times \log(228.8)} = 485.3 \text{ mm}$$

Step 4 was repeated for other AEPs of interest in the interpolated range; the results are shown in Table 4.1.

Table 4.1 : Interpolated design rainfall quantiles for a sample catchment (Using CRC-FORGE rainfall estimates)

AEP	x	R_Y	P_Y (mm)
1 : 1000	-	-	207.1
1 : 2000	(0.0)	(1.0)	228.8
1 : 50000	1.3979	1.0932	**379.6**
1 : 100,000	1.6990	1.1154	**428.4**
1 : 200,000	2.0000	1.1384	**485.3**
1 : 500,000	2.3979	1.1700	**576.1**
1 : 1,000,000	2.6990	1.1947	**659.0**
1 : 2,000,000	3.0000	1.2202	**757.0**
1 : 2,778,000	(3.1427)	(1.2326)	810.0

The above computation can conveniently be carried out on a spreadsheet (eg. Excel).

5. APPLICATION OF PROCEDURE IN ABSENCE OF CRC-FORGE DESIGN RAINFALLS

The current design practice involves sketching a frequency curve from a directly determined design estimate of 1:100 AEP to the PMP using the 'shape factors' recommended in ARR87. This chapter examines different options for a procedure that is consistent with recent design rainfall developments. The recommended methodology for the interpolation of the rainfall frequency curve from an AEP of 1:100 to the assigned AEP of the PMP is described. The methodology is then evaluated by applying it to a large number of Victorian catchments of different characteristics. The procedure is applicable to regions where CRC-FORGE design rainfalls are not available.

5.1 Alternative approaches investigated

Modified ARR87 method (parabolic function)

In accordance with recent developments and more realistic concepts in the estimation of design rainfalls in the extreme probability range, incorporation of the following modifications in the methodology for interpolation of design quantiles (between 1:100 AEP estimate and the PMP) was considered desirable (as discussed in Chapter 3).

- Subjective sketching of curves needs to be replaced by a mathematical function.
- The criterion for geometrical consistency in assigning an AEP to the PMP should be dispensed with and be based on meteorological consideration alone.
- Assigning an AEP to the PMP needs to be based on the recommendations by Kuczera and Laurenson (1998).
- No constraint be placed on the slope of the frequency curve approaching the PMP.

This involves fitting a *parabolic function* to be tangential to the slope between the 1:50 and 1:100 AEP estimates and passing through the PMP point.

Modified ARR87 method incorporating CRC-FORGE design information (cubic function)

For regions where CRC-FORGE design rainfall estimates are not available, use of additional design information for lower AEPs could be beneficial in reducing the uncertainty in the interpolated frequency curve. In this regard, the possibility of incorporating in the fitting procedure a *regional/generalised CRC-FORGE design rainfall estimate of 1:2000 AEP* was investigated. In addition to satisfying the criteria outlined above for a parabolic function, the curve should now pass through an estimate for 1:2000 AEP. To satisfy this additional constraint, a *cubic* function is required.

A mathematical function to satisfy the above criteria was formulated and the behaviour of the function in deriving design rainfall frequency curves was tested for 25 Victorian catchments. It was observed that the cubic function is generally unstable and yielded undesirable curves for some catchments. It should be noted that the ARR87 estimates for 1:50 and 1:100 AEPs, the regional or generalised CRC-FORGE estimate for 1:2000 AEP, and the PMP at its assigned AEP should be compatible with one another to produce satisfactory results. This compatibility is not readily achieved, and the additional constraint apparently reduces the range of application in comparison to that of a parabolic function. In addition, when this interpolation procedure was applied to design rainfalls for different durations, significant inconsistencies across durations were observed.

In view of limited success with the cubic function, the parabolic function is recommended for application to regions where only ARR87 estimates are available.

5.2 Functional representation of adopted procedure

The generalised procedure developed in Section 3.2 was adopted to interpolate the rainfall frequency curve from an AEP of 1:100 to the AEP of the PMP. In deducing a parabolic function from the 'generalised procedure', the estimates for $1{:}Y_1$ and $1{:}Y_2$ need to be replaced by estimates for AEPs of 1:50 and 1:100.

Following the procedure described in Section 3.2, the computational steps involved in the interpolation of the design rainfall frequency curve from an AEP of 1 in 100 to the assigned AEP of the PMP are outlined below.

For the catchment concerned and a selected storm duration, let P_{50} and P_{100} denote the design rainfall quantiles for AEPs of 1:50 and 1:100 respectively. These are determined by first deriving representative point rainfall estimates for the catchment using Chapter 2 of ARR87, or other appropriate method, and then multiplying them by corresponding CRCCH areal reduction factors (Siriwardena and Weinmann, 1996). Let P_{PMP} and $1{:}Y_{PMP}$ denote the magnitude of the probable maximum precipitation for the relevant storm duration and the AEP assigned to it (Laurenson and Kuczera, 1998) respectively.

1. Compute Y_{PMP} from the catchment area (A) in km^2.

$A \leq 100\ km^2$; $\quad Y_{PMP} = 10^7$

$A \geq 100{,}000\ km^2$; $\quad Y_{PMP} = 10^4$

$$100\ km^2 < A < 100{,}000\ km^2;\quad Y_{PMP} = \frac{1}{10^{[\log(A)-9]}}$$

2. Calculate transformed variables.

$$R_{50} = \frac{\log(P_{50})}{\log(P_{100})}$$

$$R_{PMP} = \frac{\log(P_{PMP})}{\log(P_{100})}$$

$$x_d = \log(Y_{PMP}) - \log(100)$$

3. Calculate parameters of the fitted function.

$$S_{gc} = \frac{(1-R_{50})}{\log(100)-\log(50)}$$

$$S_{gap} = \frac{(R_{PMP}-1)}{x_d}$$

$$a_1 = S_{gc}.x_d$$

$$a_2 = (S_{gap} - S_{gc})\, x_d$$

4. Determine design rainfall quantiles in the interpolated range.

For an AEP of 1:Y in the interpolated range, let P_Y denotes the design rainfall quantile that needs to be determined from the *parabolic function.*

$$x = \log(Y) - \log(100)$$

$$R_Y = 1 + \frac{a_1}{x_d}(x) + \frac{a_2}{x_d^{\,2}}(x)^2$$

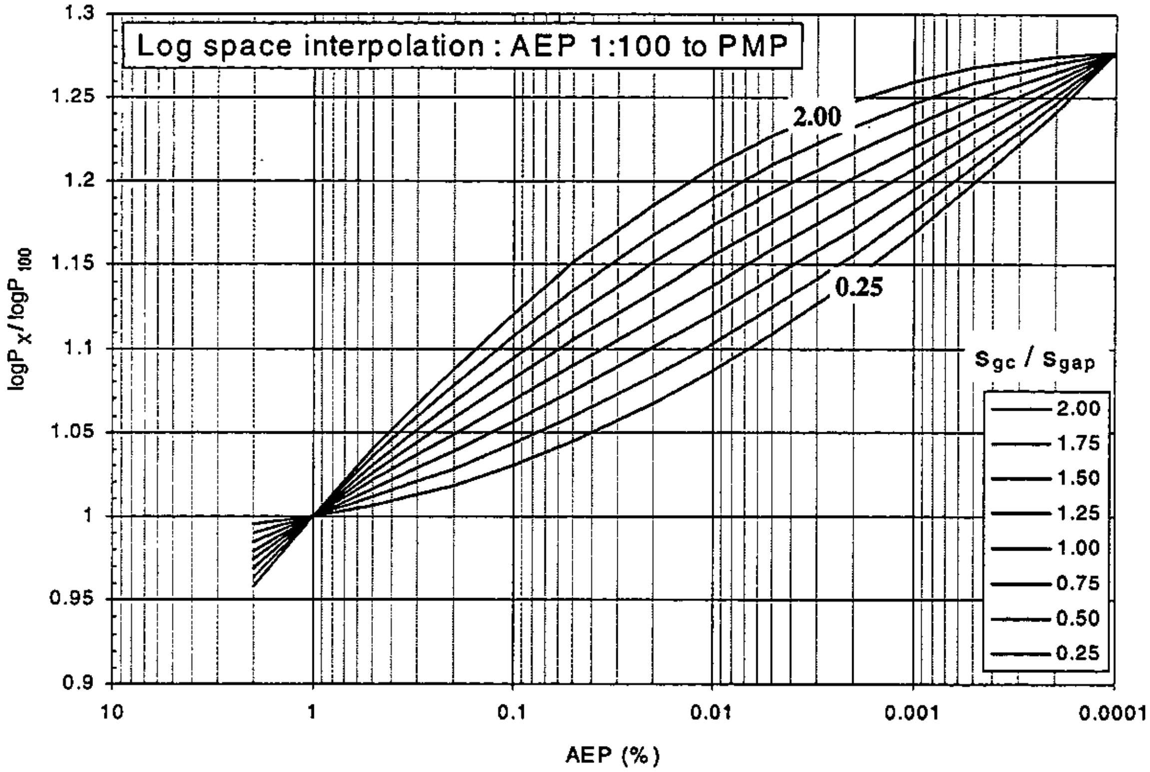
$$P_Y = 10^{R_Y \cdot \log(P_{100})}$$

Step 4 is repeated for other AEPs of interest in the interpolated range, allowing the derivation of a 'complete' frequency curve .

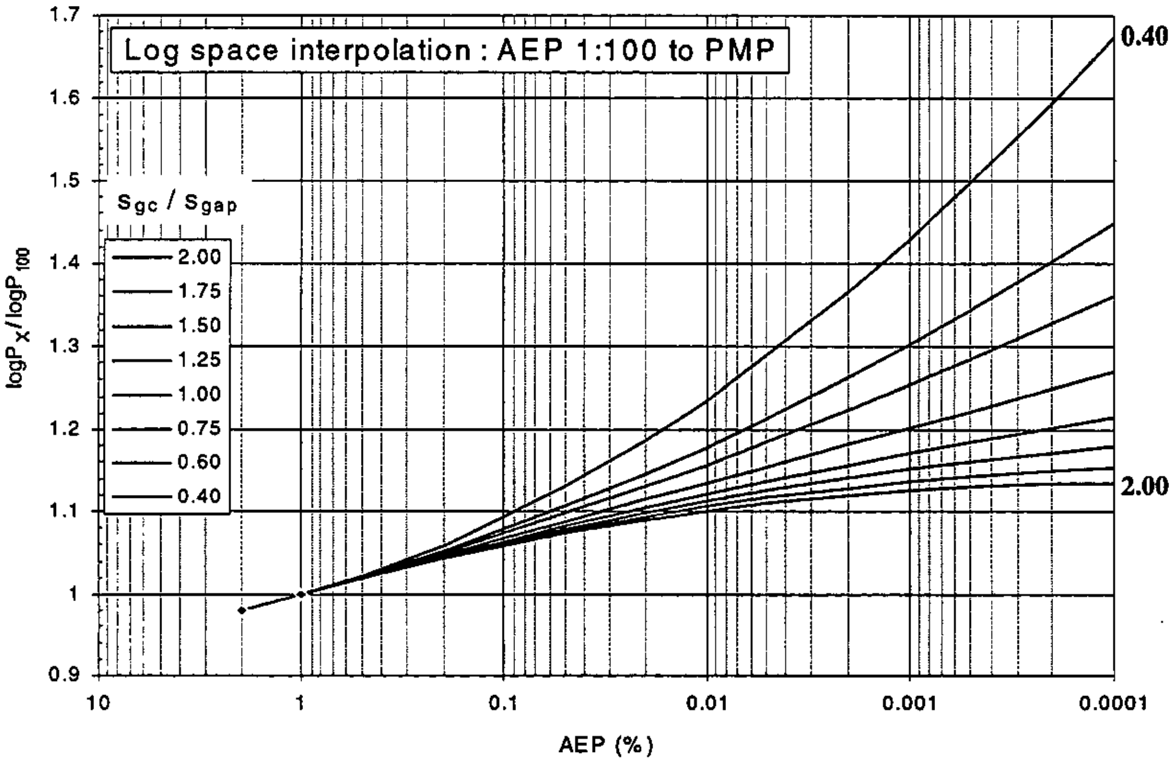
5.3 Range of application

A 'shape parameter', defined by the ratio of the slope at the upper end of the directly determined frequency growth curve, S_{gc}, and the slope between the two end points of the 'gap', S_{gap}, is used to assess the applicability of the interpolation procedure. Values of S_{gc}/S_{gap} within the range from 0.25 to 2.00 are recommended for satisfactory application. The upper limit is reached when the function becomes tangential to the horizontal line drawn through the PMP. The lower limit has been estimated from practical considerations and compatibility of design data.

Typical frequency curves encompassing a range of possible design situations are illustrated in Figure 5.1. In this figure, the behaviour of the interpolated curves for different design situations with varying slopes of S_{gc} and S_{gap} is investigated; the ratio of S_{gc}/S_{gap} ranges



(a) Fixed S_{gap} and varying S_{gc}



(b) Fixed S_{gc} and varying S_{gap}

Figure 5.1 : Typical frequency curves derived from CRCCH interpolation procedure for interpolation from the estimate of 1:100 to the PMP (applicable with ARR87 design rainfall estimates)

from a recommended minimum of 0.25 to a upper limit of 2.00. In Figure 5.1(a) arbitrarily selected 1:100 AEP and PMP design estimates determine a 'fixed' slope of S_{gap}, while the slope S_{gc} is varied by selecting a range of 1:50 AEP design estimates such that the ratio of S_{gc}/S_{gap} ranges from 0.25 to 2.00. In Figure 5.1(b) arbitrarily selected 1:50 AEP and 1:100 AEP design estimates determine a 'fixed' slope of S_{gc}; the slope S_{gap} is varied by selecting a range of PMP estimates such that the ratio of S_{gc}/S_{gap} ranges from 0.40 to 2.00.

The results indicate a satisfactorily smooth transition over the interpolated range within the recommended range of S_{gc}/S_{gap} values.

5.4 Evaluation of approach for design application

5.4.1 Application to Victorian catchments

The proposed design procedure, for situations where only ARR87 design rainfall information is available, was tested for the 25 Victorian catchments described in Section 4.3. Point rainfall quantiles ranging to an AEP of 1:100 (for 24 hours duration) at a representative location for each catchment were obtained by applying the procedure given in Chapter 2 of ARR87. These were then converted to *areal* rainfall quantiles by applying the new areal reduction factors derived for Victoria (Siriwardena and Weinmann, 1996). The interpolation procedure described in Section 5.2 was then applied to obtain 'complete' rainfall frequency curves to the assigned AEP of the PMP.

The design rainfall frequency curves were standardised with respect to the 1 in 100 design rainfall estimate, to enable direct comparison of the curves derived for 25 catchments. The 'dimensionless' curves are shown in Figure 5.2 with AEP on a log scale (as in the development of the procedure).

The design rainfall frequency curves were shown to be 'well behaved' in the interpolated range for all catchments tested (Figure 5.2). The values of the 'shape parameter', S_{gc}/S_{gap}, ranged from 0.66 to 1.43 with an average value of 1.00; these are well within the recommended range of 0.25 - 2.00. Based on the test results from the Victorian catchments, it can be concluded that the proposed interpolation procedure (Section 5.2) can be satisfactorily applied to design situations where rainfall quantiles can only be determined to a significantly higher AEP than 1 in 2000. It is noted that plausible curves with gradual transition can be achieved; inconsistencies are less likely to become apparent as the extrapolation is effected over a broader range of AEPs.

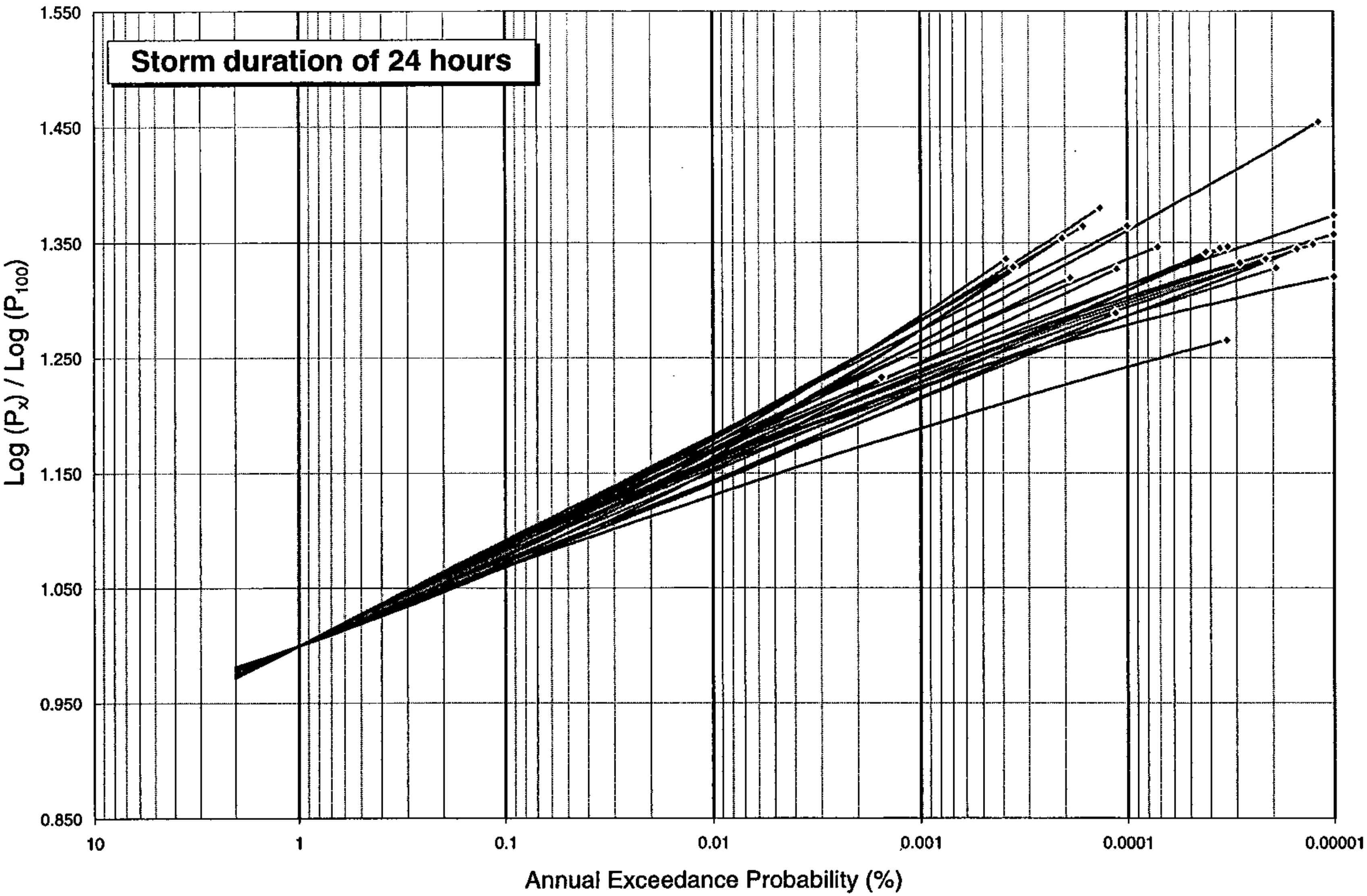


Figure 5.2 : Design rainfall frequency curves derived for 25 Victorian dam sites using CRCCH interpolation procedure based on ARR87 design rainfall estimates for AEPs from 1:50 to 1:100

5.4.2 Comparison with curves derived with availability of CRC-FORGE estimates

The 'complete' rainfall frequency curves derived using CRC-FORGE estimates were compared against those derived using ARR87 estimates (for 1:50 and 1:100 AEPs) for the 25 test catchments. In Figure 5.3, respective curves are illustrated for four representative catchments.

It was found that for the majority of catchments (13) the frequency curves derived using the ARR87 estimates plotted above the corresponding curves derived using the CRC-FORGE estimates, and the opposite was the case for 2 catchments. For 10 catchments the differences were only marginal. From these results it can be concluded that, in general, the frequency curves derived using the ARR87 estimates tend to be conservative over the interpolated range. For regions where CRC-FORGE design values have not been determined, imparting a regional measure of the CRC-FORGE estimates in the interpolation procedure would be advantageous; but the attempted implementation of such a procedure in this study was found to produce unsatisfactory results (Section 5.1).

5.4.3 Comparison with curves derived from ARR87 Interpolation Procedure

The procedure given in Chapter 13 of ARR87 was applied to interpolate rainfall estimates between an AEP of 1 in 100 and the AEP of the PMP for two test catchments (100 and 3500 km^2), selected based on a benchmarking study to evaluate the CRCCH methodology for design rainfall estimation. The resultant frequency curves are compared with those determined from the new approach in Figure 5.4. It should be noted that the AEP of the PMP was taken as 1 in 10^6 in accordance with the ARR87 slope factor criterion; the same was adopted with the new interpolation procedures to enable direct comparison.

The two frequency curves where the interpolation starts from an AEP of 1 in 100 reflect the differences due to the methodology alone, with all other factors remaining the same. It can be observed that the ARR87 assumption of the frequency curve approaching the PMP horizontally has a considerable influence on the shape of the frequency curve. As a result, for the test catchments, the estimates derived from the ARR87 interpolation procedure were shown to be considerably higher than the corresponding estimates derived from the new approach. For an AEP of 1 in 50000, design rainfall estimates derived from the ARR87 procedure are 22% and 14% higher for the smaller and larger test catchments respectively.

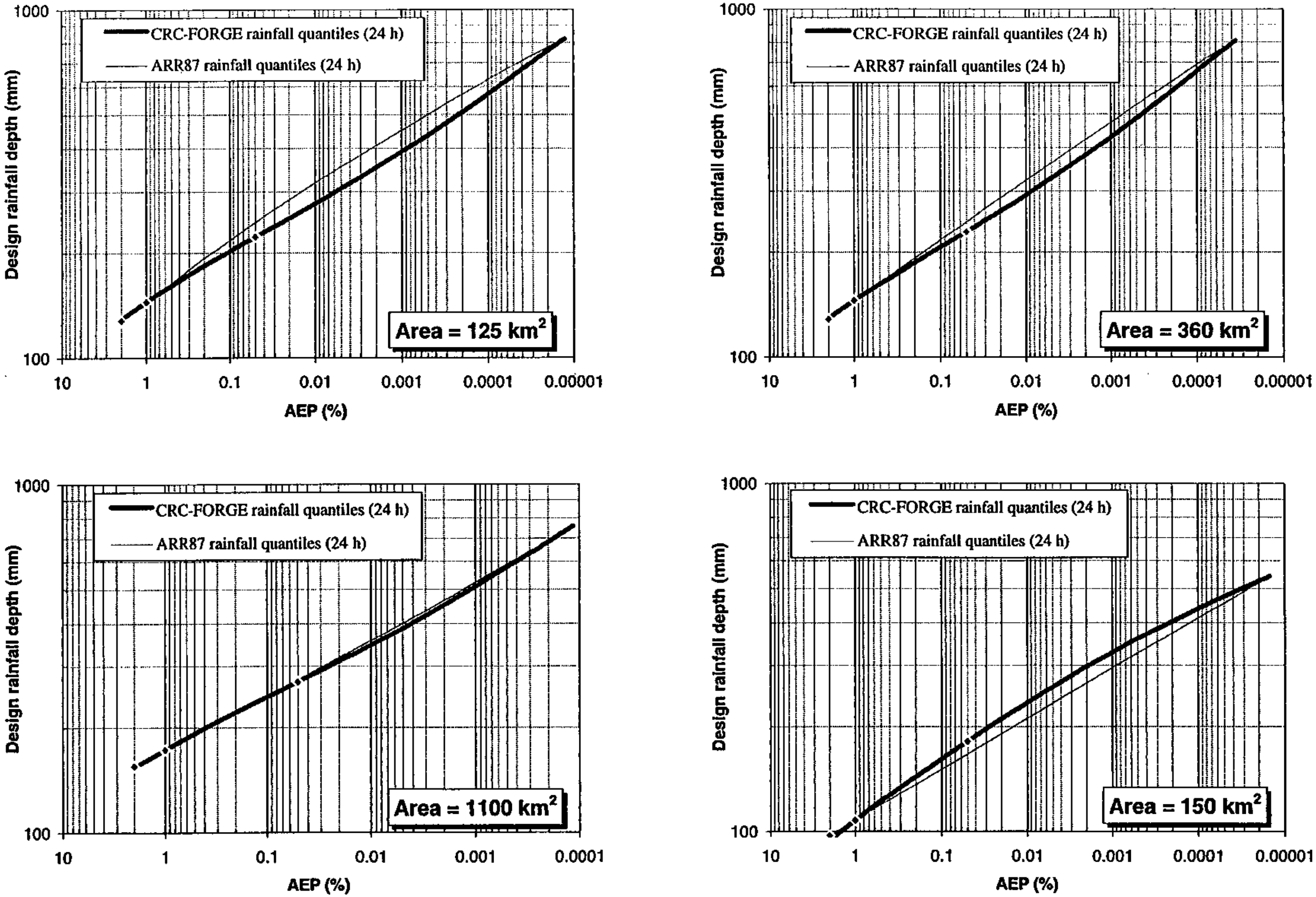
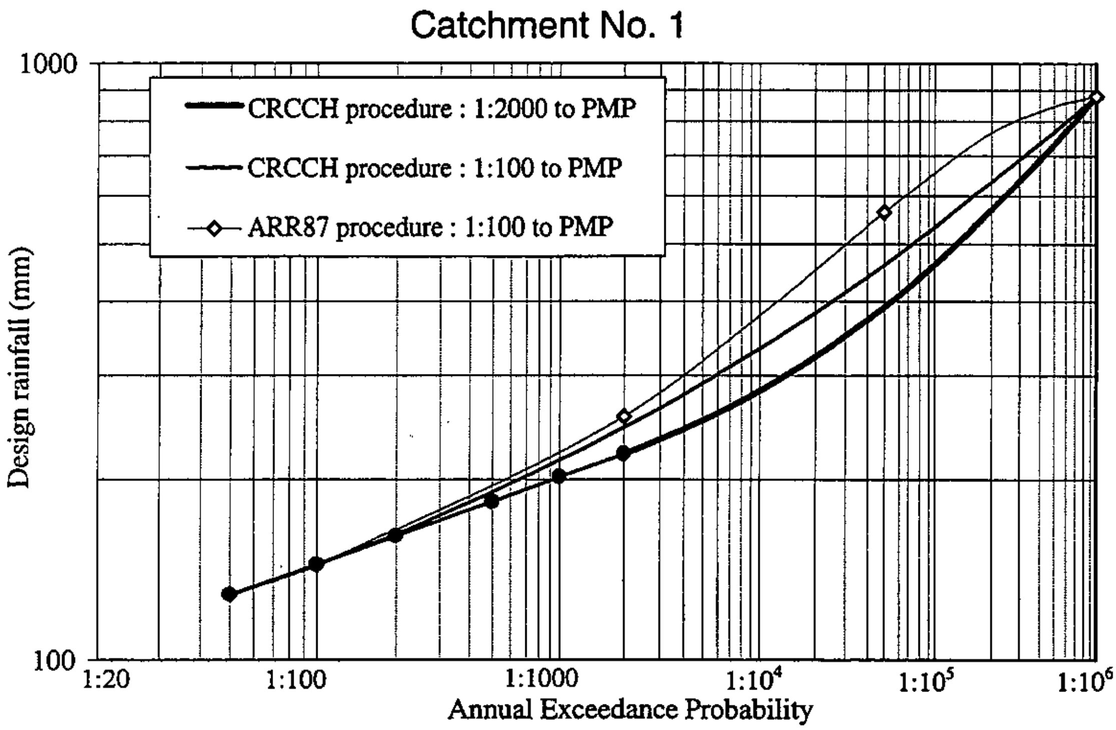


Figure 5.3 : Comparison of frequency curves derived from CRCCH interpolation procedure based on CRC-FORGE (AEPs from 1:50 to 1:2000) and ARR87 (AEPs from 1:50 to 1:100) design rainfall estimates



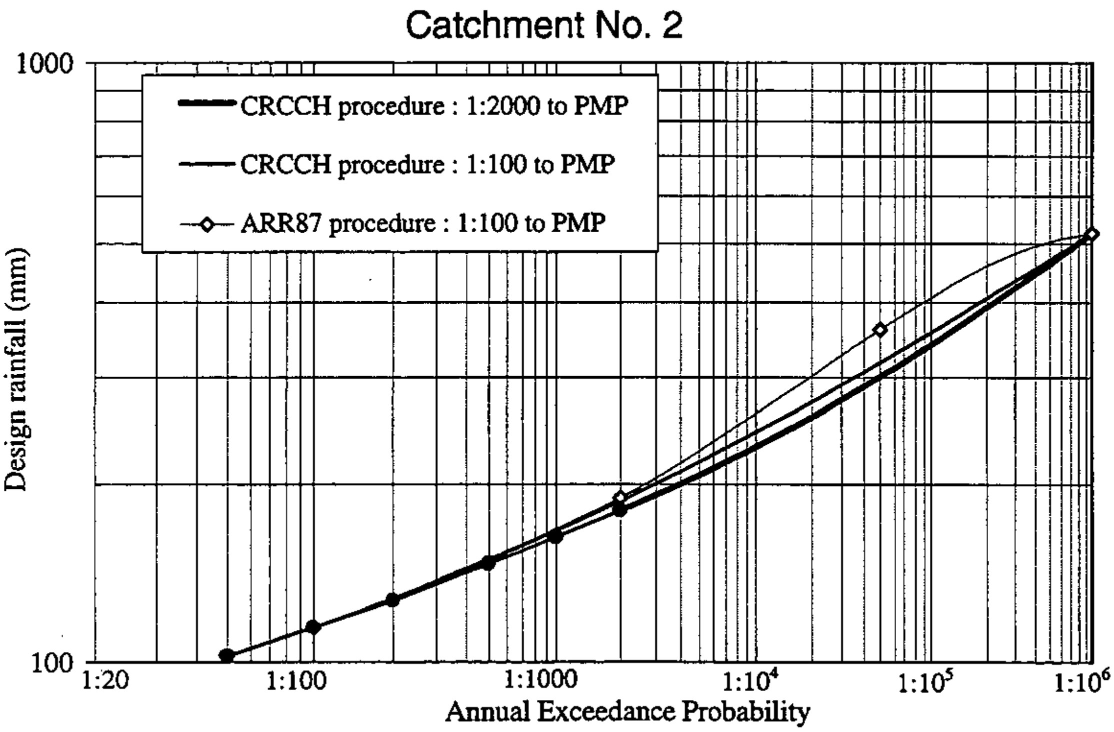


Figure 5.4 : Comparison of frequency curves derived from ARR87 interpolation procedure with those derived from CRCCH interpolation procedures (for 24h design rainfall)
(i) from 1:2000 AEP estimate to the PMP using CRCCH interpolation procedure
(ii) from 1:100 AEP estimate to the PMP using CRCCH interpolation procedure
(iii) from 1:100 AEP estimate to the PMP using ARR87 interpolation procedure

The difference between the frequency curves derived from the two procedures for different design situations may be viewed in relation to the relative magnitude of the slopes S_{gc} and S_{gap} (ref. Figure 3.1). In situations where S_{gc} is greater than S_{gap}, frequency curves derived from both procedures would generally be of upward 'convex' form; the differences between the two curves would thus not be large. However, when S_{gc} becomes less than S_{gap}, the frequency curve derived from the ARR87 interpolation procedure would be of double curvature, whereas in the new procedure, the frequency curve is defined as a smoother function of single curvature. This causes greater differences between the two frequency curves in the interpolated range.

5.4.4 Applicability to practical design situations

It appears from the test results that the CRCCH procedure for interpolation from 1:100 AEP estimate to the PMP provides estimates midway between those derived from the CRC-FORGE estimates based CRCCH interpolation procedure and the ARR87 procedure. The slight conservatism inherent in the CRCCH procedure, for application to design situations where CRC-FORGE estimates are not available, would be beneficial as it generally tends to provide 'safe' but not overly conservative estimates.

It is of some concern that the rainfall frequency curves for tropical areas may have quite different shapes than those in temperate climates, but this cannot be tested at present. For this reason, in the absence of CRC-FORGE estimates, it is recommended that the application of the CRCCH interpolation procedure from AEP 1:100 to the PMP be restricted at this stage to climatic regions of similar characteristics as Victoria.

5.5 Numerical example

This worked example illustrates the application of CRCCH interpolation procedure to a sample catchment of 360 km^2.

For a 24 hours storm duration, the design rainfall quantities for AEPs of 1:50 and 1:100, and the probable maximum precipitation were determined as:

$$P_{50} = 128.4 \text{ mm}$$
$$P_{100} = 145.1 \text{ mm}$$
$$P_{PMP} = 810.0 \text{ mm}$$

Following the computational steps in Section 5.2:

1. Compute Y_{PMP} for the catchment area (A) in km^2.

$$\text{AEP of the PMP} = 10^{[\log(360)-9]} = 3.60\times10^{-7}$$

$$Y_{PMP} = \frac{1}{AEP} = \frac{1}{3.60\times10^{-7}} = 2.778\times10^{6}$$

2. Calculate transformed variables.

$$R_{50} = \frac{\log(128.4)}{\log(145.1)} = 0.9754$$

$$R_{PMP} = \frac{\log(810.0)}{\log(145.1)} = 1.3455$$

$$x_d = \log(2.778\times10^{6}) - \log(100) = 4.4437$$

3. Calculate parameters of the fitted function.

$$S_{gc} = \frac{(1-0.9754)}{\log(100)-\log(50)} = 0.0817$$

$$S_{gap} = \frac{(1.3455-1)}{4.4437} = 0.0778$$

$$a_1 = 0.0817\times4.4437 = 0.3631$$

$$a_2 = (0.0778 - 0.0817)\times4.4437 = -0.0173$$

4. Determine design rainfall quantiles in the interpolated range.

Sample calculation for an AEP of 1:100,000:

$$x = \log(100{,}000) - \log(100) = 3.00$$

$$R_Y = 1+\frac{0.3631}{4.4437}(3.00)-\frac{0.0173}{(4.4437)^2}(3.00)^2 = 1.2372$$

$$P_Y = 10^{1.2372\times\log(145.1)} = 472.6\text{ mm}$$

Step 4 can be repeated to calculate design rainfall estimates for other AEPs in the range from 1 in 100 to the AEP of the PMP.

6. 'COMPLETE' RAINFALL FREQUENCY CURVES FOR NON-STANDARD AND SHORT DURATIONS

6.1 Derivation of complete rainfall frequency curves

CRC-FORGE design rainfall estimates for AEPs of 1 in 50 to 1 in 2000 are directly available from the database for storm durations of 24, 48, 72, 96 and 120 hours. However, in many design situations, rainfall estimates are also required for other durations. Design rainfalls for intermediate durations between 24 and 120 hours can be readily determined by interpolation, with only a small degree of approximation being involved. Design rainfalls for durations less than 24 hours are of particular importance for design flood estimation in small catchments, but are more difficult to derive. An approximate procedure to derive these estimates is described in Appendix A.

The application of the interpolation procedure described in Chapter 4 to non-standard and short rainfall durations requires approximate CRC-FORGE estimates for AEPs of 1 in 1000 and 1 in 2000, derived by the procedure outlined in Appendix A, plus PMP estimates for these durations. More importantly, the adopted design estimates should form a consistent set of frequency curves over the whole range of storm durations considered; this is particularly important as the interpolation procedure is applied independently for each duration.

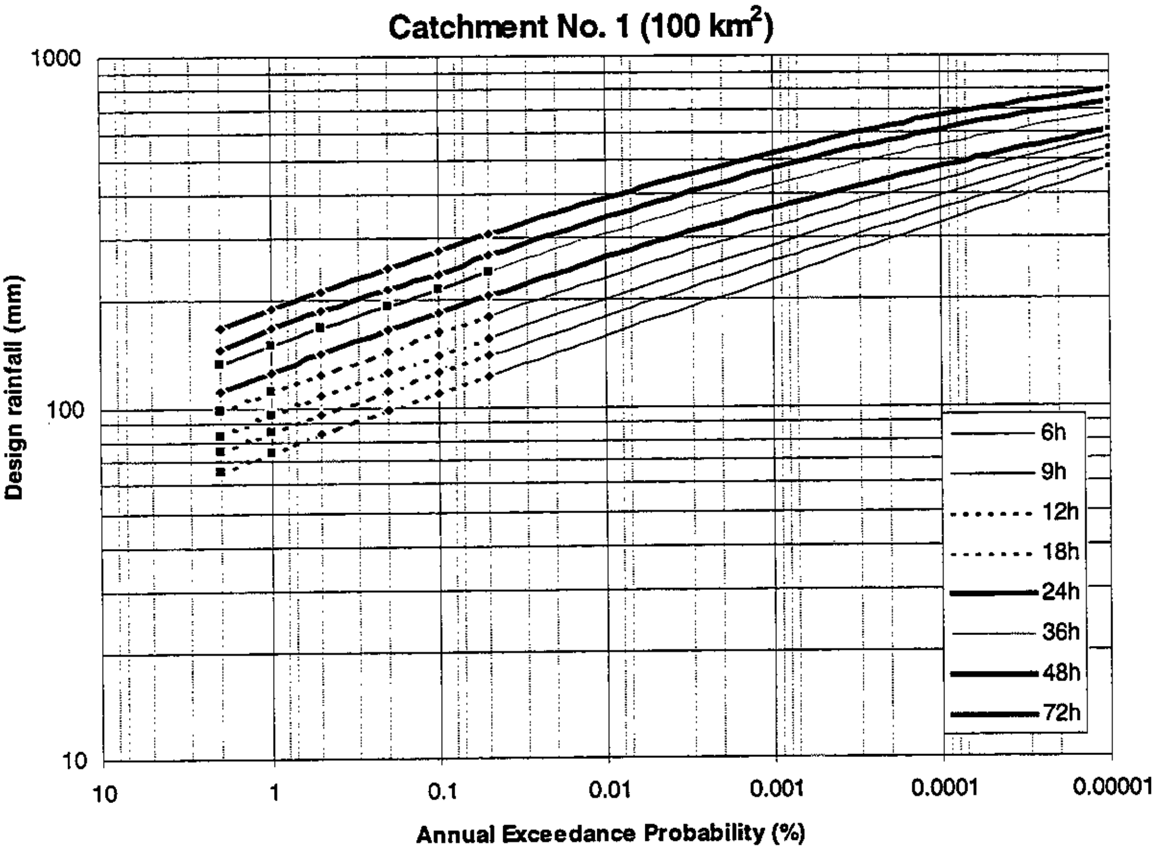
6.2 Consistency of frequency curves derived using new interpolation procedure over range of rainfall durations

The design rainfall frequency curves derived from the new interpolation procedure for different storm durations should be consistent and compatible with each other. It should be noted that the 'primary' design rainfall quantiles (AEPs of 1 in 1000 and 1 in 2000) on which the interpolation of the frequency curve is based are of diverse accuracy, ie. relatively accurate for long durations (24 hours and longer), as they have been directly determined from CRC-FORGE estimates, but only approximate for short durations.

The interpolation procedure was evaluated by applying it to two catchments, one of 100 km^2 located in Southern Victoria and the other of 90 km^2 located in North-western Victoria. The point rainfall estimates for storm durations from 6 to 72 hours, derived using the procedures given in Appendix A, were converted to areal estimates by applying new areal reduction factors for Victoria; for durations less than 18 hours, the interim ARF values were used (Siriwardena and Weinmann, 1996). Parabolic functions were then fitted for each duration, as described in Section 4.1, using design quantiles for AEPs of 1:1000 and 1:2000, the PMP and its assigned AEP. The resultant rainfall frequency curves for storm durations from 6 hours to 72 hours are shown in Figure 6.1.

These results demonstrate that the complete rainfall frequency curves derived for the range of storm durations are well behaved and consistent with each other for both test catchments (Figure 6.1). The inter-relationship of design values between storm durations is acceptable for the whole range of AEPs.

Based on the results for the test catchments, it can be concluded that the proposed procedure provides a consistent basis for deriving complete rainfall frequency curves over the range of storm durations most relevant to flood estimation in medium to large size catchments.



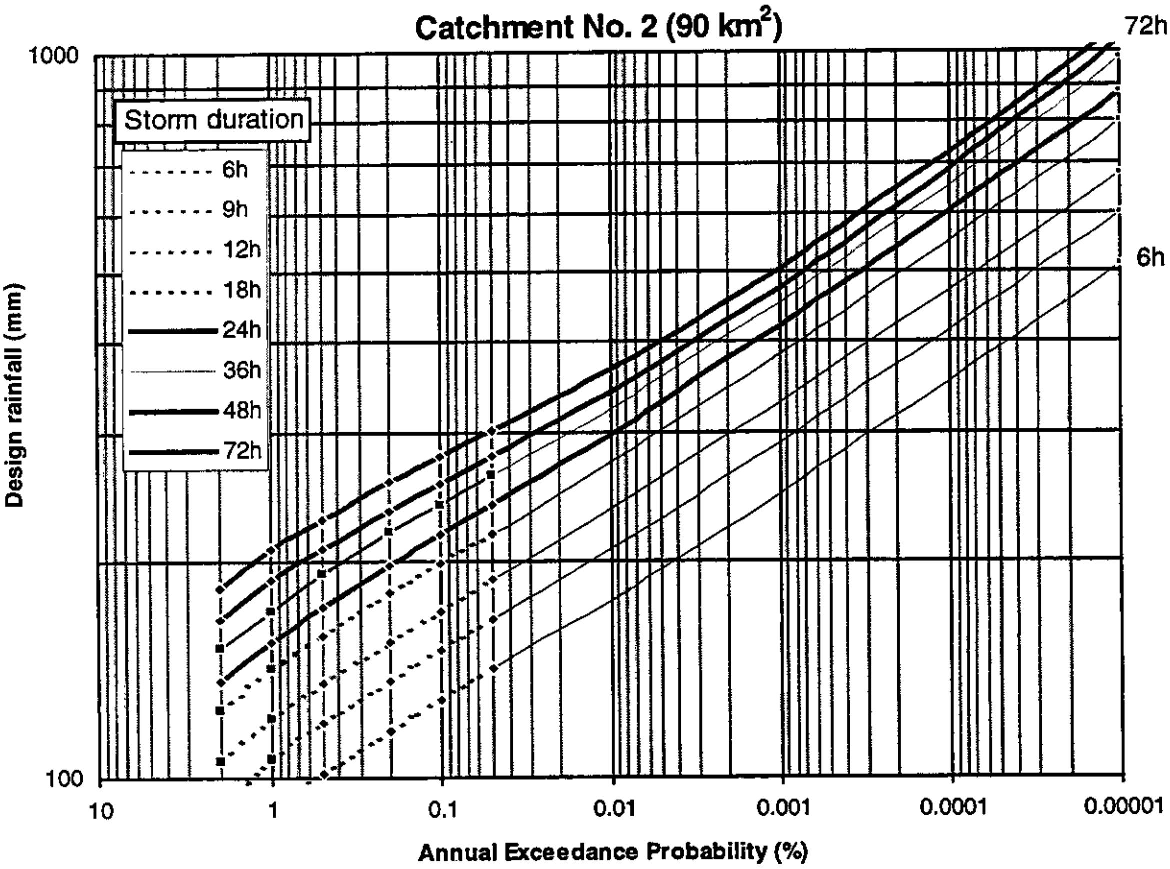


Figure 6.1: Rainfall frequency curves extrapolated to durations less than 24 hours

7. SUMMARY AND CONCLUSION

7.1 Summary

Formulating a basis for more objective spillway adequacy assessment has been the focus of attention in this study; the existing method (ARR87, Chapter 13) involves a high degree of uncertainty and tends to be too conservative. In addressing this need, the CRC-FORGE method for estimation of rare point rainfalls (Nandakumar et al., 1997) together with new CRC-developed areal reduction factors (Siriwardena and Weinmann, 1996) have provided design rainfall estimates for Victoria to an AEP of 1 in 2000, compared to the ARR87 limit of 1 in 100. Other recent developments include a new interpretation of the PMP and a new recommended plotting position assigned to it estimated from generalised procedures (Laurenson and Kuczera, 1998). An interpolation procedure developed on a basis consistent with these developments was then required to fill the 'gap' in the design rainfall frequency curve between the upper end of the directly estimated rainfall frequency curve and the PMP.

Thus, the objective of this study was to develop and test a pragmatic approach to extend the directly derived frequency curves to the probable maximum precipitation (PMP) through a well defined transition function, without making a restrictive assumption about the slope of the frequency curve at the PMP. The procedure needs to be applicable to interpolate over 'gaps' of different ranges of AEPs to satisfy different design situations.

The adopted procedure for functional representation of the frequency curve over the interpolated range was based on the work by Lowing (1995) for the UK. A 'generalised procedure' was developed by fitting a simple 2-parameter parabolic function to satisfy the requirements at either end of the interpolated curve. The development was in a log-log space, so that a smooth, well behaved function can be obtained when design quantiles are plotted against AEP on logarithmic scales. For a smooth transition of the curve the following boundary conditions were adopted:

- At the start of the transition, the slope of the interpolated curve matches the slope defined by design estimates from the uppermost segment of the directly estimated frequency curve.
- The slope of the interpolated curve through the PMP point was not constrained and self-determined by the trend of the frequency curve at the lower end. (This is in contrast to the present practice where the curve is constrained to be horizontal in approaching the PMP).

It needs to be emphasised that the interpolation is entirely determined by conditions at the two ends of the 'gap'; no additional information is introduced in fitting the curve.

The 'generalised procedure' was adapted to fill the 'gap' in the frequency curve for the following two specific situations:

- over an AEP from 1 in 2000 to the AEP of the PMP, with the lower segment of the rainfall frequency curve determined using the CRC-FORGE method;
- over an AEP from 1 in 100 to the AEP of the PMP, with design rainfalls estimated from the ARR87 procedure.

The range of different frequency curve situations to be encountered in practice can be characterised by a 'shape parameter' defined by the ratio of the slope of the upper end of the directly determined frequency growth curve, S_{gc}, and the slope between the two end points of the 'gap', S_{gap}, (the 'shape parameter' S_{gc}/S_{gap}, ranges from 0.25 to 2.0).

For satisfactory behaviour of the function, the condition of $\mathbf{S_{gc} \leq 2S_{gap}}$ should be satisfied (see Figure 3.1). This means that for satisfactory application of the interpolation procedure, the directly determined frequency curve should be compatible with the PMP and its assigned AEP.

The proposed methodology was extensively tested by deriving 'complete' rainfall frequency curves for 25 Victorian catchments (25 - 15,000 km^2) with diverse characteristics. The tests comprised design situations where CRC-FORGE rainfall quantiles are available to an AEP of 1 in 2000, as well as for situations where reliable quantiles can be determined only up to an AEP of 1 in 100. The resultant frequency curves were shown to be plausible and well behaved for all test catchments; the 'shape parameter', S_{gc}/S_{gap}, was found to be well within the recommended range of 0.25-2.00. Based on test results, it can be concluded that the new interpolation procedure can be successfully applied for diverse practical design situations. In the absence of CRC-FORGE estimates, its applicability should be restricted to climatic regions of similar characteristics of Victoria.

The methodology was also found to be satisfactory in application for shorter storm durations (< 24 hours) with design rainfalls for AEPs 1 in 200 to 1 in 2000 estimated by an approximate procedure; frequency curves were shown to be consistent over a practically significant range of storm durations.

Limited testing of rainfall frequency curves for Victorian catchments indicated that the new interpolation procedure resulted in significant reduction in the design estimates in the interpolated range compared to those derived from the current procedure of interpolation (Chapter 13, ARR87), particularly, in situations where the slope of the upper part of the directly determined frequency curve is substantially lower than the slope indicated by the probable maximum event. It is clear that the constraint imposed at the PME end in the current method, has resulted in conservative design values in the interpolated range.

The application of the procedure can be extended to interpolate *flood* frequency curves, but special care will be needed, particularly when dealing with flood frequency curves derived from short at-site records for which skewness is poorly defined.

The proposed interpolation procedure is of a purely empirical nature. While it is able to bridge significant gaps, the uncertainty of the interpolated values is expected to increase with the size of the gap. This uncertainty should be kept in mind when basing decisions on interpolated rainfall or flood data.

A summary of salient features of the new interpolation procedure and its applicability to practical design situations is given in Table 7.1, with a comparison of the corresponding features in the ARR87 procedure.

7.2 Conclusion

The following conclusions can be drawn from the study:

- Testing on a representative sample of Victorian sites has shown the CRCCH interpolation procedure to be suitable for interpolation of design rainfall frequency curves based on:
  (i) CRC-FORGE rainfall quantiles in the rare event range;
  (ii) rainfall quantiles determined from ARR87 procedures for regions where CRC-FORGE estimates are not available.

  The proposed interpolation procedure produces well-behaved and consistent design rainfall frequency curves over a practically relevant range of storm durations.

- On this basis, it is recommended that the proposed interpolation procedure replace the one given in Chapter 13 of Australian Rainfall and Runoff (IEAust, 1987) and be adopted for interpolation of design rainfall frequency curves for all practical design situations, subject to satisfactory test results from other climatic regions.

- The proposed procedure can be applied to directly interpolate flood frequency curves, but only in cases where the directly determined flood quantiles and the PMF are compatible.

Table 7.1 : Features of new interpolation procedure and its applicability to design situations

<table>
<tr><th colspan="2">Interpolation procedure</th><th colspan="3">CRCCH parabolic function</th><th>ARR87 procedure (Chapter 13)</th></tr>
<tr><td colspan="2">Development space</td><td colspan="3">Log-log space</td><td>Log-normal probability paper</td></tr>
<tr><td colspan="2">Availability of extreme rainfall estimates</td><td colspan="2">CRC-FORGE estimates available
AEP ≥ 1:2000</td><td colspan="2">Only ARR87 estimates available
AEP ≥ 1:100</td></tr>
<tr><td colspan="2">Storm duration</td><td>D ≥ 24 hours</td><td>D < 24 hours</td><td colspan="2">All storm durations</td></tr>
<tr><td colspan="2">Starting point of interpolation</td><td>AEP of 1:2000 (estimates directly determined)</td><td>AEP of 1:2000 (approximate estimates from 24-hour ratios)</td><td colspan="2">AEP of 1:100</td></tr>
<tr><td rowspan="3">End point of interpolation</td><td>magnitude</td><td colspan="3">Operational estimate of PMP (very small probability of being exceeded)</td><td>Upper limit concept of PMP</td></tr>
<tr><td>assigned AEP</td><td colspan="3">Laurenson and Kuczera (1998) recommended AEPs, solely based on empirical evidence and meteorological considerations</td><td>based on meteorological considerations and geometrical consistency</td></tr>
<tr><td>slope constraint</td><td colspan="3">no slope constraint on curve reaching PMP</td><td>curve approaching PMP horizontally</td></tr>
<tr><td colspan="2">Applicability</td><td colspan="3">Preferably with rainfall quantiles; application to flood quantiles possible in principle, if the directly determined flood quantiles and the flood derived from the PMP are compatible</td><td>Rainfall and flood quantiles</td></tr>
<tr><td colspan="2">Limiting constraints on application</td><td colspan="3">Ratio of S_{gc}/S_{gap} ranging from 0.25 to 2.00
where S_{gc} and S_{gap} are slope of upper end of directly determined frequency growth curve and slope between two end points of the 'gap' respectively (see Figure)</td><td>Range limits of Table 13.4 of ARR87</td></tr>
<tr><td colspan="2">Flexibility of procedure</td><td colspan="3">Mathematical function (least subjective)
Easily adaptable to any starting and end point</td><td>Fixed procedure requiring subjective sketching</td></tr>
<tr><td colspan="2" rowspan="2">Testing of procedure on Victorian catchments (25-15000 km^2)</td><td colspan="3">Well behaved and plausible rainfall frequency curves for all test catchments
Well within the recommended limits of S_{gc}/S_{gap}</td><td>Complex shape (double curvature) in many situations</td></tr>
<tr><td colspan="2">Most reliable as the 'gap' of interpolation is relatively small</td><td>slightly conservative in most situations</td><td>Highly conservative in most design situations</td></tr>
</table>

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APPENDIX A

CRC-FORGE RAINFALL ESTIMATES FOR NON-STANDARD AND SHORT DURATIONS

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CRC-FORGE RAINFALL ESTIMATES FOR NON-STANDARD AND SHORT DURATIONS

A.1 CRC-FORGE rainfall estimates for intermediate durations

Over a limited range of storm burst durations, the variation of point rainfall depth with duration can be closely approximated by a power function relationship. This allows application of a linear interpolation procedure to log-transformed CRC-FORGE point rainfall depths and logarithms of rainfall within the interval defined by two adjacent standard durations (e.g. 24 and 48 hours). This is illustrated in Figure A.1 for the intermediate duration of 36 hours, using rainfall data for a catchment of 100 km^2 located in Southern Victoria. Filled-in symbols denote the data available from the CRC-FORGE database for standard durations, while hollow symbols indicate interpolated values.

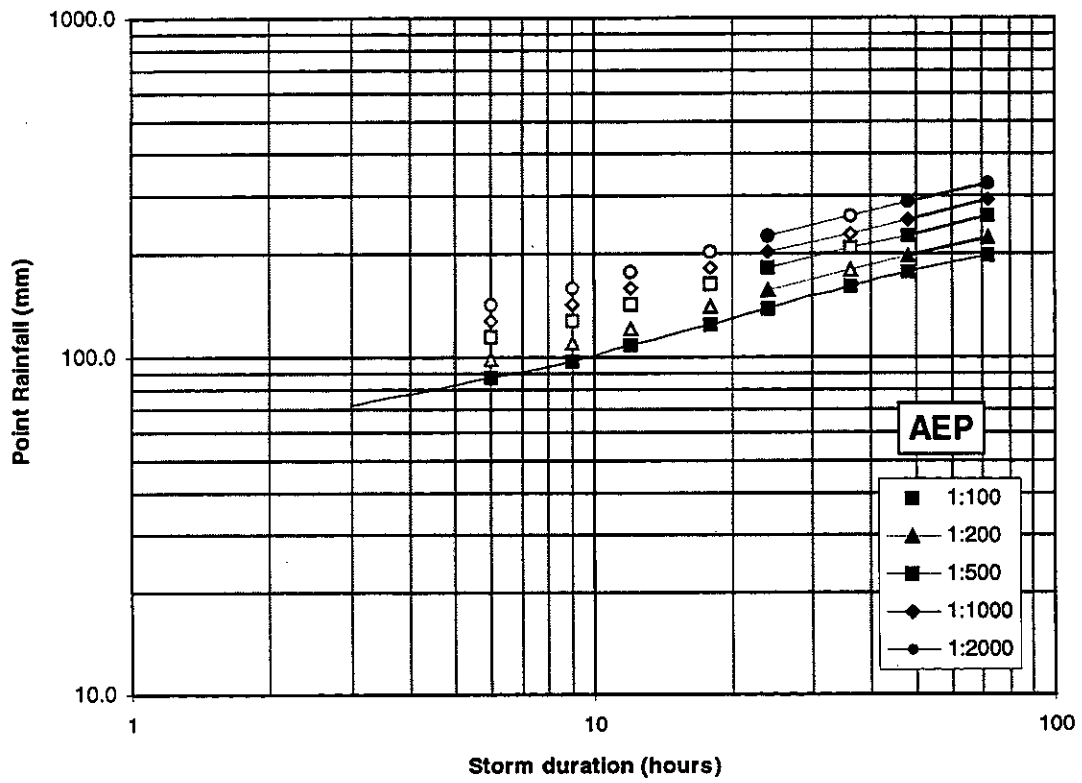

Figure A.1: Interpolation of point rainfalls for intermediate storm burst durations, and derivation of rainfall depths for durations less than 24 hours

A.2 Approximate CRC-FORGE estimates for short durations

The design rainfall information derived by the CRC-FORGE methodology is limited to the duration range from 24 to 120 hours. There is currently no reliable design rainfall information available for durations less than 24 hours and AEPs less than 1 in 100. However, in many design situations, the range of durations to be analysed to find the critical rainfall duration will extend to durations less than 24 hours. Therefore, it is important that the design rainfall frequency curves for all durations are derived on a consistent basis. In this situation, it is desirable to extrapolate CRC-FORGE design rainfalls to durations less than 24 hours.

Although a comprehensive study on estimating rare design rainfalls for short durations is outside the scope of this report, a suitable approach to extrapolate CRC-FORGE design rainfall estimates to storm durations in the range from 6 to 18 hours is illustrated in this chapter. This may be applied as an interim measure until more reliable procedures are established. The practitioner has the liberty to investigate other appropriate approaches and adapt the design procedures accordingly. Given the substantial degree of approximation involved with any such extrapolation, it is desirable to build in a degree of conservatism into the procedure.

The following considerations were involved in the development of the procedure for deriving approximate CRC-FORGE estimates for short durations (< 24 hours):

- Chapter 2 of ARR87 provides a consistent methodology for derivation of point rainfall intensities for a range of storm durations (6 min - 72 hours) and for AEPs to a lower limit of 1 in 100. The original CRC-FORGE procedure for deriving design rainfalls at ungauged sites (Nandakumar et al., 1997) has been adjusted to ensure consistency with these ARR87 values. The CRC-FORGE design rainfalls for lower AEPs are now derived by applying 'adjusted CRC-FORGE growth factors' to an ARR87 based 'scaling variable' (the ARR87 estimate for an AEP of 1 in 50 or 1 in 100). This consistency needs to be preserved in deriving 'approximate' rainfall quantiles for shorter durations.

- For a limited range of rainfall durations shorter than 24 hours (say to a lower limit of 6 hours), the 24-hour CRC-FORGE growth factors can be assumed to provide a sufficiently close approximation to the growth factors that would result from a CRC-FORGE analysis of short duration rainfall data. This is equivalent to assuming that all the variation of design rainfalls with duration is explained by the variation in the 'scaling variable', and that the short duration rainfall frequency curves between AEPs of 1 in 50 and 1 in 2000 are parallel to the 24-hour frequency curve (when rainfall is plotted on a logarithmic scale).

- The designer will need to check that the extrapolated design rainfall quantiles for shorter durations produce a consistent set of complete rainfall frequency curves to the AEP of the PMP and, in the case of any inconsistency, modify the procedure as required.

In consideration of the above factors, the following simple procedure to extrapolate CRC-FORGE estimates to short durations has been developed:

1. For the site of interest, extract the 24-hour rainfalls for AEPs from 1 in 50 to 1 in 2000 from the CRC-FORGE design rainfall database.

2. Compute the 24-hour CRC-FORGE growth factors as the ratio of the rainfall for a selected AEP to that of the 'scaling variable'. (In the example used here, the ARR87 estimate for an AEP of 1 in 100 was applied as the 'scaling variable'; alternatively, the 1 in 50 ARR87 estimate could have been used.)

3. Derive ARR87 estimates of the 'scaling variable' (in this case the 1 in 100 AEP rainfalls) at the site of interest for durations from 6 to 18 hours.

4. For each rainfall duration of interest, multiply the scaling variable obtained in step 3 with the growth factors derived in step 2 to obtain the corresponding point rainfalls for AEPs less than 1 in 100.

5. Convert the point rainfalls to areal design rainfalls by multiplying them by the corresponding areal reduction factors for the range of storm durations and AEPs of interest.

The application of the proposed procedure is illustrated in Figure A.1 for the catchment described in Section A.1. In this figure, the directly determined design rainfalls, the ARR87 based scaling variables (1 in 100 AEP) for storm durations from 6 to 72 hours, and the 24-hour CRC-FORGE point rainfall estimates for AEPs of 1 in 200 to 1 in 2000, are shown as 'filled-in' symbols. The extrapolated point design rainfalls for durations less than 24 hours are shown as 'hollow' symbols.

In estimating rainfall quantiles for durations less than 24 hours, the uncertainties associated with the procedure should be borne in mind as, at this stage, the derived quantiles for AEPs less than 1 in 100 cannot be verified against estimates that are directly based on the analysis of short duration rainfall data.

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